BER Performance of A Communication System Using BPSK Signaling Scheme with Smart Antenna at Base Station

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Abstract

This paper analyzes Bit Error Rate (BER) performance of a wireless communication system using BPSK modulation technique with a smart antenna at base station. The channels under consideration are Additive White Gaussian Noise (AWGN) channel or a slow, flat Rayleigh fading channel with AWGN. Under the assumptions that the MMSE beamformer is used and the time delays of all users are approximately equal, we first analytically derive closed-form expressions for the BER of the desired user. Then, computer simulation is carried out to verify the theoretical results.

I. Introduction

Wireless communications are now playing a vital role in supporting a variety of voice and data services. The limitation ofavailable radio frequency, however, poses a major challenge to these systems. Consequently, approaches for enlarging the communications capacity are of great interest. One promising solution to cope with the problem is the use of smart antenna. Smart antennas or adaptive antenna arrays offer a wide range of ways to enhance performance of wireless communication systems. Some key benefits of smart antenna technology are provided in [1].

The heart of a smart antenna is an adaptive beamforming algorithm, i.e.,the technique to combine the array outputs. A reasonable strategy is to find the best weight vector so as to optimally combine the array outputs under some suitable criterion. Several optimization criteria that have been widely used are Minimization of mean squared error (MMSE), maximization of signal-to-interference-plus-noise ratio (SINR), and maximum likelihood (ML) [2].

The objective of this paper is to derive closed-form expression for computing the error probabilities of a coherent BPSK receiver a with smart antenna in AWGN channel and slow, flat Rayleigh fading channel with AWGN so as to provide readers with some knowledge about the use of smart antenna to improve the BER performance of a wireless communication system. Computer simulation results will be provided to confirm the derived probabilities of error of the system.

II. Ideal BER for a wireless communications system using BPSK signaling with a smart antenna

In this system, the transmitted signal is given by:

$$s(t)=\sqrt{E_b}\sum_{n=-\infty}^{\infty}b(n)g(t-nT_b) \tag{1}$$

where E_b is the signal energy per bit, $\{b(n)\}$ is the information sequence, $b(n)=\pm 1$ for all n, $g_k(t)$ is a basis function of duration T_b for the k^{th} user. For the BPSK system with co-channel interference, it is undoubted that all users have the same basis function, i.e., $g_k(t)=g(t)$ for all

k. Obviously, the energy of the basis function is equal to unity, that is:

$$E_g = \int_0^{T_b} g^2(t)dt = 1 \tag{2}$$

Consider $K+1$ users transmitting signals through a fading channel to a base station with an array of M elements. Suppose that the angle spread is negligibly small such that all the irresolvable paths, which make contribution to the received signal will essentially arrive from one direction, the received signal at the array corrupted by AWGN is given by:

$$\underline{\mathbf{r}}(t) = \sum_{k=0}^{K}\sum_{n=-\infty}^{\infty} \underline{\mathbf{a}}(\theta_k)\alpha_k e^{-j\phi_k}\sqrt{E_{bk}}\, b_k(n) g(t - nT_b - \tau_k) + \underline{\mathbf{n}}(t) \tag{3}$$

where, $\underline{\mathbf{a}}(\theta_k) = [a_0(\theta_k) \;\; a_1(\theta_k) \;\; \ldots \;\; a_{M-1}(\theta_k)]^T$ is the array response for k^{th} user; θ_k is the direction of arrival (DOA) of the k^{th} signal path; T denotes the transpose of a vector; α_k, ϕ_k and τ_k are respectively the fading coefficient, the phase shift and the time delay associated with the k^{th} signal path, ϕ_k is uniformly distributed over the interval $(-\pi, \pi)$; and $\underline{\mathbf{n}}(t) = \underline{\mathbf{n}}_i(t) + j\underline{\mathbf{n}}_q(t)$ is a vector of the complex-valued white Gaussian noise process, whose covariance matrix is given by

$$E\left[\underline{\mathbf{n}}(t)\underline{\mathbf{n}}^H(t)\right] = \frac{N_0}{2}\mathbf{I} + \frac{N_0}{2}\mathbf{I} = N_0\mathbf{I} \tag{4}$$

where $E[.]$ denotes the ensemble average; H denotes the complex conjugate transpose of a vector; N_0 is the one-sided noise power density; and $\mathbf{I}$ is the $M \times M$ identity matrix.

Let us assume that the channel fading is sufficiently slow such that the phase shift of the desired user, who is assumed to be the user $k=0$, can be estimated from the received signal without error, i.e., ideal coherent detection. Moreover, all users are distributed in such a way that their signal time delays are almost the same, i.e., $\tau_0 \approx \tau_1 \approx \cdots \approx \tau_K \approx \tau$, and can be estimated precisely. Under those assumptions, without loss of generality, we may consider the received signal within bit duration T_b. The received data vector at the output of the matched filters, sampled at $t = T_b + \tau$, is given by:

$$\underline{\mathbf{x}}(1) = \underline{\mathbf{x}}_0(1) + \underline{\mathbf{i}}(1) + \underline{\mathbf{n}}(1) = \underline{\mathbf{x}}_0(1) + \underline{\mathbf{u}}(1) \tag{5}$$

where,

$$\underline{\mathbf{x}}_0(1) = \underline{\mathbf{a}}(\theta_0)\alpha_0\sqrt{E_{b0}}\, b_0(1) \tag{6}$$

Similarly, the co-channel interference term is equal to:

$$\underline{\mathbf{i}}(1) = \sum_{k=1}^{K} \underline{\mathbf{a}}(\theta_k)\alpha_k e^{-j\phi_k}\sqrt{E_{bk}}\, b_k(1) \tag{7}$$

And the noise term, $\underline{\mathbf{n}}(1)$, in Equation (5) has mean value:

$$E[\underline{\mathbf{n}}(1)] = \int_{\tau}^{T_b+\tau} E[\underline{\mathbf{n}}(v)]g(v-\tau)dv = \mathbf{0} \tag{8}$$

and the covariance matrix

$$E\left[\underline{\mathbf{n}}(1)\underline{\mathbf{n}}^H(1)\right] = N_0\mathbf{I} \tag{9}$$

The covariance matrix of the undesired signal component in Equation (5) is:

$$\mathbf{R}_{uu} = E\left[\underline{\mathbf{u}}(1)\underline{\mathbf{u}}^H(1)\right] = \sum_{k=1}^{K} \underline{\mathbf{a}}(\theta_k)\underline{\mathbf{a}}^H(\theta_k)E\left[\alpha_k^2\right]E_{bk} + N_0\mathbf{I} \tag{10}$$

The output of the array is obtained by multiplying the received data vector by the optimum weight vector, thus yielding:

$$y(1) = \underline{\mathbf{w}}_o^H\underline{\mathbf{x}}_0(1) + \underline{\mathbf{w}}_o^H\underline{\mathbf{i}}(1) + \underline{\mathbf{w}}_o^H\underline{\mathbf{n}}(1) \tag{11}$$

At convergence, the optimum weight vector based on the MMSE criterion is given by:

$$\underline{\mathbf{w}}_o = \zeta\mathbf{R}_{uu}^{-1}\underline{\mathbf{v}}(\theta_0) \tag{12}$$

Then, Equation (11) can be written as:

$$y(1)=\zeta\alpha_0\sqrt{E_{b0}}b_0(1)\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_0)$$
$$+\sum_{k=1}^{K}\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_k)\zeta\alpha_k e^{-j\phi_k}\sqrt{E_k}b_{bk}(1)$$
$$+\zeta\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{n}}(1) \tag{13}$$

When the number of users is large enough, the central limit theorem allows us to model the second term in Equation (13) as a sample value of a zero-mean complex Gaussian random variable with variance equal to:

$$\sigma_i^2=\zeta^2\sum_{k=1}^{K}\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_k)\right|^2 E\left[\alpha_k^2\right]E_{bk} \tag{14}$$

The third term in Equation (16) resulting from the corrupted AWGN at the array has mean value equal to:

$$E\left[\zeta\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{n}}(1)\right]=\zeta\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}E\left[\underline{\mathbf{n}}(1)\right]=0 \tag{15}$$

and variance equal to:

$$\sigma_{n'}^2=\zeta^2 N_0\underline{\mathbf{a}}^H(\theta_0)\left[\mathbf{R}_{uu}^{-1}\right]^2\underline{\mathbf{a}}(\theta_0) \tag{16}$$

Therefore, $y(1)$ is a sample value of a complex Gaussian random variable with mean

$$m_y=\zeta\alpha_0\sqrt{E_{b0}}b_0(1)\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_0)\right| \tag{17}$$

The variance of $y(1)$ is equal to:

$$\sigma_y^2=\sigma_i^2+\sigma_{n'}^2 \tag{18}$$

The transmitted bit $b_0(1)$ can be recovered by simply comparing the real part of the array output, i.e., $\mathrm{Re}[y(1)]$, with a threshold. Specifically, if $\mathrm{Re}[y(1)]>0$, the receiver will decide $b_0(1)=1$, otherwise it will decide $b_0(1)=-1$. Now the newvariable based on which the receiver makes decision is:

$$y_r(1)=\mathrm{Re}[y(1)]=\zeta\alpha_0\sqrt{E_{b0}}b_0(1)\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_0)+n'' \tag{19}$$

where n'' is a sample value of a zero-mean Gaussian random variable, defined by:

$$n''=\mathrm{Re}\left[\begin{array}{l}\sum_{k=1}^{K}\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_k)\zeta\alpha_k e^{-j\phi_k}\sqrt{E_k}b_{bk}(1)\\+\zeta\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{n}}(1)\end{array}\right] \tag{20}$$

If we suppose that the real part and the imaginary part of $y(1)$ are sample values of two independent Gaussian random variables with the equal variance, then the variance of random variable whose sample value is n'' is equal to $\sigma_{n''}^2=\sigma_y^2/2$.

As a result, $y_r(1)$ is a sample value of a Gaussian random variable having mean value $m_{y_r}=\zeta\alpha_0\sqrt{E_{b0}}b_0(1)\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_0)$ and variance $\sigma_{y_r}^2=\sigma_{n''}^2$. Consequently, the conditional probability of the receiver making decision in favor of symbol 1, or $b_1(1)=1$, given that symbol 0 was transmitted is:

$$P_e(0)=\int_0^\infty f_{y_r}(x\mid b_0(1)=-1)dx$$
$$=\frac{1}{\sqrt{2\pi\sigma_{y_r}^2}}\int_0^\infty \exp\left[-\frac{(x-m_{y_r})^2}{2\sigma_{y_r}^2}\right]dx \tag{21}$$

After some manipulation, the conditional probability of error given that symbol 0 was transmitted is equal to:

$$P_e(0)=\frac{1}{2}erfc\left(\sqrt{\frac{\alpha_0^2\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_0)\right|^2E_{b0}}{\left[\begin{array}{l}\sum_{k=1}^{K}\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}_{uu}^{-1}\underline{\mathbf{a}}(\theta_k)\right|^2E\left[\alpha_k^2\right]E_{bk}\\+N_0\underline{\mathbf{a}}^H(\theta_0)\left[\mathbf{R}_{uu}^{-1}\right]^2\underline{\mathbf{a}}(\theta_0)\end{array}\right]}}\right) \tag{22}$$

where $erfc()$ is the complementary error function defined by:

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-v^2) dv \tag{23}$$

By a similar way, we can show that the conditional probability of error given that symbol 1 was transmitted is also equal to:

$$P_e(1) = P_e(0) \tag{24}$$

If symbol 1 and symbol 0 are transmitted with equal probability, averaging the conditional error probabilities $P_e(0)$ and $P_e(1)$, we find that, for time-invariant channels, i.e., fixed attenuations α_0, the average probability of bit error for the coherent BPSK system equals:

$$P_e = \frac{1}{2} erfc(\eta_0) \tag{25}$$

where

$$\eta_0 = \frac{\alpha_0^2 \left[\underline{\mathbf{a}}^H(\theta_0) \mathbf{R'}_{uu}^{-1} \underline{\mathbf{a}}(\theta_0)\right]^2 \gamma_{b0}}{\left[\begin{array}{l} \sum_{k=1}^{K} \left|\underline{\mathbf{a}}^H(\theta_0) \mathbf{R'}_{uu}^{-1} \underline{\mathbf{a}}(\theta_k)\right|^2 E\left[\alpha_k^2\right] \gamma_{bk} \\ + \underline{\mathbf{a}}^H(\theta_0) \left[\mathbf{R'}_{uu}^{-1}\right]^2 \underline{\mathbf{a}}(\theta_0) \end{array}\right]} \tag{26}$$

where $\mathbf{R'}_{uu} = \sum_{k=1}^{K} \underline{\mathbf{a}}(\theta_k) \underline{\mathbf{a}}^H(\theta_k) E\left[\alpha_k^2\right] \gamma_{bk} + \mathbf{I}$, and $\gamma_{bk} = E_{bk}/N_0$ is the SNR per bit for the k^{th} user.

There are two types of channel being considered by this paper. The first type is AWGN channel, and the second one is slow, flat Rayleigh fading channel.

2.1 AWGN channel

In this kind of channel, the bit error rate of the coherent BPSK system given by Equation (25) with $\alpha_0 = \alpha_1 = \cdots = \alpha_K = 1$.

2.2 Rayleigh fading channel

In such kind of channel, η_0 has a chi-square probability distribution with two degrees of freedom with the probability density function given by [3]. As a consequence, the BER of the desired user in Rayleigh channel is equal to:

$$P_{e,BPSK-FADING}(\bar{\gamma}_{b0}) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + 1/\bar{\eta}_0}}\right) \tag{27}$$

where

$$\bar{\eta}_0 = \frac{\left[\underline{\mathbf{a}}^H(\theta_0) \mathbf{R'}_{uu}^{-1} \underline{\mathbf{a}}(\theta_0)\right]^2 \bar{\gamma}_{b0}}{\left[\sum_{k=1}^{K} \left|\underline{\mathbf{a}}^H(\theta_0) \mathbf{R'}_{uu}^{-1} \underline{\mathbf{a}}(\theta_k)\right|^2 \bar{\gamma}_{bk} + \underline{\mathbf{a}}^H(\theta_0) \left[\mathbf{R'}_{uu}^{-1}\right]^2 \underline{\mathbf{a}}(\theta_0)\right]}$$

III. Simulation results and discussion

In this section, the analytical results are verified by computer simulation. Some main parameters for the simulation are as follows. The symbol rate is $256 \times 10^3\ bauds/s$. For Rayleigh fading channel maximum Doppler shift is $f_d = 160Hz$.

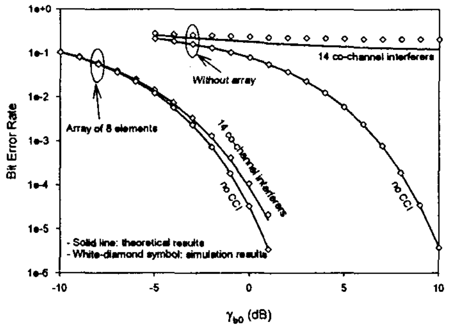

Fig. 1: BER versus SNR per bit of the desired user in the BPSK system for different numbers of antenna elements and co-channel interferers - AWGN channel

The DOAs of users are taken from the set: (-12°, 0°, -25°, 30°, -30°, 25°, 50°, 29°, 18°, 23°, 36°, 70°, -40°, -35°, -70°). The DOA of the desired user is 12°. In the transmitter and receiver, square root Nyquist filters are used so as to eliminate Intersymbol Interference (ISI). The roll-off factors are set to $\alpha = 0.5$ for both transmitter and receiver filters. A uniform, linear, half-wavelength spacing array is employed at based station.

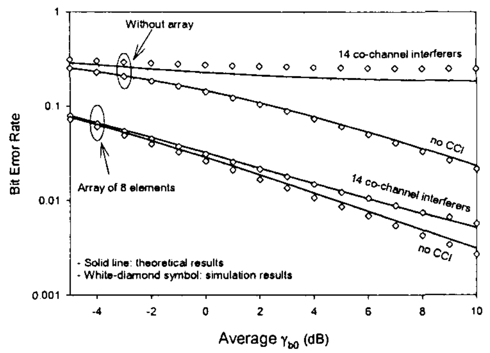

Fig. 2: BER versus SNR per bit of the desired user in the BPSK system for different numbers of antenna elements and co-channel interferers Flat Rayleigh fading channel with AWGN

The weight vector is updated by using the well-known LMS beamforming algorithm [4]. The step sizes of the LMS algorithm is chosen to be $\mu = 0.0001$ and $\mu = 0.00001$ for Rayleigh fading channel and AWGN channel, respectively. In these simulations, all CCI powers are equal and are set at a value that is 10dB less than the power of the desired user.

Fig. 1 and Fig. 2 respectively show the BER versus SNR for different numbers of users and various numbers of elements in AWGN channel and Rayleigh fading channel contaminated by AWGN. As can be seen from these two figures, the BER performance of the system using smart antenna is much improved compared with that of the system without array. Moreover, the simulation results show a very good agreement with the theoretical results given by Equation (25) for AWGN channel, and by Equation (27) for Rayleigh fading channel. This means that, if the CCI terms are Gaussian distributed, all the expressions derived in this paper can be used toobtain the exact bit error rates of desired user in a coherent BPSK system as the time delays of all users are approximately equal.

IV. Conclusions

In this paper, we consider the BER performance of a wireless communication system using BPSK modulation scheme with a smart antenna at base station. We have derived close form equations that enable us to determine the BER of the system for arbitrary number of antenna elements. As a consequence, one does not have to spend time on simulation to obtain the result. Our analytical results were confirmed to be relatively accurate by the simulation results.

Reference

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