

Design of 2-D MA FIR Filters for Channel Estimation in OFDM Systems

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Abstract

The accuracy of channel estimation significantly affects the performance of coherent OFDM receiver. It is desirable to employ a good channel estimator while requiring low implementation complexity. In this paper, we propose a channel estimator that employs a simple two-dimensional (2-D) moving average (MA) filter as the channel estimation filter. The optimum tap size of the 2-D MA FIR filter is analytically designed in the time and frequency domain in association with the channel condition and pilot signal to interference power ratio. The analytic results can be applied to the design of adaptive channel estimator. Finally, the performance of the proposed channel estimator is verified by computer simulation.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) system usually employs coherent detection to improve the detection performance. The performance of coherent detection significantly depends upon the accuracy of channel estimation (CE).

There have been a number of studies on CE for OFDM systems. Two-dimensional (2-D) Wiener filter is considered as the optimum channel estimation filter (CEF) in a linear minimum mean square error (MMSE) sense [1]. However, it may not be practical because of large implementation complexity. Moreover, it requires the correlation matrix of the channel impulse response (CIR) as well as the signal to interference power ratio (SIR) of the received symbol. The use a single 2-D Wiener filter can be replaced with the use of two 1-D Wiener filters for each domain because it can reduce the implementation complexity with a small performance loss [1]. However, it is still too complex to employ two 1-D Wiener filters in practice.

Least square (LS) scheme can be considered as a simple one [2]. The LS CE, however, may result in significant performance degradation since it does not consider the correlation of the CIR between the adjacent symbols in the time and frequency domain. The use of filtering or windowing method is proposed to mitigate

this problem [2].

The Doppler spectrum of the channel is usually spread to the maximum Doppler frequency of an experiencing channel. Thus, a brick-wall type lowpass filter with the cut-off frequency equal to the maximum Doppler frequency of the channel can be used as the CEF [3]. However, such a CEF may not be practical because it is not implementable using a small number of filter taps. Similarly, the estimation accuracy can be enhanced in the frequency domain by neglecting the time-domain CIR corresponding to the out-of cyclic prefix (CP) symbol duration [2]. However, it still requires a large implementation complexity since it requires additional fast Fourier transform (FFT) and inverse FFT (IFFT) process.

In this paper, we consider the use of a 2-D MA FIR filter as the CEF, which can be implemented using only addition operation while providing relatively good receiver performance. There are a few results on the effect of the tap size of the MA FIR CEF on the channel estimation performance [4,5]. However, few results have been reported on analytic design of the MA FIR CEF. Analytic design of the optimum 1-D MA FIR CEF was considered for the direct sequence code division multiple access (DS-SS) systems [6], but it cannot directly be applied to the design of optimum 2-D MA FIR CEF because the optimum tap sizes in the time and frequency domain are closely related to each other. We analytically design the optimum 2-D MA FIR CEF by minimizing the MSE of the CIR.

Following Introduction, Section II describes an OFDM system, where pilot symbols are regularly scattered in the time and frequency grid. The optimum 2-D MA FIR CEF is analytically derived in Section III. The performance of the proposed scheme is evaluated in Section IV. Finally, conclusions are summarized in Section V.

II. System model

Consider an OFDM transmission system whose baseband equivalent model is depicted in Fig. 1. In the

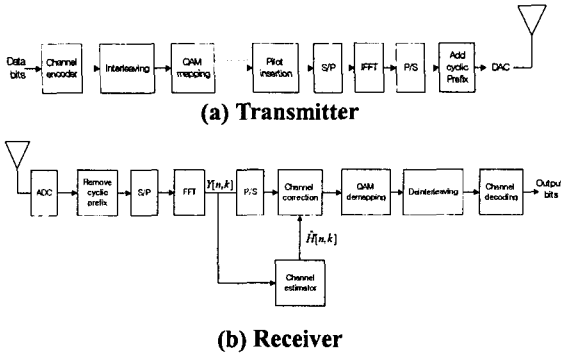


Fig. 1. OFDM transceiver with channel estimation
 transmitter, K data symbols at the n -th symbol time, $\{X[n, k]\}$, $k=1, 2, \dots, K-1$, are converted into time domain signals using the IFFT and then a CP is added. We assume that the pilot symbols are regularly scattered in a rectangular style, i.e., located apart by A and B symbols in the time and frequency grids, respectively.

We assume a wireless channel whose impulse response is represented as

$$h_m(t, \tau) = \sum_{l=0}^{L-1} h_{m,l}(t) \delta(\tau - \tau_l) \quad (1)$$

where L is the number of multipaths, m denotes the receive antenna index, $\delta(\cdot)$ is the Kronecker delta function, τ_l is the delay of the l -th path and $h_{m,l}(t)$ is the corresponding complex CIR at time t .

The CIR can be modeled as a complex zero-mean additive white Gaussian noise (AWGN) process. Since CE is performed at each receive antenna, we omit the index m for ease of description. We assume that $h_l(t)$ is statistically independent for each path and has the same normalized correlation function $r_l(\Delta t)$ for all l . Then,

$$r_l(\Delta t) = E\{h_l(t + \Delta t)h_l^*(t)\} = \sigma_l^2 r_l(\Delta t) \quad (2)$$

where σ_l^2 is the average power of the l -th path. Assuming Rayleigh fading channel, the time-domain correlation function $r_l(\Delta t)$ can be represented as

$$r_l(\Delta t) = \begin{cases} J_0(2\pi f_d \Delta t) & , \text{ for the classic spectrum} \\ \frac{\sin(2\pi f_d \Delta t)}{2\pi f_d \Delta t} & , \text{ for the flat spectrum} \end{cases} \quad (3)$$

where f_d is the maximum Doppler frequency of the channel and $J_0(\cdot)$ is the first kind, zero-th order Bessel function. The frequency response of the channel at time t can be represented as

$$H(t, f) = \int_{-\infty}^{\infty} h(t, \tau) \exp(-j2\pi f \tau) d\tau \quad (4)$$

$$= \sum_{l=0}^{L-1} h_l(t) \exp(-j2\pi f \tau_l) .$$

Assuming the normalized average path power (i.e., $\sum_{l=0}^{L-1} \sigma_l^2 = 1$), the correlation function of the frequency

response can be represented as

$$r_H(\Delta t, \Delta f) = r_t(\Delta t) r_f(\Delta f) . \quad (5)$$

where

$$r_f(\Delta f) = \sum_{l=0}^{L-1} \sigma_l^2 \exp(-j2\pi \Delta f \tau_l) . \quad (6)$$

In an OFDM symbol with the symbol time T_s and subcarrier spacing Δf , the correlation function (5) can be represented as

$$R_H[n, k] = R_t[n] R_f[k] \quad (7)$$

where $R_t[n] = r_t(nT_s)$ and $R_f[k] = r_f(k\Delta f)$.

In the receiver, the CP is removed before the FFT process. Assuming ideal synchronization at the receiver, the signal of the k -th subcarrier at the n -th symbol time can be represented by

$$Y[n, k] = X[n, k] H[n, k] + Z[n, k] \quad (8)$$

where $H[n, k]$ is the channel frequency response of the k -th subcarrier at the n -th symbol time and $Z[n, k]$ is the background noise and interference and can be approximated as zero mean AWGN with variance σ_z^2 .

In vector notation, it can be rewritten as

$$\mathbf{Y}[n] = \mathbf{X}[n] \mathbf{H}[n] + \mathbf{Z}[n] \quad (9)$$

where

$$\mathbf{Y}[n] = [Y[n, 0] \ Y[n, 1] \ \dots \ Y[n, K-1]]^T, \quad \mathbf{X}[n] = \text{diag}(X[n, 0], X[n, 1], \dots, X[n, K-1]),$$

$$\mathbf{H}[n] = [H[n, 0] \ H[n, 1] \ \dots \ H[n, K-1]]^T,$$

and $\mathbf{Z}[n] = [Z[n, 0] \ Z[n, 1] \ \dots \ Z[n, K-1]]^T$.

III. Optimum 2-D MA FIR CEF

The pilot symbols are first estimated using the LS method [2], i.e.,

$$\tilde{H}[n, k] = Y[n, k] / X[n, k] = H[n, k] + Z'[n, k] . \quad (10)$$

Assuming that $|X[n, k]|^2 = 1$ for all pilot symbols, $Z'[n, k]$ is also zero mean AWGN with variance σ_z^2 .

When a 2-D MA FIR CEF has $(2M_f + 1)$ - and $(2M_t + 1)$ -taps in the time and frequency domain, respectively, the CIR at the n' -th symbol time and k' -th subcarrier can be obtained by

$$\hat{H}[n', k'] = \frac{1}{(2M_f + 1)(2M_t + 1)} \sum_{m=-M_f}^{M_f} \sum_{m_2=-M_t}^{M_t} \tilde{H}[n + m, k + m_2] \quad (11)$$

where $n - A/2 \leq n' < n + A/2$ and $k - B/2 \leq k' < k + B/2$. Since there is negligible difference between the CIRs corresponding to the pilot symbol and adjacent data symbol, we consider the case of $n' = n$ and $k' = k$ for simple description. Note that (11) needs only addition operation except normalization process.

By changing the summation form into an integral form, the MSE of the estimated CIR can be approximated by

$$\varepsilon(m_1, m_2) = 1 - 2q_{2,1}(m_1)q_{f,1}(m_2) + q_{2,2}(m_1)q_{f,2}(m_2) + \sigma_z^2 q_{2,3}(m_1)q_{f,3}(m_2) \quad (12)$$

where $m_t = T_s(M_t + 0.5)$, $m_f = \Delta f(M_f + 0.5)$ and

$$q_{t,1}(m_t) = \frac{1}{2m_t} \int_{-m_t}^{m_t} r_t(At) dt, \quad q_{f,1}(m_f) = \frac{1}{2m_f} \int_{-m_f}^{m_f} r_f(Bf) df$$

$$q_{t,2}(m_t) = \frac{1}{(2m_t)^2} \int_{-m_t}^{m_t} \int_{-m_t}^{m_t} r_t(A(t_1 - t_2)) dt_1 dt_2$$

$$q_{f,2}(m_f) = \frac{1}{(2m_f)^2} \int_{-m_f}^{m_f} \int_{-m_f}^{m_f} r_f(B(f_1 - f_2)) df_1 df_2 \quad (13)$$

$$q_{t,3}(m_t) = \frac{T_s}{2m_t}, \quad q_{f,3}(m_f) = \frac{\Delta f}{2m_f}.$$

Since $\varepsilon(m_t, m_f)$ is a concave function of m_t and m_f , the optimum tap size can uniquely be determined by solving

$$\left. \frac{\partial \varepsilon(m_t, m_f)}{\partial m_t} \right|_{m_t = \hat{m}_t} = 0 \quad \text{and} \quad \left. \frac{\partial \varepsilon(m_t, m_f)}{\partial m_f} \right|_{m_f = \hat{m}_f} = 0. \quad (14)$$

Substituting the polynomial approximations of (3) and $\cos(t)$ for $0 \leq t \leq 2\pi/3$ and neglecting the third-order term, the optimum tap size can be approximated by $\hat{M}_t = (\hat{m}_t + 0.5)/T_s$,

$$\hat{M}_t \approx \begin{cases} \left(\frac{15q_{f,3}(m_f)\sigma_z^2}{16T_s^4 k_{f,2} [8q_{f,2}(m_f) - 3q_{f,1}(m_f)]} \right)^{\frac{1}{5}}, & \text{for the classic spectrum} \quad (15) \\ \left(\frac{225q_{f,3}(m_f)\sigma_z^2}{128T_s^4 k_{f,2} [8q_{f,2}(m_f) - 3q_{f,1}(m_f)]} \right)^{\frac{1}{5}}, & \text{for the flat spectrum} \end{cases}$$

$$\hat{M}_f \approx \left(\frac{15q_{t,3}(m_t)\sigma_z^2}{16(\Delta f)^4 k_{t,2} [8q_{t,2}(m_t) - 3q_{t,1}(m_t)]} \right)^{\frac{1}{5}} \quad (16)$$

where

$$k_{t,1} = \pi^2 A^2 f_d^2, k_{t,2} = \frac{1}{4} \pi^4 A^4 f_d^4$$

$$k_{f,1} = 2\pi^2 B^2 \tau_{v1}^2, k_{f,2} = \frac{2}{3} \pi^4 B^4 \tau_{v2}^2 \quad (17)$$

$$\tau_{v1}^2 = \sum_{i=0}^{L-1} \tau_i^2 \sigma_i^2, \tau_{v2}^2 = \sum_{i=0}^{L-1} \tau_i^4 \sigma_i^2.$$

Note that the optimum tap size in the time domain is affected by the tap size in the frequency domain and vice versa. Since the optimum tap size satisfying (14) may not be represented in a closed form, it can be obtained iteratively as follows.

As illustrated in Fig. 2, the initial tap size is set to $\hat{M}_{t,0} = \hat{M}_{f,0} = 1$. For $\hat{M}_{t,0} = 1$, the optimum tap size $\hat{M}_{f,1}$ in the frequency domain is set to 25 by (16). Using $\hat{M}_{f,1}$, the optimum tap size $\hat{M}_{t,1}$ in the time domain is set to 7 by (15). In this manner, the values of $\hat{M}_{f,i}$ and $\hat{M}_{t,i}$, $i=2, 3, \dots$, are iteratively obtained, converging to a globally optimum value. Since the MSE is a concave function of M_t and M_f , the optimum tap size can always be found. The global optimum tap size

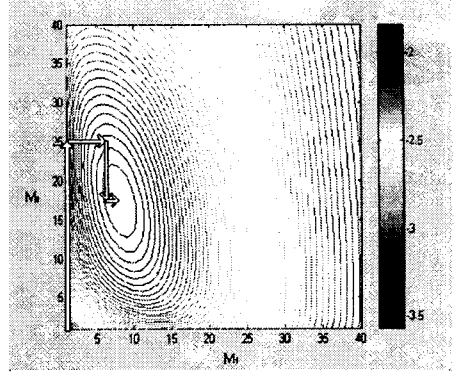


Fig. 2. Iterative procedure for finding the optimum tap size of the 2-D MA FIR CEF

can usually be obtained within five iterations.

IV. Performance evaluation

To investigate the effect of channel estimation on the receiver performance, the performance is evaluated in terms of bit error rate (BER). The simulation condition is summarized in Table 1. For performance comparison, we also consider the performance of 2-D Wiener CE and 2-D linear interpolation (LI) CE. The tap size of 2-D Wiener CEF is set to 80 and 20 in the time and frequency domain, respectively, since the use of additional taps provides negligible performance improvement. We also consider the use of a fixed 2-D MA FIR CEF whose tap sizes are determined to be $M_t = 2$ and $M_f = 1$ considering the maximum sample delay of 64, $f_{d,\max}$ of 1342.6 Hz and maximum SIR of 20 dB.

Fig. 3 and 4 depict BER performance with the convolutional coding whose code block is composed of the data bits in A OFDM symbols. Note that BER performance with $L=32$ is better than that with $L=2$ due

Table 1. Simulation condition

Parameters	Values
Total bandwidth	80 MHz
FFT/Guard interval	6.4 μ s/1.6 μ s
FFT size	512
Pilot redundancy	11.1 % (A=3, B=3)
Carrier frequency	5.8 GHz
Allowable maximum Doppler frequency	250 km/h (1342.6 Hz)
Channel coding	Convolutional coding (Code rate=1/2, constraint length=9)
Modulation	QPSK, 64QAM
Channel	Rayleigh (classic) Exponential profile (RMS delay = 5 symbols)

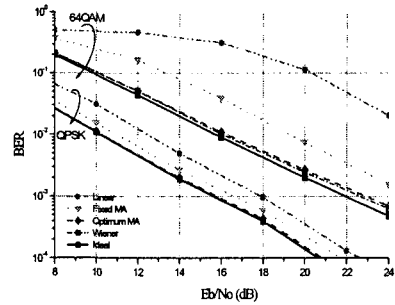
to increased frequency diversity effect. The optimum 2-D MA FIR CEF can provide BER performance similar to the 2-D Wiener CEF in most of channel condition. It can also be seen that the performance of the fixed MA FIR CEF significantly degrades, particularly when the number of multipaths and/or the maximum Doppler frequency is small and high-order modulation is employed.

V. Conclusions

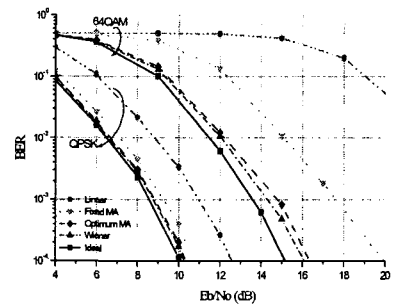
In this paper, we consider the design of 2-D MA FIR filters as the CEF for ease of implementation while providing relatively good receiver performance. We have proposed a simple procedure for the design of the optimum tap sizes of the 2-D MA FIR CEF. It can be seen that the optimum tap size depends on the channel condition parameters including the maximum Doppler frequency, power-delay profile and SIR. Simulation results show that the use of optimum 2-D MA FIR CEF is quite practical considering both the implementation complexity and performance. The analytic results can also be applied to the design of adaptive channel estimators.

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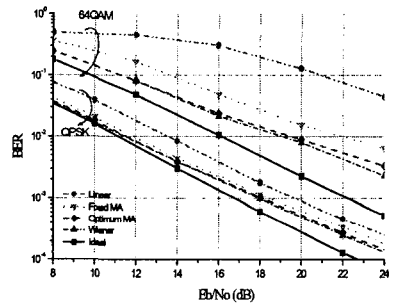


(a) $L=2$

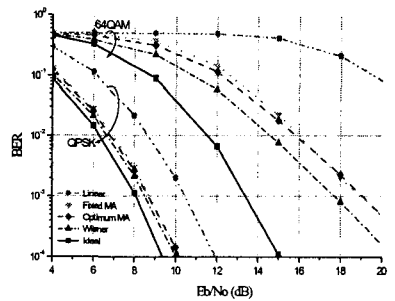


(b) $L=32$

Fig. 3. BER performance when $f_d T_S = 0.00043$.



(a) $L=2$



(b) $L=32$

Fig. 4. BER performance when $f_d T_S = 0.0107$.