# SPREADING FACTOR SELECTION FOR RETRANSMISSIONS OF NON-REAL TIME DATA IN DS/CDMA SYSTEMS

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Abstract-In this paper, it is shown that, in DS/CDMA mobile systems, halving or doubling the spreading factor (HSF or DSF) when retransmission is requested possibly improves the throughput. Given transmit power, DSF essentially decreases probability of packet error (PPE) by increasing the signal energy per information bit. It, however, doubles the time duration needed for transmitting the original packet. On the other hand, HSF increases PPE. It, however halves the time duration required to carry the original packet. Thus, the efficiency of HSF or DSF as a retransmission strategy depends on the amount of increased or reduced PPE after HSF or DSF is selected. With achieving given residual error probability (REP) in CDMA systems, the effective throughput is evaluated in this paper to find conditions with which HSF or DSF achieves better performance than using the original one. Analytic results show that HSF or DSF performs better when relatively small or big changes in their PPE's are present, respectively.

Keywords-Spreading factor, retransmissions, DS/CDMA mobile systems, effective throughput.

#### I. INTRODUCTION

In wireless data communications, the channel experiences burst errors and packet dropouts due to hostile fading, over-rising interference, inadequate handoffs and so forth. For the error recovery, retransmission of the lost information is widely accepted with forward error correction (FEC) mechanisms [1]. The retransmitted is sometimes the whole part or certain partial information of the lost packet, depending on different automatic repeat request (ARQ) strategies [2], [3]. In practical systems, since the number of allowable retransmissions (NAR) is limited to avoid unacceptable time delay, residual packet error is inevitably produced [4]. The retransmission efficiency is often evaluated by residual errors and the number of resulting retransmissions [5].

On ARQ schemes, the retransmission number is restricted according to different characteristics of two data service types. One of two types is real time data service, which has tight limitation of transmission delay. The other is non-real time data service, which has some tolerance to transmission delay. As a result, the NAR involves in acceptable transmission delay. Also, limited NAR reluctantly causes residual error probability (REP). In this paper, non-real time data trans-mission is assumed

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to evaluate widely a throughput, depending on NAR and REP

Considering data transmission in wireless channel, the characteristic of the time-varying multipath fading channel must be reflected, because a practical wireless channel induces a time-varying response with bursty errors due to multipath fading and shadowing effects. Then, a throughput is changed according to the fading channel conditions. It means that it is difficult to achieve an optimum throughput. Besides, it can say that a variable is needed to be matched to the prevailing channel conditions. In this paper, a spreading factor, as the variable for matching, is considered in DS/SSMA systems [6].

In this study, it is shown that doubling or halving the spreading factor (DSF or HSF) possibly improves the throughput that is the amount of successful packets per delay. If the spreading factor doubles, the probability of packet error (PPE) per transmission decreases. It, however, increases the time duration that is required to carry an original packet. On the other hand, if the spreading factor halves, the PPE per transmission increases, but the demanded time duration diminishes. We use the expected number of retransmissions (ENR), as a performance measure, that depends on the PPE per trans-mission and time duration. Thereby, an effective throughput, which is based on ENR and REP, is considered to compare fairly with performances in point of the ENR.

The effective throughput for DSF, HSF or original spread-ing factor (OSF) is obtained when target REP is given in this paper. Then, it is shown that HSF or DSF works better, for the respective small or big changes in their PPE's per transmission, than using OSF. Consequently, at the beginning of retrans-mission, an adequate spreading factor can be formed for improving the retransmission efficiency based on the ratio of the PPE's per transmission between HSF and OSF as well as between DSF and OSF.

#### II. SYSTEM DESCRIPTION

We consider transmission of equal size packets in a mobile wireless communication channel. One packet transmission may incur errors. Let us assume that the PPE per transmission is unchanged during retransmissions. Also, we assume error-free feedback since it is usually short and well protected. The system deliberates the cases of HSF and OSF in fig. 1 as well as

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DSF in fig. 2 when the retransmission is required. In the case of using OSF or HSF, fig. 1 shows that one packet advances to be accepted at receiver. Similarly, fig.2 illustrates an advance of one packet in the case of using DSF. In fig. 1 and 2, the number in the circles represents the frequency of transmissions.  $x_s$  and  $x_{2s}$  denote NAR's in OSF and DSF, respectively.  $P_s$  and  $P_{2s}$  denote the probabilities of the packet

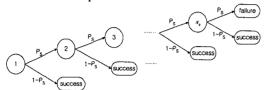


Figure 1. The state diagram for retransmission of one packet in OSF.

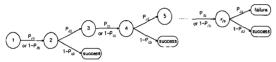


Figure 2. The state diagram for retransmission of one packet in DSF.

error in OSF and DSF, respectively. When HSF is selected, the system is the same as that for OSF but NAR since NAR must be changed by the difference of PPE's between OSF and HSF. In fig. 2, it is considered that two successful trans-missions are needed since DSF reduces the information rate in half. Also, the first of two transmission can not be decided whether it is successful in transmission since cyclic redundancy check (CRC) is located in the second transmitted packet. In this paper, we would like to present that it exits the range of PPE where the system for HSF or DSF has better performance than OSF, which is based on given REP.

#### III. EVALUATION OF THE PERFORMANCE

PPE is defined as the probability of packet error in a single transmission and denoted by  $P_s$  where spreading factor s is selected. Thus, spreading factor s/2 and 2s mean HSF and DSF, respectively. According to the characteristics of HSF and DSF, PPE's are expressed as

$$P_{s/2} = \alpha P_s$$
, and  $P_s = \beta P_{2s}$ . (1)

where  $\alpha$  and  $\beta$  ( $1 \le \alpha$ ,  $\beta \le \infty$ ) denote the increased amount of the PPE in HSF, OSF and DSF, respectively. Then, the performance of HSF, OSF or DSF depends on  $\alpha$ ,  $\beta$  and  $P_{2s}$ .

## A. Residual Error Probability (REP)

REP is defined as the probability of the transmission failure after NAR is exceeded and denoted by  $E_r$ . Form fig. 1 and 2, REP's are obtained as

$$E_r = P_{s/2}^l$$
,  $E_r = P_s^m$ , and  $E_r = \left(2P_{2s} - P_{2s}^2\right)^{n/2}$ . (2) where  $l$ ,  $m$  and  $n$  are NAR's in HSF, OSF and DSF, respectively. Using (2), NAR's for given REP are

$$l = \frac{\log_{10} E_r}{\log_{10} P_{s-2}}, \quad m = \frac{\log_{10} E_r}{\log_{10} P_s}, \text{ and } n = \frac{\log_{10} E_r^2}{\log_{10} (2P_{2s} - P_{2s}^2)}.$$
 (3)

NAR generally increases as PPE increases, and spreading factor reduces.

#### B. Expectation Number of Retransmission (ENR)

Let  $N_s$  denote the number of retransmission where original spreading factor s is selected.  $x_{s/2}$  and  $x_{2s}$  denote the numbers of retransmission in HSF and DSF, respectively. From fig. 1 and 2, ENR's in HSF, OSF and DSF are obtained by

$$E[N_{s/2}] = lP_{s/2}^{l} + (l-1) \cdot P_{s/2}^{l-2} (1 - P_{s/2}) + \dots + (1 - P_{s/2})$$

$$= \frac{1 - P_{s/2}^{l}}{1 - P_{s/2}},$$

$$E[N_{s}] = mP_{s}^{m-1} + (m-1)P_{s}^{m-2} (1 - P_{s}) + \dots + (1 - P_{s})$$

$$= \frac{1 - P_{s}^{m}}{1 - P_{s}}, \text{ and}$$
(5)

$$E[N_{2s}] = n(1 - (1 - P_{2s})^2)^{(n-2)/2}$$

$$+ (n-2)(1 - (1 - P_{2s})^2)^{(n-4)/2} (1 - P_{2s})^2$$

$$+ \dots + 2(1 - P_{2s})^2$$

$$= \frac{1 - \frac{n}{2}(2P_{2s} - P_{2s}^2)^{(n-2)/2} + (\frac{n-2}{2})(2P_{2s} - P_{2s}^2)^{n/2}}{(1 - P_{2s})^4}$$

$$+ n(2P_{2s} - P_{2s}^2)^{(n-2)/2}$$
(1)

Using (1), (3) and (4)~(6), ENR's for given REP are obtained by

$$E[N_{s/2}] = \frac{1 - (\alpha P_s)^{\frac{\log_{10} E_r}{\log_{10} \alpha P_s}}}{1 - \alpha P_s} , \qquad (7)$$

$$E[N_s] = \frac{1 - P_s^{\frac{\log_{10} E_r}{\log_{10} P_s}}}{1 - P_s} \quad , \tag{8}$$

$$= \frac{1 - (\beta P_{2s})^{\frac{\log_{10} E_r}{\log_{10} \beta P_{2r}}}}{1 - \beta P_{2s}} \quad \text{, and}$$
 (9)

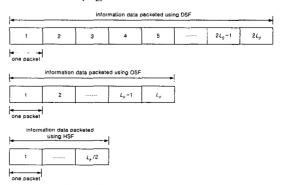


Figure 3. The packeted information data for DSF, OSF and HSF, respectively.

$$E[N_{2s}] = \frac{1 - \frac{\log_{10} E_r}{\log_{10} (2P_{2s} - P_{2s}^2)} (2P_{2s} - P_{2s}^2)^{\frac{\log_{10} E_r}{\log_{10} (2P_{2s} - P_{2s}^2)^{-1}}}}{(1 - P_{2s})^4} + \frac{\left(\frac{\log_{10} E_r}{\log_{10} (2P_{2s} - P_{2s}^2)} - 1\right) (2P_{2s} - P_{2s}^2)^{\frac{\log_{10} E_r}{\log_{10} (2P_{2s} - P_{2s}^2)}}}{(1 - P_{2s})^4}}{(1 - P_{2s})^4} + \frac{2\log_{10} E_r}{\log_{10} (2P_{2s} - P_{2s}^2)} (2P_{2s} - P_{2s}^2)^{\frac{\log_{10} E_r}{\log_{10} (2P_{2s} - P_{2s}^2)^{-1}}}}$$

# C. Effective Throughput

In order to compare with the efficiencies of ENR's for respective HSF, OSF and DSF under given REP, an effective throughput is reasonably considered. The effective throughput for OSF, which represents the amount of successful retransmission per delay in information data, is written as [7]

$$\eta_s = \frac{(1 - E_r) \cdot B}{L_R \cdot E[N_s] \cdot (2\tau + \delta)} \quad . \tag{11}$$

where

B = information data bits,

 $L_B$  = the number of packets in information data,

 $E[N_s]$  = the ENR when s, OSF, is selected,

 $\tau$  = the propagation delay,

 $\delta$  = the time required to transmit a packet plus receiver processing time for checking an erroneous packet, plus the time to transmit an acknowledgement, and

 $E_r$  is REP.

In the case of HSF or DSF, the number of packets in information data and ENR are considered since REP and delay are constant. In fig. 3, it is shown that the number of packets is doubled or halved since information rate is doubled or halved in DSF or HSF, respectively. Thus, if HSF, OSF or DSF is selected, the number of packets is  $L_B/2$ ,  $L_B$ , or  $2L_B$ , respectively. As a result, using (11), the effective throughputs for respective HSF and DSF are obtained by

$$\eta_{s/2} = \frac{(1 - E_r) \cdot B}{(L_B / 2) \cdot E[N_{s/2}] \cdot (2\tau + \delta)}$$
, and (12)

$$\eta_{2s} = \frac{(1 - E_r) \cdot B}{(2L_B) \cdot E[N_{2s}] \cdot (2\tau + \delta)} \quad . \tag{13}$$

#### IV. NUMERICAL RESULTS

In (10) and (11), the effective throughputs include the constant part which can be ignored to compare with them. Thus, (10) and (11) can be rewritten simply as

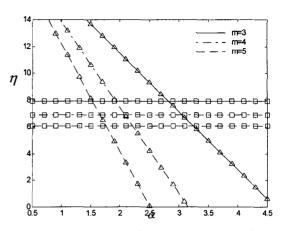


Figure 4. Throughput  $\eta$  versus  $\alpha$  for HSF and OSF when REP is 1 percent; " $\square$ " and " $\triangle$ " mean the cases of OSF and HSF, respectively.

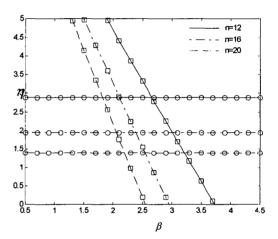


Figure 5. Throughput  $\eta$  versus  $\beta$  for OSF and DSF when REP is 1 percent; " $\square$ " and " $\square$ " mean the cases of OSF and DSF, respectively.

$$\eta_{s/2} = C \frac{1}{(1/2)E[N_{s/2}]},$$
(14)

$$\eta_{s} = C \frac{1}{E[N_{c}]} \quad , \text{and}$$
 (15)

$$\eta_{2s} = C \frac{1}{2E[N_{2s}]} . \tag{16}$$

where C is constant, and 
$$C = \frac{(1 - E_r) \cdot B}{L_B \cdot (2\tau + \delta)}$$
.

Therefore, using (7)~(10) and (14)~(16), in fig. 4 and 5, we numerically compare with the obtained throughputs for HSF, OSF and DSF for 1 percent REP, respectively. Fig. 4 illustrates the change of the throughputs versus  $\alpha$  for m, HSF and OSF.

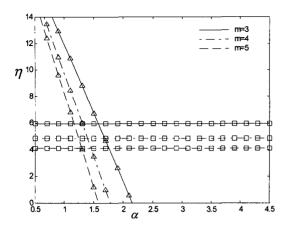


Figure 6. Throughput  $\eta$  versus  $\alpha$  for HSF and OSF when REP is 10 percent; "□" and "△" mean the cases of OSF and HSF. respectively.

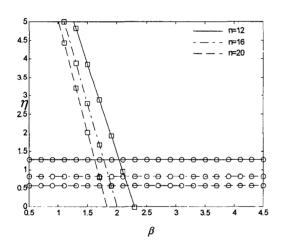


Figure 7. Throughput  $\eta$  versus  $\beta$  for OSF and DSF when REP is 10 percent; "D" and "O" mean the cases of OSF and DSF, respectively.

In fig. 4, the throughputs for OSF are unchanged since they are not related to  $\alpha$ . Those for HSF, however, decline as  $\alpha$  increases. In addition, the throughputs are obtained under three kinds of NAR for OSF, and increasing m essentially implies that PPE in OSF increases. Fig. 5 illustrates the change of the throughputs versus  $\beta$  for n, OSF and DSF. Unlike fig. 4, in fig. 5, the throughputs for OSF decline as  $\beta$  increases, and those for DSF are unchanged. Moreover, the throughputs are obtained under three kinds of NAR for DSF, and increasing n essentially means that PPE in DSF increases. In fig. 4 and 5, the intersecting points are the standards to select the efficient one of HSF, OSF and DSF. Then, that is shown that the specific spreading factor exists to obtain the increasing throughput which is changed by the NAR,  $\alpha$  and  $\beta$ . We found that the throughput for HSF increases as  $\alpha$  and  $\beta$  decrease. The throughput, however, for DSF

increases as  $\alpha$  and  $\beta$  increase. Accordingly, if the PPE's per transmission in HSF, OSF and DSF are estimated respectively, the adequate spreading factor can be selected for improving the throughput by the  $\alpha$  and  $\beta$  that are the error probability ratios. In addition, for the observation of the throughput response by the change of the REP, fig. 6 and 7 are inserted. Like fig. 4 and 5, fig. 6 and 7 show that the throughputs are changed, but REP is 10 percent. In the comparison with fig. 4 and 6, it indicates that the efficiency of HSF for 1 percent REP is better than for 10 percent REP. Also, fig. 5 and 7 show that the efficiency of DSF for 1 percent REP is better than for 10 percent REP.

#### V. CONCLUSION

In this paper, we have studied spreading factor selection to improve the retransmission performance in wireless com-munication systems. The effective throughputs for respective HSF, OSF and DSF were evaluated, based on ENR on ARQ scheme. Using the results in simulations, it was shown that the throughput depends on NAR and the ratio of PPE's between HSF and OSF as well as between DSF and DSF. These results also demonstrate the need to select the adequate spreading factor in order to improve the retransmission throughput. Simulation results indicate that the efficient selection of the spreading factor gives more benefit for the throughput when  $\alpha$  or  $\beta$  is smaller or larger, respectively. In addition, the efficiency of HSF or DSF increases as REP reduces. For considering the changing PPE with time, when the selection of a spreading factor is required, it is needed that the present and future channel information which is based on variable  $P_{2s}$ ,  $\alpha$  and  $\beta$ between the first retransmission and the allowable retransmission. In the approach, accurate prediction on PPE in time-varying channels is prerequisite for improving the performance. However, it is very difficult to predict the information in the far future since the fading channel can be rapidly changed. Therefore, first of all, further study may be conducted to make a strategy for the accuracy of the prediction in the near future.

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