Comparison of the Normalized SNRs between the LPA Beamformer and the Conventional Beamformer for a Moving Source

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Abstract — The DOA(Direction Of Arrival) estimation to select a best beam for receiving a particular signal in switched beam antenna systems, and to shape the optimal beam in adaptive array antenna systems, is typically performed under the assumption that the target user motion is almost negligible. In this paper, we model the user as the time-varying source and adopt the LPA(Local Polynomial Approximation) tracking algorithm, proposed by Katkovnik, to solve the time-varying DOA estimation problem. Then, we compare the power spectrum functions between the LPA beamformer and the conventional beamformer, also, the normalized SNRs of each beamformer. The results show that the LPA beamformer is robuster than the conventional beamformer in time-varying environments. In addition, in case of the conventional beamformer, more array elements give rise to more degradation in the aspect of SNR.

Ⅰ. INTRODUCTION

For a long time, smart antenna systems have been receiving a lot of interests as techniques to improve the performance of wireless communication systems [3]. Certainly, the smart antennas are useful for increasing channel capacity and spectrum efficiency, and reducing multipath fading and co-channel interference. In order to obtain these benefits, the DOA information is essential since it is used to select the best beam for receiving a particular signal in the switched beam antenna systems, and to track the target signals and shape the optimal beam to their directions in the adaptive array antenna systems, especially. Accordingly, many DOA tracking and estimating methods have been developed to satisfy these demands [3]. However, most of existing DOA estimations in the smart antenna systems are typically performed under the assumption that the target user motion is almost negligible, that is, the DOA of the user for tracking is time-invariant. This time-invariant DOA in the conventional tracking algorithm is used for the simplicity of modeling [1], [4]. However, practically, the DOAs of the users are time-varying. Therefore, we adopt the LPA tracking algorithm, proposed by Katkovnik, as the alternative tracking algorithm for the time-varying users [1], [2].

In this paper, we compare the variations of the SNR between the LPA beamformer and the conventional beamformer when the velocity of the moving source changes. To do that, we first apply the LPA algorithm to obtain the DOA of a single moving source, and observe the difference of the power functions between the LPA beamformer and the conventional beamformer in time-varying environments. This paper is organized as follows: In Section Ⅱ, the signal model of a moving source is explained. In Section Ⅲ, the LPA tracking algorithm are described, and the power spectrum functions of the LPA and the conventional beamformer are discussed. In Section Ⅳ, first, the power spectrum of each beamformer is depicted, and then the normalized SNR values are compared according to the user velocity. Finally, the results are summarized and concluded in Section Ⅴ.

Ⅱ. SIGNAL MODEL FOR A TIME-VARYING SOURCE

We consider a case of a single source. To develop the array model below, we use some assumptions.

First, the source is located in the far field of the array antenna and the plain wave is propagated. Also, both the user and the array elements are in the same horizontal plain. Thus, we only consider the azimuthal DOA of a user. Further, we assume that the mutual coupling between the array elements is ignored.

Consider a uniform linear array(ULA) with L -elements depicted in Fig. 1. This array antenna receives a single narrowband signal propagated from the unknown time-varying source with a direction ϕ at time t. Then, the $L\times 1$ signal vector, $R(t)$, at the array output is modeled as

$$R(t)=A(\phi(t))s(t)+N(t) \tag{1}$$

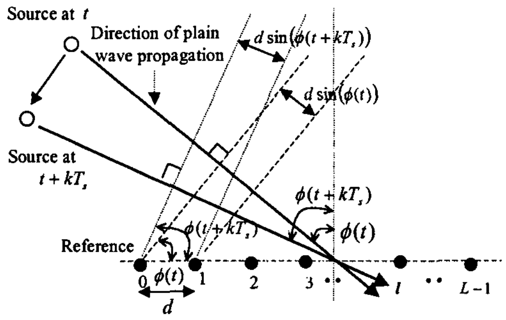

Fig. 1. Array model for LPA tracking algorithm

where

$$A(\phi(t))=\{a_l(\phi(t)): l=0,1,\cdots,L-1\} \tag{2}$$

is defined as the $L\times 1$ steering vector. Here,

$$a_l(\phi(t))=e\{-j2\pi dl\sin\phi(t)/\lambda\} \tag{3}$$

represents the array output for the l-th element at time t where d and λ are the inter-element distance and the signal wavelength, respectively. Also, $s(t)$ denotes the incident signal, and $N(t)$ is the $L\times 1$ additive noise vector given by a zero-mean complex Gaussian random variable with variance σ_n^2 .

Now, to model a single moving user for the LPA tracking algorithm, we consider that the azimuthal angle of a target user is linearly increasing as time increases. The LPA beamformer performs the time-based sampling of the received signal per period T_s to obtain the user motion information. If we model the user motion within the total sampling interval T, the DOA value at time $t+kT_s$, is represented, by using Taylor series, as

$$\begin{aligned}\phi(t+kT_s)&=\phi(t)+\phi^{(1)}(t)kT_s+\frac{\phi^{(2)}(t)}{2}(kT_s)^2+\cdots\\&=z_0+z_1kT_s+z_2(kT_s)^2\cdots\end{aligned} \tag{4}$$

where k is a integer number denoting k-th sampling within the total sampling interval. Here, assuming that the sampling interval, T_s, is sufficiently short, we can neglect the third and later terms in Eq. (4). Therefore, we obtain the following approximated form [2]:

$$\phi(t+kT_s)=z_0+z_1kT_s \tag{5}$$

where

$$z_0=\phi(t),\qquad z_1=\phi^{(1)}(t) \tag{6}$$

are the instantaneous user DOA and the user velocity, respectively. Now, the problem of user-DOA-tracking is to find the estimate vector of

$$Z=\begin{pmatrix}z_0\\z_1\end{pmatrix} \tag{7}$$

from the time-varying array processing within the total sampling interval.

III. LPA TRACKING ALGORITHM IN TIME-VARYING ENVIRONMENTS

In this section, we observe how the LPA tracking algorithm is formulated with the L-elements uniform linear array model for a single user. For this goal, a LS(least square) approach, one of the most popular tracking techniques in adaptive array antenna systems, is used to estimate the DOA for tracking the user [5], [6].

First, consider the following LS based function from Eq. (1).

$$\begin{aligned}F_{LPA}(t,Z)=\sum_k w_g(kT_s)\|R(t+kT_s)\\-A(Z,kT_s)s(t+kT_s)\|^2\end{aligned} \tag{8}$$

where $A(Z,kT_s)$ is given by Eq. (5)~(7). The total sampling interval is given by the following window function

$$w_g(kT_s)=\frac{T_s}{g}w\left(\frac{kT_s}{g}\right). \tag{9}$$

Here, $w(x)$ is a function satisfying the following properties

$$w(x)\geq 0,\quad w(0)=\max w(x),\quad \int_{-\infty}^{\infty}w(x)dx=1 \tag{10}$$

and the parameter, g (>0), determines the window length.

Finding the DOA of a target user is equal to looking for the DOA estimate vector $\hat{Z}$ minimizing $F_{LPA}(t,Z)$, that is, seeking for the solution, $\hat{Z}$, of the problem given by

$$\hat{Z}=\arg(\min F_{LPA}(t,Z)). \tag{11}$$

In order to minimize $F_{LPA}(t,Z)$ on $s(t)$, we utilize the derivative of Eq. (8) represented by

$$\frac{\partial F_{LPA}}{\partial s^*(t+kT_s)}=0 \tag{12}$$

where $(\cdot)^*$ stands for the complex conjugate value. After we use the property $A^H(Z,kT_s)A(Z,kT_s)=L$, we insert the signal estimate $\hat{s}(t)$, obtained from Eq. (12), into Eq. (8). Then, we get the following result represented as

$$\begin{aligned}F_{LPA}(t,Z)=\sum_k w_g(kT_s)\{|R(t+kT_s)|^2\\-\frac{1}{L}|A^H(Z,kT_s)R(t+kT_s)|^2\}.\end{aligned} \tag{13}$$

Now, we obtain the fact that minimizing Eq. (8) is equivalent to maximizing the LPA power spectrum function of the array output given by

$$P_{LPA}(t, Z) = \frac{1}{L}\sum_{k} w_g(kT_s)|A^H(Z, kT_s)R(t+kT_s)|^2 \quad (14)$$

In conclusion, for the user-DOA-tracking in time-varying environments, the LPA beamformer find the DOA value maximizing the LPA power spectrum function of Eq. (14).

Here, the conventional power spectrum function is briefly introduced as means to be compared to the LPA beamformer. After performing the same procedure mentioned above, we obtain the power spectrum function of the conventional beamformer in time-varying environments given by

$$P_{con}(\phi) = \frac{1}{L} A^H(\phi)\hat{Q}(t)A(\phi) \quad (15)$$

where

$$\hat{Q}(t) = \sum_{k} w_g(kT_s)R(t+kT_s)R^H(t+kT_s). \quad (16)$$

Here, $\hat{Q}(t)$ is the estimate value of the $L\times L$ covariance matrix of the array output vector $R(t)$. We refer this power spectrum function as a special case of the LPA algorithm assuming $z_0=\phi$ and $z_1=0$, that is, it means the localized version of the conventional power spectrum function.

IV. PERFORMANCE COMPARISON

In this section, we compare the performance between the LPA beamformer and the conventional beamformer. First, each power spectrum function is compared according to the velocity, in order to observe the tracking ability in time-varying environments. After that, We compare the normalized SNR in array output, obtained from the true user, in order to show the performance degradation. The results are explained in terms of the robustness to the time-varying environments

A. Power Spectrum Function

We consider a uniform linear array with 10 elements that the inter-element distance is the half wavelength, $d=\lambda/2$ in Eq. (3). The rectangular symmetric window, which has 50 as the total sampling number, is used. Also, the sampling period, T_s, is 0.1 second. It is assumed that the sensor noise is a zero-mean complex Gaussian random variable with a variance $\sigma_n^2=1$. In addition, we assume that the channel effects such as the path gain, the Doppler shift, the phase offset, and the time delay are negligible. Now, we consider the following situation in order to show an adaptation to the cellular communications. A single user is moving along the azimuthal angle 100m apart from a base station and the true user location is $\phi=0^0$. Also, it is considered that the transmitted real signal, $s(t)$, always has $E[|s(t)|^2]=1$ within the total sampling interval.

Fig. 2 and Fig. 3 show the results when the user velocities are 0km/hour and 60km/hour, respectively. From the results, we obtain the following facts.

When the user does not move, the power spectrums of the LPA and the conventional beamformer are similar. However, when the user moves with a constant velocity, the power spectrum shape of the conventional beamformer is distorted while that of the LPA beamformer is maintained. It means that the tracking ability of the conventional beamformer for the true DOA eminently degrades in time-varying environments. In case of the conventional beamformer, we can observe that the maximum value of the power spectrum points out a wrong angle. Contrary to the conventional beamformer, the LPA beamformer shows the remarkable ability for the true DOA tracking in time-varying environments because the maximum value of its power spectrum gives the accurate indication of the true user location regardless of the velocity. This is an outstanding benefit of LPA beamformer for the application to mobile communications.

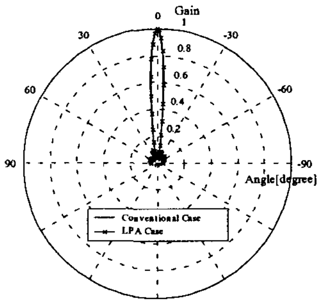

Fig. 2. Power spectrums when the user velocity is 0km/hour

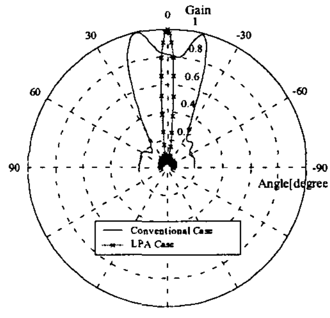

Fig. 3. Power spectrums when the user velocity is 60km/hour

B. Normalized SNR in Array Output

When the source moves rapidly, the LPA algorithm exactly tracks the source DOA whereas the conventional algorithm cannot chase it. In addition, the DOA estimation ability of the conventional beamformer remarkably degrades as the velocity of the target source increases [1]. This fact results in the performance degradation of the system. One of the methods to examine this system degradation is to observe the SNR value. Therefore, we compare the normalized SNR variation obtained by using the power spectrum functions between the LPA beamformer and the conventional beamformer as the source velocity changes. We consider the uniform linear array antennas with 2, 6, and 10 elements, respectively, that the inter-element distance is the half wavelength, $d=\lambda/2$. Other factors are equal to the previous assumptions. Once again, we consider the situation where a single user is moving along the azimuthal angle 100m apart from a base station. Also, it is assumed that $E[|s(t)|^2]/\sigma_n^2=1$ within the total sampling interval where $s(t)$ is the transmitted signal and σ_n^2 is the noise variance.

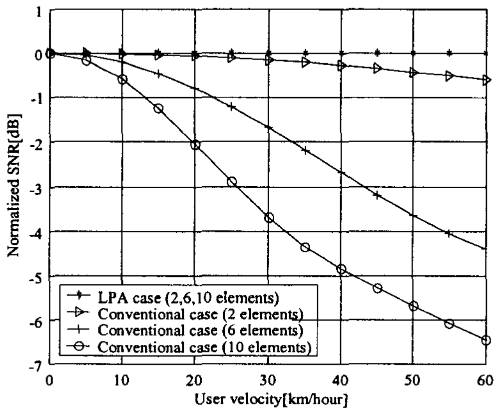

Fig. 4. Normalized SNR of the LPA beamformer and the conventional beamformer versus the source velocity

Fig. 4 shows the normalized SNR in dB of the LPA beamformer and the conventional beamformer with 2, 6, and 10 elements, respectively, according to a user velocity. Here, the normalized SNR is defined as

$$Nor=\frac{SNR_{velocity}}{SNR_{non-velocity}} \quad (17)$$

As shown in Fig. 4, the SNR of the LPA beamformer maintains the same value regardless of the velocity of the user, whereas that of the conventional beamformer has the remarkable degradation as the velocity of the user increases. Especially, we note that the SNR degradation rate of the conventional beamformer becomes large as the number of array elements increases. When the velocity is 50km/hour, the cases of 2, 6, and 10 elements show about $-0.5dB$, $-3.5dB$, and $-5.5dB$ degradation, respectively. This means that the conventional beamformer with more array elements is more sensitive to the velocity magnitude. Generally, more array elements give the more power gain. However, pay attention to the fact that the results do not show the power gains but represent the normalized SNR values, here.

IV. CONCLUDING REMARKS

In this paper, we studied the performance of the LPA tracking algorithm and the conventional tracking algorithm in time-varying environments. Especially, the normalized SNR of each beamformer was compared to observe the degradation rate according to the velocity. In conclusion, the LPA tracking algorithm is robuster than the conventional tracking algorithm in time-varying environments, since LPA tracking algorithm is developed by using the user velocity information. The results of the normalized SNR prove this. Contrary to the LPA case, we confirm that the conventional tracking algorithm is very frail in time-varying environments through the normalized SNR results. Moreover, in the conventional beamformer case, as the number of array elements increases, the degradation rate of the normalized SNR becomes large.

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