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Chaotic Forecast of Time-Series Data Using Inverse Wavelet Transform

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Abstract: Recently, the chaotic method is employed to forecast a near future of uncertain phenomena. This method makes it possible by restructuring an attractor of given time-series data in multi-dimensional space through Takens' embedding theory. However, many economical time-series data are not sufficiently chaotic. In other words, it is hard to forecast the future trend of such economical data on the basis of chaotic theory. In this paper, time-series data are divided into wave components using wavelet transform. It is shown that some divided components of time-series data show much more chaotic in the sense of correlation dimension than the original time-series data. The highly chaotic nature of the divided component enables us to precisely forecast the value or the movement of the time-series data in near future. The up and down movement of TOPICS value is shown so highly predicted by this method as 70%.

Keywords: Chaos theory, Short-term forecasting, Wavelet transform.

1 Introduction

The chaotic short-term forecasting method[1][2] based on time-series data enables us to know a value, which we could not predict before. Nevertheless, it is still difficult to definitely forecast a value even in near future because many kinds of data are less chaotic. Even though such data are less chaotic, it is possible to abstract and pull out the partial chaotic portion out of the data[3][4][5].

In this research, wavelet transform[6] is employed to take chaotic portions out of the original time-series d and we can find the more highly chaotic component out of the original data by measuring these correlated dimension. Once we can successfully find the highly chaotic portion out of the original data, it enables us to improve the forecasting precision by the wavelet transformation.

The correlation dimension[7][8] of the divided components should be measured smaller than the one of the original data, if the divided components are more highly chaotic than the original data.

2. Chaotic Approach and Forecasting

Forecasting can base on the Takens' embedding theory [10] which tells us that it is possible to restructure the trajectory of a dynamic system in a high dimensional space by using only the information (that is, time-series data) of partial component dimensions (variables).

Using time-series data $x(t)$, let us define vector $Z(t)$ as follows:

$$Z(t)=(x(t),x(t-\tau),x(t-2\tau),\cdots,x(t-(n-1)\tau)) \quad (1)$$

where τ denotes an arbitrary constant time interval. The vector $Z(t)$ shows one point in n dimensional space (Data Space). Therefore, changing t generates a trajectory in the n dimensional data space. When n is sufficiently large, this trajectory shows a smoothly changed one of the high dimensional dynamic system. That is, if the dynamic system has some attractor, the attractor obtained from the original one should come out on the data space. In other words, the original attractor of the dynamic system can be embedded in the n dimensional topological space. Number n is named an embedded dimension. Denoting the dimension of the original dynamic system by m, it can be proved that this dimension n is sufficiently large if n holds the following:

$$n \geq 2m+1 \quad (2)$$

Equation (2) is a sufficient condition on the embedded dimension. It is required to employ data with more than $3m+1$ to $4m+1$ samples within a certain time length in short-term forecasting.

Next, let us illustrate the deterministic structure using a restructured trajectory. There are several methods. Let us embed discrete time-series data with equal time interval $\tau=15$ in embedded dimension $n=3$. Observed discrete time-series samples can be mapped into a topological space with embedded 3-dimensional space As a result, the mapped vector is denoted in the following:

$$Z(t) = (x(t), x(t-\tau), x(t-2\tau)) \quad (3)$$

Let $Z(i)$ denote a 3-dimensional vector of that maps observed data including the most recent time into a topological space.

These data in the neighborhood of $Z(i)$ are ones observed in the past.

The trajectory of $Z(i+1)$ at one step future has been observed.

These relations enable us to forecast behavior $Z(i+1)$ in near future.

The future trajectory $x(i+1)$ of the given time-series data $(x(i), x(i-1),...)$ can be calculated..

3. Correlation Dimension

The measurement of correlation dimension is employed to evaluate whether the time-series data are chaotic or not. The evaluation of the correlation dimension is pursued by checking whether the time-series data distribute in the less dimensional space than m-dimensional space, if the data is embedded in m-dimensional space.

At first, let us embed the time-series data into m dimensional space. Then, the procedure is written as follows:

(1) The procedure:

STEP1: draw the circle with radius r at the center of the points which each embedded vector has.

STEP2: count how many points are included within the drawn circle and measure its number C.

When the radius is large, then the large number of points should be included in the circle with radius r. As C is an increasing function of r, let us denote it as $C(r)$. If plotted points are distributed evenly in the m-dimensional space, the number of points included within the circle should increase proportionally to the area of the circle, as radius r increases.

$$C(r) = ar^{m} \quad (4)$$

On the other hand, if the structure has any regularity, $C(r)$ should increase proportionally to the less value than m powered value.

$$C(r) = br^{(m-x)} \quad (5)$$

The value $(m-x)$ is named correlation dimension. In the case of random data, the regularity could not be found in the space even if the embedded dimension is increased. Therefore, the correlation dimension should increase even if the embedded dimension does. When the time-series data have the deterministic structure in the embedded space, the correlation dimension can not increase and should be matured at a certain value, even if the embedded dimension increases.

4. Wavelet Transformation

Fast Fourier Transform is a widely employed method to transform signal into the portions of each frequencies. A sin function is employed as a base function. The sin function is an infinitive smooth function. Therefore, the information obtained by the Fast Fourier Transform does not include the local information such as the place

and the frequency where and which frequency the original signals have.

On the other hand the wavelet transform employs a compact portion of a wavelet as a base function. Therefore, it is a time and frequency analysis such as it enables us to determine the signal using time and frequency.

The mother wavelet transform $\psi(x)$ of a function $f(x)$ can be defined as follows:

$$(W_{\psi} f)(b,a) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{|a|}} \overline{\psi\left(\frac{x-b}{a}\right)} f(x) dx \quad (6)$$

Where a is a scale of the wavelet, b is a translate. $\overline{\psi(x)}$ is a conjugate of a complex number. It is also possible to recover the original signal $f(x)$ using wavelet transform. That is, we can realize the inverse wavelet transform as follows:

$$f(x) = \frac{1}{C_{\psi}} \iint_{R} (W_{\psi} f)(b,a) \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right) \frac{dadb}{a^2} \quad (7)$$

The wavelet transform is a useful method to know the characteristics of the signal but not an efficient one. It is because the signal has a minimum unit and the wavelet method expresses many-duplicated information's. This point can be resolved by discrediting a dimensional axis. Let us denote a dimension as $(b, 1/a) = (2^{-j}k, 2^{j})$, then the discrete wavelet transform can be rewritten as

$$d_k^{(j)} = 2^j \int_{-\infty}^{\infty} \overline{\psi(2^j x - k)} f(x) dx \quad (8)$$

Inverse wavelet transform is

$$f(x) \sim \sum_j \sum_k d_k^{(j)} \psi(2^j x - k) \quad (9)$$

Let us denote the summation $\sum_k d_k^{(j)}\psi(2^j x - k)$ of the right term as

$$g_j(x) = \sum_k d_k^{(j)}\psi(2^j x - k) \tag{10}$$

Then let us define $f_j(x)$ as

$$f_j(x) = g_{j-1}(x) + g_{j-2}(x) + \cdots \tag{11}$$

where an integer j is named a level. If we can denote $f(x)$ as $f_0(x)$, then

$$f_0(x) = g_{-1}(x) + g_{-2}(x) + \cdots \tag{12}$$

This equation illustrates that the function $f_0(x)$ is transformed into wavelet components $g_{-1}(x)$, $g_{-2}(x)$, ..., . It is required that the left side should be transformed uniquely into the right side and also the left side should be realized by composition from the right side components. They can be realized by using a mother wavelet ψ as a base function.

Function $f_j(x)$ can be rewritten using a recursive forms

$$f_j(x) = g_{j-1}(x) + f_{j-1}(x) \tag{13}$$

This equation means that the original signal $f_j(x)$ can be transformed into wavelet components $g_{j-1}(x)$ and $f_{j-1}(x)$. This equation enables us to decompose the original into the wavelet components step by step. This method is named multi-resolution signal decomposition.

5. Moving Wavelet Transform

When time-series data are analyzed on the basis of wavelet transform, the number of employable samples should decrease such as 1/2 of the number of the original samples for j=-1 and 1/4 for j=-2, respectively. Accordingly the time interval should be expanded to the 2 times and 4 times wider, respectively. This is because wavelet transform produce lower frequency components according that j becomes smaller. This situation makes it difficult to inverse-transform the forecasted components of wavelet transform into the original time-series value.

This paper proposes a method to move the starting and terminated points and wavelet transform them so as to obtain the same focal future time-point. This wavelet transform is named a moving wavelet transform in this paper. This method enables us to inverse-wavelet transform forecasted component values for each j at next future time-point into the original time-series...

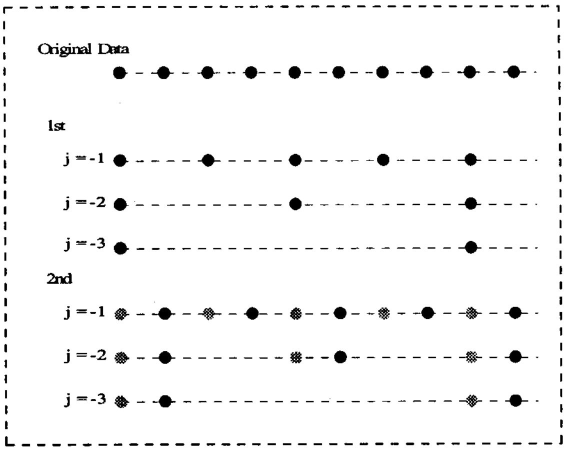

Fig. 1 Moving Wavelet Transform

For example, let us wavelet transform original time-series data until j=-3 as in Figure. 1.. In this case, the number of employable samples for j=-1 should be 1/2 of the total number of samples and one for j=-2 is 1/4 and one for j=-3 is 1/8. If we move the starting point one by one, we can interpolate the lacked samples or components which should be inverse-wavelet-transforms and calculate the inversed forecasted value.

6. Forecast Based on Inverse-Wavelet-Transform

This section will be spent to explain the short-term forecast based on inverse-wavelet-transform.

The wavelet transform decomposes time- series data into each frequency band. Decomposed time-series data is more convenient to forecast the short-term future comparing the original time-series data. Therefore, it is much reasonable to forecast the short-term future using components decomposed by wavelet transform and to inverse-wavelet-transform these forecasted components values into the original time-series value which means the short-time forecasted value of the original time-series data. This method enables us to forecast the short-term future based on wavelet transform with more precision than on the original time-series data.

Basically, the chaotic forecast is to evaluate value in the one term future from the present. Nevertheless, the decomposing by wavelet transform divides the original data into each of frequency bunds. This bring out that the length, τ, of one term for each component should be different. Therefore, it is impossible to inverse-wavelet-transform the forecasted component values into the original value. We should employ moving wavelet transform to bring the forecasted component values at one term future and using these forecasted component values at the one term future can be inverse-wavelet-transformed into the original time-series.

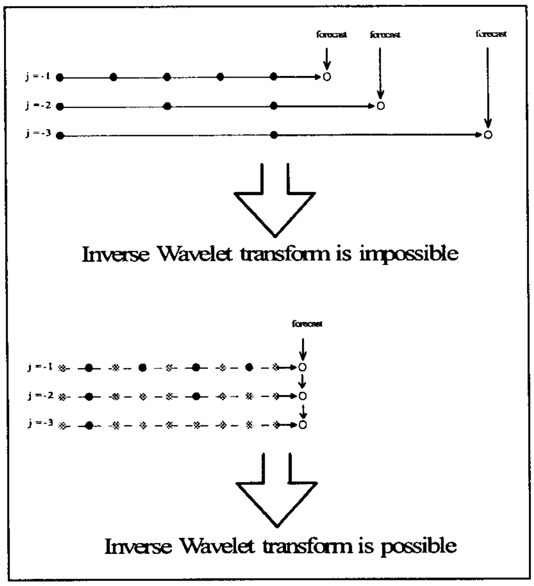

Fig. 2 Forecast based on Inverse-Wavelet-Transform

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