2003 International Symposium on Advanced Intelligent Systems
September 25-28, 2003, Jeju, Korea

Fuzzy-Model-Based Kalman Filter for Radar Tracking

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Abstract - **In radar tracking, since the sensor measures range, azimuth and elevation angle of a target, the measurement equation is nonlinear and the extended Kalman filter (EKF) is applied to nonlinear estimation. The conventional EKF has been widely used as a nonlinear filter for radar tracking, but the considerably large measurement error due to the linearization of nonlinear function in highly nonlinear situations may deteriorate the performance of the EKF. To solve this problem, a fuzzy-model-based Kalman filter (FMBKF) is proposed for radar tracking. The FMBKF uses a local model approximation based on a TS fuzzy model instead of a Jacobian matrix to linearize nonlinear measurement equation. The hybrid GA and RLS method is used to identify the premise and the consequent parameters and the rule numbers of this TS fuzzy model. In two-dimensional radar tracking problem, the proposed method is compared with the conventional EKF.**

I. INTRODUCTION

For the past three decades, the tracking problem of a moving target with radar measurements has been a fruitful application area for the state estimation. In general, the objective of target tracking is to estimate accurately the target trajectory dependent on the noisy measurements from the sensor. In radar tracking problems, since the sensor measures the range, azimuth and elevation angle of a target, the measurement equation is nonlinear and the extended Kalman filter (EKF) is applied to nonlinear estimation. The EKF has been widely used as a nonlinear filter for radar tracking. However, the usual tracking filters relying on the linear approximation lead to poor convergence and erratic behavior in highly nonlinear situations.

To resolve this problem, a fuzzy-model-based Kalman filter (FMBKF) is proposed for radar tracking. The FMBKF uses a local model approximation based on a TS fuzzy model instead of a Jacobian matrix to linearize nonlinear measurement equation. In the proposed method, to identify the premise and the consequent parameters and the rule numbers of this TS fuzzy model, the hybrid GA and RLS method is used as an optimization learning method to search more globally optimal solution.

The proposed FMBKF is applied and simulated in two-dimensional radar tracking problem, Computer simulation is divided by two parts-one is simulation for offline optimization of TS fuzzy model and the other is the Monte Carlo simulation for radar tracking using the optimized TS fuzzy model. The FMBKF is compared with the conventional EKF.

II. PROBLEM STATEMENTS

In a two-dimensional Cartesian co-ordinate system, the target motion model is described by the following linear discrete-time difference equation with additive noise that models unpredictable disturbances [1, 2]:

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + G_k \mathbf{v}_k$$

where the state $\mathbf{x}_k = [x_k \; y_k \; \dot{x}_k \; \dot{y}_k]^T$ consists of the position and the velocity of a moving target, and process noise $\mathbf{v}_k$ is assumed to be white and zero-mean with covariance

$$E[\mathbf{v}_k \mathbf{v}_k^T] = Q_k .$$

The target is tracked by radar on the origin and the sensor measures range r and azimuth θ of the target as shown in Fig. 1.

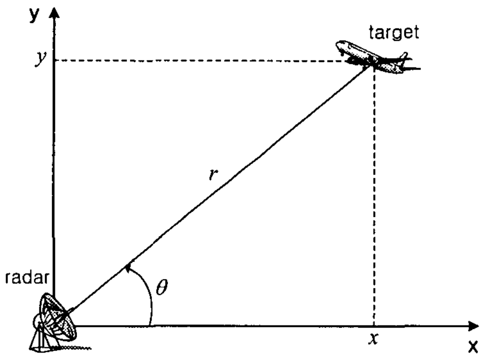


Fig. 1 Configuration of radar tracking

The measurement equation is described by the following nonlinear discrete equation

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{w}_k = \begin{bmatrix} (x_k^2 + y_k^2)^{1/2} \\ \tan^{-1}(y_k / x_k) \end{bmatrix} + \begin{bmatrix} w_r \\ w_\theta \end{bmatrix} \quad (1)$$

where the measurement noise w_r and w_θ are assumed to be white, Gaussian, mutually uncorrelated, and zero-mean with covariance

$$R_k = \mathrm{diag}\{\sigma_r^2,\ \sigma_\theta^2\}.$$

The basic task of this paper is to optimize the TS fuzzy model to be used in the linearization of nonlinear function and to estimate as accurately as possible the target trajectory from the radar measurements using the optimized TS fuzzy model.

III. FUZZY MODEL BASED KALMAN FILTER

A. EKF

In the EKF, the nonlinear measurement function of (1) is approximated as follows:

$$\mathbf{h}(\mathbf{x}_k) \approx \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}) + H_k(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}) \tag{2}$$

where H_k is the Jacobian of $\mathbf{h}(\cdot)$ evaluated at the predicted state estimate $\hat{\mathbf{x}}_{k|k-1}$:

$$H_k = \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k|k-1}} = \begin{bmatrix} \cos\bar{\theta}_k & \sin\bar{\theta}_k & 0 & 0 \\ -\dfrac{\sin\bar{\theta}_k}{\bar{r}_k} & \dfrac{\cos\bar{\theta}_k}{\bar{r}_k} & 0 & 0 \end{bmatrix}$$

where the prediction of range $\bar{r}_k$ and azimuth $\bar{\theta}_k$ are defined by

$$\bar{r}_k = (\hat{x}_{k|k-1}^2 + \hat{y}_{k|k-1}^2)^{1/2}$$

$$\bar{\theta}_k = \tan^{-1}(\hat{y}_{k|k-1} / \hat{x}_{k|k-1})$$

The EKF algorithm using Jacobian linearization is summarized as follows:

State prediction and its covariance

$$\hat{\mathbf{x}}_{k|k-1} = F_{k-1}\hat{\mathbf{x}}_{k-1|k-1} \tag{3}$$

$$P_{k|k-1} = F_{k-1}P_{k-1|k-1}F_{k-1}^T + G_{k-1}Q_{k-1}G_{k-1}^T \tag{4}$$

Kalman Gain

$$K_k = P_{k|k-1}H_k^T[H_kP_{k|k-1}H_k^T + R_k]^{-1} \tag{5}$$

Updated state estimate and its covariance

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})] \tag{6}$$

$$P_{k|k} = [I - K_kH_k]P_{k|k-1} \tag{7}$$

B. FMBKF

The FMBKF uses a local model approximation based on a TS fuzzy model instead of a Jacobian matrix to linearize nonlinear measurement equation. The j th $(j = 1\cdots n)$ TS fuzzy rule for each nonlinear measurement equation is as follows:

IF χ_1 is A_1^j and χ_2 is A_2^j, THEN $y = p_1^j\chi_1 + p_2^j\chi_2$

where input vector $\bar{\mathbf{x}}_k$ is the measured position in each axis as follows:

$$\begin{aligned}\bar{\mathbf{x}}_k &= [\chi_1\ \chi_2]^T = [x_k^m\ y_k^m]^T \\ &= [r_k^m\cos\theta_k^m\ \ r_k^m\sin\theta_k^m]^T\end{aligned} \tag{8}$$

y means the output of each nonlinear measurement equation and is represented in the form of linear combination of $\bar{\mathbf{x}}_k$. A_i^j $(i = 1,2)$ is the Gaussian membership function with membership grade $\mu_i^j(x_i)$ described as

$$\mu_i^j(x_i) = \exp\left[-\frac{1}{2}\left(\frac{x_i - c_i^j}{\sigma_i^j}\right)^2\right] \tag{9}$$

where c_i^j and σ_i^j are the center and the standard deviation of the Gaussian membership function respectively.

By product inference and weighted average defuzzification, the output of this fuzzy system y for the input $\bar{\mathbf{x}}_k$ is obtained as

$$y = \frac{\sum_{j=1}^{n}[\mu_1^j(x_1)\mu_2^j(x_2)]\cdot[p_1^jx_1 + p_2^jx_2]}{\sum_{j=1}^{n}[\mu_1^j(x_1)\mu_2^j(x_2)]}$$

Let β^j be

$$\beta^j = \frac{[\mu_1^j(x_1)\mu_2^j(x_2)]}{\sum_{j=1}^{n}[\mu_1^j(x_1)\mu_2^j(x_2)]}$$

then

$$\begin{aligned}y &= \sum_{j=1}^{n}\beta^j[p_1^jx_1 + p_2^jx_2] \\ &= \sum_{j=1}^{n}[\beta^jp_1^jx_1 + \beta^jp_2^jx_2]\end{aligned}$$

Thus the nonlinear measurement function $\mathbf{h}(\mathbf{x}_k)$ is approximated in the FMBKF as follows:

$$\mathbf{h}(\mathbf{x}_k) \approx [h_1^j(\bar{\mathbf{x}}_k)\ h_2^j(\bar{\mathbf{x}}_k)]^T = \sum_{j=1}^{n}\mathrm{H}_k\bar{\mathbf{x}}_k \tag{10}$$

where $h_l^j(\bar{\mathbf{x}}_k)$ means the l th $(l = 1,2)$ measurement equation of $\mathbf{h}(\mathbf{x}_k)$ for j th fuzzy rule and H_k is the measurement matrix denoted as follows.

$$\mathrm{H}_k = \begin{bmatrix} \beta_1^jp_{11}^j & \beta_1^jp_{12}^j & 0 & 0 \\ \beta_2^jp_{21}^j & \beta_2^jp_{22}^j & 0 & 0 \end{bmatrix} \tag{11}$$

where β_l^j means the value of β^j for the l th measurement equation of $\mathbf{h}(\mathbf{x}_k)$.

Finally, we can rewrite the measurement equation in the following form

$$\begin{aligned}\mathbf{z}_k^F &= \sum_{j=1}^{n}\mathrm{H}_k\mathbf{x}_k + \mathbf{w}_k \\ &= \sum_{j=1}^{n}\begin{bmatrix} \beta_1^jp_{11}^j & \beta_1^jp_{12}^j & 0 & 0 \\ \beta_2^jp_{21}^j & \beta_2^jp_{22}^j & 0 & 0 \end{bmatrix}\mathbf{x}_k + \mathbf{w}_k\end{aligned} \tag{12}$$

Now, we can apply the standard Kalman filter algorithm and the FMBKF algorithm is as follows:

State prediction and its covariance

$$\hat{\mathbf{x}}_{k|k-1} = F_{k-1}\hat{\mathbf{x}}_{k-1|k-1} \tag{13}$$

$$P_{k|k-1} = F_{k-1}P_{k-1|k-1}F_{k-1}^T + G_{k-1}Q_{k-1}G_{k-1}^T \tag{14}$$

Kalman Gain

$$K_k^F = \sum_{j=1}^{n}(P_{k|k-1}H_k^{j^T})[\sum_{j=1}^{n}(H_k^j P_{k|k-1}H_k^{j^T}) + R_k]^{-1} \tag{15}$$

Updated state estimate and its covariance

$$\hat{\mathbf{x}}_{k|k}^F = \hat{\mathbf{x}}_{k|k-1} + K_k[\mathbf{z}_k - \sum_{j=1}^{n}(H_k^j \hat{\mathbf{x}}_{k|k-1})] \tag{16}$$

$$P_{k|k}^F = [I - \sum_{j=1}^{n}(K_k H_k^j)]P_{k|k-1} \tag{17}$$

C. Identification of TS fuzzy model

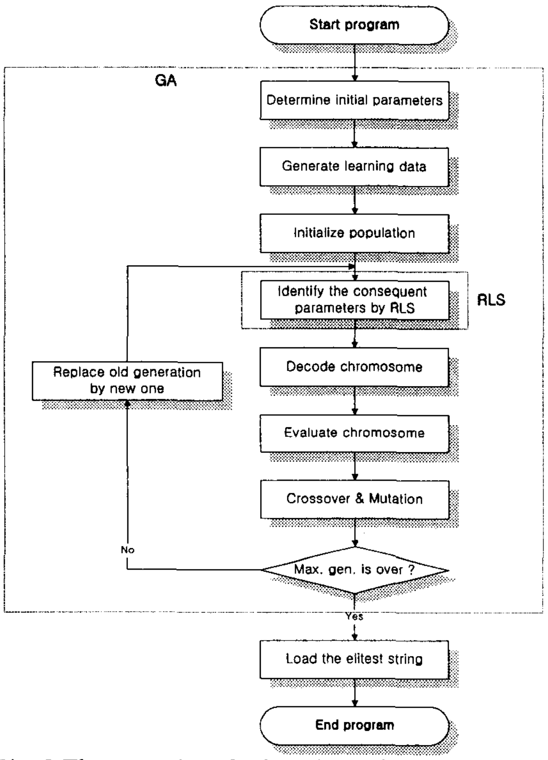


Fig. 2 The procedure for learning using the hybrid GA and RLS method

In this paper, the hybrid GA and RLS method is used to identify the TS fuzzy model. The GA is used to identify the premise parameters and the rule numbers of TS fuzzy model, and the RLS method is used to identify the consequent parameters. The proposed hybrid GA and RLS method is shown in Fig. 2 and is summarized as follows [3-5].

Step 1: Set the parameters for the GA (maximum generation number, maximum rule number, population size, crossover rate, and mutation rate).

Step 2: Randomly generate the initial population such that all searching variables exist within the search space.

Step 3: Identify the consequent parameters by RLS

Step 4: Decode the chromosome of each individual in the population and determine the fuzzy systems for sub-models. Evaluate the determined fuzzy systems by (6) and give a fitness value to each individual in the population by (7).

Step 5: Evolve a new population by reproduction, crossover, and mutation.

Step 6: Increase the generation number by one, and replace the old generation with the new one. During the replacement, preserve an individual that has the maximum fitness value by the elitist reproduction.

Step 7: Repeat Steps 3 through 6 until one of the following is satisfied:
(1) the satisfactory population shows up,
(2) the generation number reaches the maximum generation number, or
(3) the fitness function value is not increased for the predetermined generations.

The GA represents the searching variables of the given optimization problem as a chromosome containing one or more sub-strings. In this case, the searching variables are the center c_i^j and the standard deviation σ_i^j for a Gaussian membership function of the fuzzy set A_i^j and the consequent parameter p_i^j. A convenient way to convey the searching variables into a chromosome is to gather all searching variables associated with the j th fuzzy rule into a string and to concatenate the strings as

$$S_j = \{c_1^j,\ \sigma_1^j,\ c_2^j,\ \sigma_2^j,\ p_1^j,\ p_2^j\}$$

$$S = \{S_1,\ S_2,\ \cdots,\ S_M\}$$

where S_j is the real coded parameter sub-string of the j th fuzzy rule in an individual S. At the same time and to identify the number of fuzzy rules, we utilize the binary coded rule number string, which assigns a 1 or 0 for a valid or invalid rule, respectively.

The fitness of the individual is determined in inverse proportion to the square error (SE) and the number of rules.

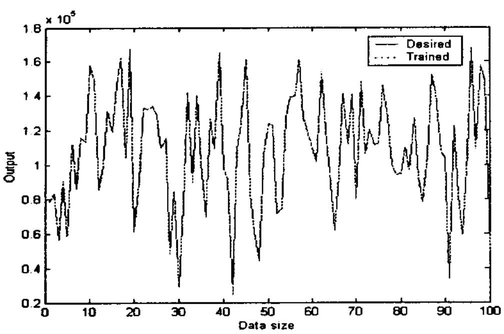
$$\text{fitness} = \lambda\frac{1}{\text{SE}+1} + (1-\lambda)\frac{1}{\text{rule number}+1} \tag{18}$$

where λ means the relative value between MSE and rule number.

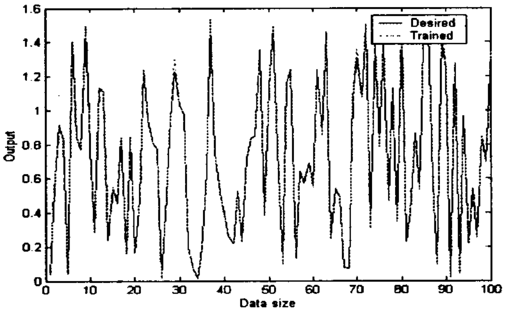
IV. SIMULATION RESULTS

Computer simulation was divided by two parts-one was simulation for offline optimization of TS fuzzy model and the other was the Monte Carlo simulation for radar tracking using the optimized TS fuzzy model.

Figure 3 shows the training results using the hybrid GA and RLS method.



(a) 1st function ($(x_k^2 + y_k^2)^{1/2}$)



(b) 2nd function ($\tan^{-1}(y_k / x_k)$)

Fig. 3 Training results

Incoming target was considered and the initial state of the target was

$$[150km \quad 120km \quad -125m \quad -125m]^T,$$

and the standard deviation of process noise was set at $0.5m/s^2$ for each axis, and those of measurement noises were assumed to be $50m$ and 2° for range and azimuth, respectively. The radar is located at the origin and measures the tracker-to-target range and azimuth at a sampling time $1\,s$.

The result for a Monte Carlo simulation of 200 runs is shown in Fig. 4.

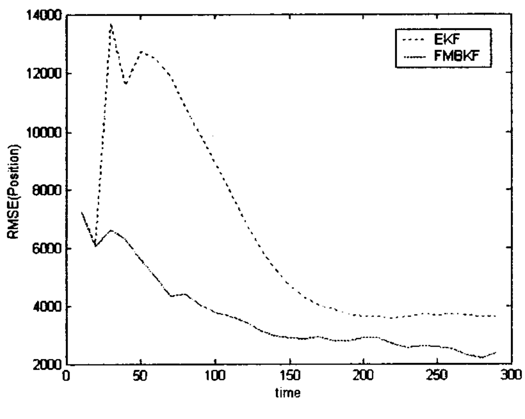


Fig. 4 Simulation result

V. CONCLUSIONS

In this paper, we proposed the fuzzy-model-based Kalman filter (FMBKF) using a local model approximation based on a TS fuzzy model. The hybrid GA and RLS method was used to identify the premise and the consequent parameters and the rule numbers of the TS fuzzy model. The proposed FMBKF was applied and simulated in two-dimensional radar tracking problem. The simulation results have shown that the FMBKF had much superior performance to the conventional EKF.

ACKNOWLEDGEMENT

This work was supported by KOSEF R01-2001-000-00316.

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