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A Rule Merging Method for Fuzzy Classifier Systems and Its Applications to Fuzzy Contorl Rules Acquisition

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Abstract— **This paper proposes a fuzzy classifier system (FCS) using hyper-cone membership functions (HCMFs) and rule reduction techniques. The FCS can generate excellent rules which have the best number of rules and the best location and shape of membership functions. The HCMF is expressed by a kind of radial basis function, and its fuzzy rule can be flexibly located in input and output spaces. The rule reduction technique adopts a decreasing method by merging the two appropriate rules. We applay the FCS to a fuzzy rule generation for the inverted pendulum control.**

Keywords— **Fuzzy Classifier System, Fuzzy Rule Generation, Genetic Algorithm, Inverted Pendulum.**

I. Introduction

Fuzzy classifier systems (FCSs) applying the Michigan approach type genetic algorithms (GAs) are proposed [1],[2]. These methods are composed of compact systems, because GA is done in one fuzzy system. Also, it is considered that these can be applied to on-line learning of autonomous robots, etc.. However, these methods are fixed membership functions. Therefore, these are methods which choose necessary fuzzy rules among the large number of rules without tuning of the membership functions.

A purpose of our study is the development of the automatic generation technique of fuzzy rules by FCSs with the decision of the rule number, location and rule shape. We had presented an automatic generation method of fuzzy rules using hyper-cone membership functions (HCMFs) by FCS [3]. The HCMF[4],[5] is expressed by a kind of radial basis function, and its fuzzy rule can be flexibly located in input and output spaces. In this method, however, the number of rules is fixed, and it is not the optimum rule number. Also, the similer rules exist in fuzzy systems.

In this paper, a rule reduction technique using rule merging is introduced in order to choose the optimum rule number. We applay these methods to fuzzy rules generation of the inverted pendulum control.

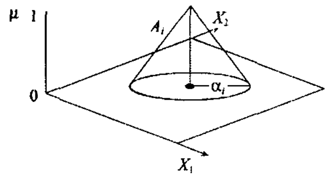

Fig. 1. Shape of hyper-cone membership function ($l = 2$)

II. Fuzzy Classifier System Using Hyper-Cone Membership Functions

A. *Fuzzy Rules Using HCMFs*

In this method, we give fuzzy rule R^i as below:

$$R^i : \text{if } \boldsymbol{x} \text{ is } A_i \text{ then } \boldsymbol{y} \text{ is } B_i,\ i = 1, 2, \cdots, n \quad (1)$$

where i is rule number, n is the number of rules, $\boldsymbol{x}$ and $\boldsymbol{y}$ are the input and output vectors, respectively, and A_i and B_i are fuzzy subsets. In this method, fuzzy subsets A_i and B_i are defined by a hyper-cone membership function (HCMF). In this fuzzy system, since there are n fuzzy rules, n HCMFs are located in input and output each space.

The HCMF $\mu_{A_i}(\boldsymbol{x})$ of A_i is defined by Eqs.(2) and (3).

$$\mu_{A_i} : A_i \rightarrow [0, 1] \quad (2)$$

$$\mu_{A_i}(\boldsymbol{x}) = \left(1 - \frac{\|\boldsymbol{x} - \boldsymbol{a}_i\|}{\alpha_i}\right) \vee 0 \quad (3)$$

where $\boldsymbol{a}_i$ and α_i are the center vector and the radius of the fuzzy subsets A_i. The membership function μ_{A_i} has a grade 1.0 at the center $\boldsymbol{a}_i \in \boldsymbol{R}^l$, and the membership value decreases in proportion to the distance from the center $\boldsymbol{a}_i$. Fig.1 shows the HCMF in case of $l = 2$ (l is the number of inputs). HCMF μ_{B_i} is defined in m dimensional output space by the same way as the input.

We find the reasoning result μ_{B^*}, defuzzy it, and calculate the real output value $\boldsymbol{y}^*$. The truth value ω_i of a rule R^i for input vector $\boldsymbol{x}^*$ is calculated by Eq.(3). In next step, we use the truth value ω_i to

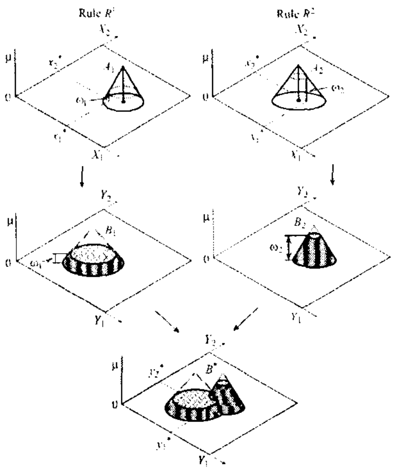

Fig. 2. An example of the reasoning method

define a $\mu_{B_i^*}(y)$ as shown in Eq.(4).

$$\mu_{B_i^*}(y) = \omega_i \wedge \mu_{B_i}(y) \quad (4)$$

We find the composite reasoning result $\mu_{B^*}(y)$ for each rule in Eq.(5).

$$\mu_{B^*}(y) = \bigvee_{i=1}^{n} \mu_{B_i^*}(y) \quad (5)$$

In the final step, output y^* is given by the center of gravity of the membership function $\mu_{B^*}(y)$.

$$y^* = \frac{\int_{D_y} \mu_{B^*}(y) y dy}{\int_{D_y} \mu_{B^*}(y) dy} \quad (6)$$

Fig.2 shows an example of the reasoning in the case of $l = 2,\ m = 2,\ n = 2$.

B. Fuzzy Classifier System Using HCMFs

FCS consists of four blocks as shown in Fig.3. In this method, fuzzy systems using HCMFs are used. Fundamental operations of FCS are as follows.

[Fuzzy Rule Base]

There existed n fuzzy rules based on HCMFs.

[Fuzzy Inference System]

Senses (inputs) are received from an environment, and fuzzy rules which suit these senses are chosen from the Fuzzy Rule Base. Then, fuzzy reasoning described in before section is carried out from those rules, and actions (outputs) to the environment are decided.

[Apportionment of Credit System]

In the Apportionment of Credit System, rewards are provided from the environment to the FCSs for

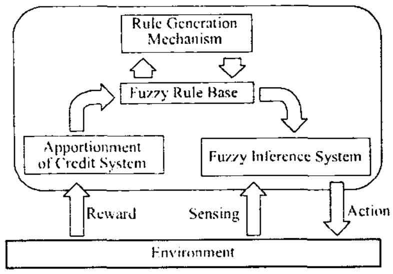

Fig. 3. Fuzzy Classifier System

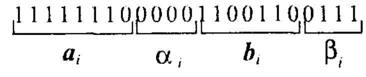

Fig. 4. Example of chromosome

actions. In other words, reward is an evaluation for the fuzzy system. Also, rewards are distributed to each rule as a credit. The credit is an evaluation of each rule, and higher the rule contributes to obtain the reward, higher the evaluation of the rule increases. In this method, the credit cf_i of each rule is provided following method. When there are J actions in one trial, a reward $re_j (j = 1, 2, \ldots, J)$ is given in each action. The credit $cf_i (i = 1, 2, \ldots, n)$ is given from the reward in proportion to the truth value of the rule as follows:

$$cf_i = \sum_{j=1}^{J} \frac{\mu_{ij}}{g_j} \times re_j \quad (7)$$

$$g_j = \sum_{i=1}^{N} \mu_{ij} \quad (8)$$

where μ_{ij} is the truth value of fuzzy rule R^i in reasoning action (output) j.

[Rule Generation Mechanism]

New rules are generated based on credit cf_i by GA. A coding of one rule in this GA is done as one individual. Therefore, each credit is given a fitness of each individual. Also, population is composed of n individuals because there are n fuzzy rules. Genetic parameters of fuzzy rule R^i are as follow

- Center coordinate a_i of fuzzy subset A_i,
- Radius α_i of fuzzy subset A_i,
- Center coordinate b_i of fuzzy subset B_i, and
- Radius β_i of fuzzy subset B_i.

These parameters are coded as one chromosome like Fig.4.

This method is used genetic operations (see Fig.5) based on the simple GAs. The procedure of genetic operations is as follows:

[Step1] An initial population is randomly produced. Also, the fitness (the credit cf_i) of each individual

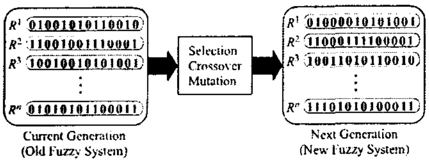

Fig. 5. Genetic operations

is calculated.

[Step2] We produce the population of the next generation by following operations until a population size n is completed.

Selection: Two individuals (rules) are selected by the roulette wheel model.

Crossover: Two individuals cross each other. This method is used one point crossover.

Mutation: Each gene is mutated by a mutation rate.

Reproduction: Place new offspring in a new population.

[Step3] Carrying out the new fuzzy system, and the credit cf_i of each rule is calculated.

[Step4] If a prespecified stopping condition is not satisfied, return to Step2.

III. Rule Reduction Technique Using Rule Merging

In the above method, a generated fuzzy system does not always have the optimum rule number because the number of rules is fixed. Also, the similer rules seem to exist. In this study, a technique for reductions of the rule number by rules merging in every certain generation is introduced.

[Step1] The fuzzy reasoning is curried out using n rules during $G-1$ generations from initial generation.

[Step2] In G-th generation, the following operations are carried out.

[Step2-1] Similar rules are merged.

[Step2-2] Fuzzy reasoning is carried out based on merged rules, and rewards are given. The credits are apportioned these rules.

[Step2-3] Individuals of next generation are made based on these rules by GA.

[Step3] For the next $G-1$ generations, the operation same as Step1 is done. In $2G$-th generation, Step2 is done.

Next, conditions for two rules merging in Step2-1 is as follows:

1) The merging is not executed, if output membership functions of two rules are located in the positive and the negative regions in output space.

2) The merging is not executed, if two input membership functions of two rules don't include the center of each membership function (See Fig.6).

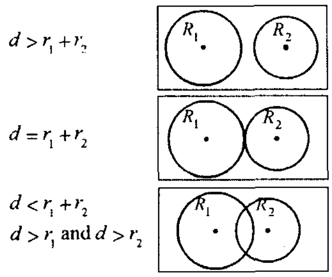

Fig. 6. Not Merging Condition

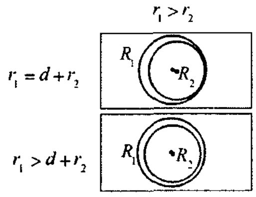

Fig. 7. Absorbing Condition

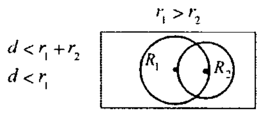

Fig. 8. Merging Condition

3) The bigger membership function absorbs the smaller one, if a membership function completely include another membership function (See Fig.7).

4) The merging is executed, if a membership function is include the center of another membership function. (See Fig.8).

In Figs. 6 - 8, d is the distance between two input membership functions of two rules, and r_1 and r_2 are radiuses. When (x_1, y_1) and (x_2, y_2) are center coordinates of two membership function, the radius r of new membership functions and its center coordinate (X, Y) are calculated by Eqs. (9) - (11).

$$r = \frac{\frac{r_1}{2} + \frac{r_2}{2} + d}{2} \tag{9}$$

$$X = \frac{\frac{r_1(x_1 - x_2)}{2d} + \frac{r_2(x_2 - x_1)}{2d} + x_1 + x_2}{2} \tag{10}$$

$$Y = \frac{\frac{r_1(y_1 - y_2)}{2d} + \frac{r_2(y_2 - y_1)}{2d} + y_1 + y_2}{2} \tag{11}$$

The rule merging procedure is executed as long as rule pair satisfies these conditions exists.

IV. Application to the Inverted Pendulum Control

A. Simulation Model of the Inverted Pendulum

We generated fuzzy rule for the inverted pendulum system by the presented method. Fig.9 is showed its simulation model. In this system, the objective is to control the translational forces in order to position the cart at the center of finite width

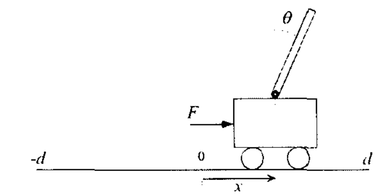

Fig. 9. Model of Inverted Pendulum

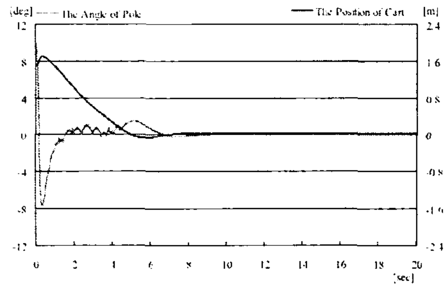

Fig. 10. Simulation result (θ_0, x_0) = (10.0, 1.5)

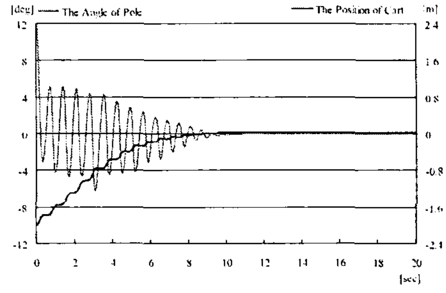

Fig. 11. Simulation result (θ_0, x_0) = (12.0, -2.0)

rail while balancing the pole on the cart simultaneously.

The state variables of the inverted pendulum are following:
θ: The angle of pole for verticality [deg],
$\dot{\theta}$: The angular velocity of pole [deg/s],
x: The distance of the cart from center [m],
$\dot{x}$: The velocity of cart [m/s].
In generated fuzzy systems, inputs are θ, $\dot{\theta}$, x and $\dot{x}$, and output is the force added to the cart F[N].

B. Simulation Result

Parameters of the inverted pendulum were set as follow: the length of pole was 0.5 m, weights of cart and pole were 1.0 kg and 0.1 kg, and d was 2.4 m.

We set the number of rules (the population size) $n = 20$. The crossover rate was 25%, the mutation rate was 3.0%, and the number of generation was 10000. The credit cf_i was given by evaluations of four initial positions ((θ_0, x_0) = (-10.0, -1.5), (-10.0, 1.5), (10.0, -1.5) and (10.0, 1.5)). The mergin parameter G was set in 10. Therefore, the mergin operatopm was carried out every 10 generations. Also, the reward re_j is given in the following equation.

$$re_j = (12.0 - |\eta_j|) + (2.4 - |x_j|) \qquad (12)$$

We tried 20 times for different initial populations, and obtained fuzzy rule sets. The rule number decreases in two by mergin method 12 times within 20 simulations. Figs.10, 11 show simulation results of fuzzy rule sets (the number of rules: 2) obtained by the FCS with mergin method. From these figures, our proposed method could control the inverted pendulum in such a way that the pole would not fall, and the angle of the pole has been converged on 0. We also applied FCS without the mergin method under same condition. However, in many cases, fuzzy rules which the control from some initial positions failed were obtained. On the other hand, FCS with the mergin method is obtained compact and effective fuzzy systems.

V. Conclusions

In this paper, we presented an automatic generation technique for fuzzy rules using hyper-cone membership function by FCS. Also, we introduced the rule merging method in presented FCS in order to reduce the number of rules. These methods were applied in the inveted pendulum control. These simulation results using obtained fuzzy rules showed skillful performance. Also, in rule merging method, the compact fuzzy system with small rule number was obtained.

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