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The Triple I Method for Fuzzy Reasoning

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Abstract— A new method, the Triple I method, is proposed for solving the problem of fuzzy reasoning. The Triple I method for solving fuzzy modus ponens is compared with the CRI method, i.e., Compositional Rule of Inference, and reasonableness of the Triple I method is clarified. Moreover, the Triple I method can be generalized to provide a theory of sustentation degrees. Lastly, the Triple I method can be bring into the framework of classic logics.

Fuzzy Modus Penens (briefly, FMP) is the foundamental form of fuzzy reasoning. FMP answers the question of how to deduce a conclusion B^* when a deduction rule " A implies B" is known and an input A^* similar to A is given. That is

suppose that	$A \quad \to B$	
and given	A^*	
calculate		B^*

The most famous method for solving the question of FMP is the CRI (Compositional Rule of Inference) method proposed by L.A.Zadeh[2]. The basic idea of CRI method is as follows:

(i) Choose X and Y as discourses of universe and transform A, A^* and B, B^* to be fuzzy subsets of X and Y respectively.

(ii) Transform the deduction rule $A \to B$(major premise) to be a fuzzy relation $R(A(x), B(y))$ on $X \times Y$ by means of certain implication operator $R : [0,1]^2 \to [0,1]$.

(iii) Define the conclusion B^* to be the composition of the minor premise A^* and $R(A(x), B(y))$, i.e., $B^*(y) = \sup\{A^*(x) \circ R(A(x), B(y)) | x \in X\} (y \in Y)$, where "$\circ$" usually denotes minimum.

CRI method has been accepted and popularly used in dealing with fuzzy control problems. But CRI method seems differ from the methodology of Artificial Intelligence (briefly, AI), because AI emphasizes symbolic manipulation and rooted in logic, and very much neglected anything pertaining to "number crunching", and hence it seems that there is a **gap** between FMP and AI[1]. The aim of the present paper is to try to provide a new method, which will be called the Triple I method, for solving the question of FMP so that the above mentioned gap can be filled up in certain sense. The main points of the Triple I method are as follows.

(1) The procedure (i) of CRI method is preserved but instead of (ii) and (iii) of CRI method the concept of tautology in logic is employed. The basic idea of the Triple I method is that the conclusion B^* should be deduced from both $A \to B$ and A^* jointly but not separately, i.e., based on the major premise $A \to B$ the minor premise A^* implies the conclusion. In other words, $A \to B$ should fully support the fact that $A^* \to B^*$ or in logic terminology, $(A \to B) \to (A^* \to B^*)$ should be a tautology. Precisely, $(A(x) \to B(y)) \to (A^*(x) \to B^*(y)) = 1$ holds for every pair $(x, y) \in X \times Y$ and B^* is the smallest fuzzy subset of Y.

(2) The implication operator $\rightarrow$ used in the Triple I method is always accompanied by a corresponding t-norm $\otimes$ such that $a \otimes b \leq c$ if and only if $a \leq b \rightarrow c$. We say that $(\otimes, \rightarrow)$ is an adjoint pair, or briefly, $\rightarrow$ is adjoint. Diverse adjoint pairs can be used to solve the question of FMP, and the conclusion B^* obtained by means of the Triple I method possesses a universal form as follows:

$$B^*(y) = \sup\{A^*(x) \otimes (A(x) \rightarrow B(y))\}(y \in Y). \quad (1)$$

For example, $\otimes$ can be any one of the following t-norms:

$$a \otimes_L b = (a + b - 1) \vee 0,$$
$$a \otimes_G b = a \wedge b,$$
$$a \otimes_\Pi b = ab,$$
$$a \otimes_0 b = \begin{cases} a \wedge b, & a + b > 1, \\ 0, & a + b \leq 1, \end{cases}$$

And the corresponding $\rightarrow$ are as follows:

$$a \rightarrow_L b = (1 - a + b) \wedge 1,$$
$$a \rightarrow_G b = \begin{cases} 1, & a \leq b, \\ b, & a > b, \end{cases}$$
$$a \rightarrow_\Pi b = \begin{cases} 1, & a = 0, \\ \frac{b}{a} \wedge 1, & a > 0, \end{cases}$$
$$a \rightarrow_0 b = \begin{cases} 1, & a \leq b, \\ (1 - a) \vee b, & a > b. \end{cases}$$

They are called Łukasiweicz operator, Gödel operator, Product operator, and R_0-operator respectively.

(3) It is proved that $a \rightarrow b = 1$ if and only if $a \leq b$ whenever $\rightarrow$ is adjoint, and the popularly used implication operators are the Łukasiewicz operator R_L, the Gödel operator R_G, the product operator R_Π, and the operator R_0.

(4) The Triple I solution possesses reversibility property whenever A is normal and R_0 is used, that is, if the input A^* equals A, and there exists $x_0 \in X$ such that $A(x_0) = 1$, then the output B^* equals B. That is

suppose that	A	$\rightarrow$	B
and given	A		
then obtain			B

(5) The Triple I method is easy to be generalized to form the so called sustentation degree theory, i.e., when a positive number $\alpha \leq 1$ is given, the smallest fuzzy subset B^* of Y satisfying

$$(A(x) \rightarrow B(y)) \rightarrow (A^*(x) \rightarrow B^*(y)) \geq \alpha \quad x \in X, y \in Y$$

can be calculated and

$$B^*(y) = \sup\{\alpha \otimes A^*(x) \otimes (A(x) \rightarrow B(y)) | x \in X\}, y \in Y. \quad (2)$$

It is clear that formula (2) turns to be (1) whenever $\alpha = 1$.

(6) The Triple I method can be transplanted in multiple-valued logic, even it can be transplanted in classic mathematical logic.

References

[1] D.Dubois, H.Prade, Fuzzy sets in approximate reasoning, part I, Fuzzy Sets and Systems, 40, 143-202,1991.

[2] L.A.Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Trans. Systems. Man, Cybernet., 3, 28-44,1973.