2003 International Symposium on Advanced Intelligent Systems
September 25-28, 2003, Jeju, Korea

Incomplete Information Recognition Using Fuzzy Integrals Aggregation: With Application to Multiple Matchers for Image Verification

Seong H. Kim* and M. Kamel

Systems Design Engineering, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1

email: well48@hanmir.com (*corresponding author)

Abstract - In the present work, a main purpose is to propose a fuzzy integral-based aggregation framework to complementarily combine partial information due to lack of completeness. Based on Choquet integral (CI) viewed as monotone expectation, we take into account complementary, non-interactive, and substitutive aggregations of different sources of defective information. A CI-based system representing upper, conventional, and lower expectations is designed for handling three aggregation attitudes towards uncertain information. In particular, based on Choquet integrals for belief measure, probability measure, and plausibility measure, CI_{bl}-, CI_{pr}-, and CI_{pl}-aggregator are constructed, respectively. To illustrate a validity of proposed aggregation framework, multiple matching systems are developed by combining three simple individual template-matching systems and tested under various image variations. Finally, compared to individual matchers as well as other traditional multiple matchers in terms of an accuracy rate, it is shown that a proposed CI-aggregator system, {CI_{bl}-aggregator, CI_{pr}-aggregator, CI_{pl}-aggregator}, is likely to offer a potential framework for either enhancing completeness or for resolving conflict or for reducing uncertainty of partial information.

Keywords: Fuzzy system, uncertainty reduction; fuzzy integral; information fusion; template matching

I. Introduction

Since fuzzy integrals defined by fuzzy measures have been introduced [Sugeno 1977], fuzzy integrals associated with fuzzy measures have been applied to human reasoning systems for handling uncertain information, particularly incomplete information due to fuzziness. It is partly because fuzzy measures can deal with an interaction among partial information sources and partly because fuzzy integrals can generate global information that is more complete, resolve conflict among different sources of partial information, or increase reliability of defective information via complementary aggregation, non-interactive aggregation, and substitutive aggregation [Bloch and Hunter 2001].

Generally, according to aggregation operator properties [Marichal 2000], it is known that Choquet integrals are appropriate for aggregating a cardinal scale such as a grade of similarity between two objects. Moreover, Choquet integral with respect to an arbitrary fuzzy measure is viewed as an expectation by the fuzzy measure. Thus, this integral is sometimes called monotone expectation [Bolanos et al. 1989]. A CI-based aggregation system representing conventional, lower, and upper expectations can take into account those three aggregation attitudes towards uncertain information. More precisely, CI for belief measure (i.e., possibility expectation) may be used as an optimistic aggregation, CI for plausibility measure (i.e., necessity expectation) as a pessimistic aggregation, and CI for probability measure (i.e., conventional expectation) as an additive non-interacting aggregation.

In the fields of pattern recognition/analysis and image understanding [2;3;6;11;13;16], multiple criteria decision-making [8;9], and fuzzy logic control [10], as far as CI aggregation is concerned, several works available to us are obviously focused on Choquet integrals for Sugeno measure (i.e., λ-fuzzy measure) or for probability measure. From a practical standpoint, it is worthwhile to develop a CI-based framework encompassing three aggregation attitudes to deal with incomplete image information caused by temporal or spatial changes. To illustrate a validity of the aggregation, a multiple matching system is developed by combining different individual template matching systems.

This work is organized as follows: Section II briefly describes theoretical background and practical interpretation of aggregations. Section III is devoted to specific application of three CI-aggregators to an object verification problem using template matching. Finally, conclusive remarks and a research direction in a near future are mentioned in Section IV.

II. Aggregation Attitude

Given a finite discrete whole set $X = \{x_1, x_2, \ldots, x_n\}$, for any A, $B \subseteq X$ and $A \cap B = \phi$, a probability $P: 2^X \to [0,1]$ has the following properties: $P(\phi) = 0$, $P(X) = 1$, $P(A \cup B) = P(A) + P(B)$.

A belief measure and a plausibility measure can be represented in terms of a basic probability assignment (BPA). BPA is a set function $m(A): 2^X \to [0,1]$. Here $m(\phi) = 0$ and $\sum_{A \subseteq X} m(A) = 1$.

A belief measure for $\forall A \subseteq X$ is defined as $Bel(A) = \sum_{B \subseteq A \subseteq X} m(B)$. A plausibility measure as a duality of a belief measure is defined as $Pl(A) = \sum_{B \cap A \neq \phi} m(B)$. It is known that belief and plausibility measures are subsets of lower probability and upper probability measures, respectively [Shafer 1976].

In case focal elements are nested (i.e., consonant or ordered by the inclusion relation of sets), plausibility and belief measures become possibility and necessity measures, respectively [Wang 1991]. Here, a focal element is a set A such that $m(A) > 0$. Namely,

$$Pl(A \cup B) = max(Pl(A), Pl(B)) \text{ and}$$

$$Bel(A \cap B) = min(Bel(A), Bel(B)).$$

Fuzzy measures represent relevance degrees of partial sources yielding subjective evidence.

Suppose that $h(\{x_{(1)}\}) \leq h(\{x_{(2)}\}) \leq \ldots \leq h(\{x_{(n)}\})$. For $A_{(i)} = \{x_{(i)}, x_{(i+1)}, \ldots, x_{(n)}\}$, $A_{(n+1)} = \phi$, CI with respect to any fuzzy measure g is defined as

$$CI(h_1, \ldots, h_n) = \sum_{i=1,\ldots,n} [g(A_{(i)}) - g(A_{(i+1)})] \cdot h(x_{(i)}).$$

As a particular case, CI with respect to probability measure, CI_{pr}, is equivalent to a weighted arithmetic mean operator. As for probability,

$$CI_{pr}(h_1, \ldots, h_n) = \sum_{i=1,\ldots,n} pr(\{x_{(i)}\}) \cdot h(x_{(i)})$$

Here $\sum_{i=1,\ldots,n} pr(\{x_{(i)}\}) = 1$, $pr(\{x_{(i)}\}) \geq 0$ for any i.

Assume that fuzzy measures be arranged as $0 \leq g(\{x_{(1)}\}) \leq g(\{x_{(2)}\}) \leq \ldots \leq g(\{x_{(n)}\}) = 1$. By dividing given fuzzy densities by a maximal fuzzy density, we obtain $\max_i g(\{x_{(i)}\}) = 1$ for any i. For nested focal elements $A_{(i)} \subseteq X$ such that $A_{(1)} \supseteq A_{(2)} \supseteq \ldots \supseteq A_{(n)}$, with $g(\{x_{(0)}\}) = 0$, the BPA is presented by

$$m(A_{(i)}) = g(x_{(i)}) - g(x_{(i-1)}).$$

In addition, discrete upper expectation (UE) and lower expectation (LE) are defined by Lebesgues-Stieljes integrals [Grabisch et al. 1995] as follows:

$$UE = \sum m(A_{(i)}) \cdot \max_{k=i}^{n} h(x_{(k)})$$

and

$$LE = \sum m(A_{(i)}) \cdot \min_{k=i}^{n} h(x_{(k)}).$$

Discrete CI for belief measure, $CI_{bl}(h_1, \ldots, h_n)$, based on optimistic functions, is formulated as

$$CI_{bl} = \sum_{A_i \subseteq X} [g(x_{(i)}) - g(x_{(i-1)})] \cdot \max_{k=i}^{n} h(x_{(k)}).$$

Similarly, based on pessimistic functions, discrete CI for plausibility measure, $CI_{pl}(h_1, \ldots, h_n)$, is given as

$$CI_{pl} = \sum_{A_i \subseteq X} [g(x_{(i)}) - g(x_{(i-1)})] \cdot \min_{k=i}^{n} h(x_{(k)}).$$

In **Table 1**, according to three fuzzy measures, aggregator attitudes toward aggregation of partial sources are categorized.

Table 1 Attitudes towards aggregation of sources of incomplete evidence

Aggregator	Objective evidence source	Subjective evidence source	Interpretation	Aggregation attitude
CI_{bl}	Upper confidence	Belief measure	Possibility expectation	Optimistic; Complementary; Synergistic; Supporting
CI_{pr}	Regular	Probability measure	Conventional expectation	Neutral; Non-interactive; Additive
CI_{pl}	Lower confidence	Plausibility measure	Necessity expectation	Pessimistic; Substitutive; Redundant; Destructive

III. Application to Template Matching

To illustrate a validity of proposed aggregation framework, {CI_{bl}-aggregator, CI_{pr}-aggregator, CI_{pl}-aggregator}, multiple matching systems are developed by combining three simple individual template matching systems. Individual matchers are based on the maximum of absolute differences (MOAD), on the sum of absolute differences (SOAD), and on the sum of squared differences (SOSD). These matchers are characterized by using the intensity-based feature functions and distance-typed similarity functions [Aschwanden and Guggenbuehl 1992]. MOAD, SOAD, and SOSD distances can be described by orders of infinite, of one, and of two in the family of Minkowski metrics, respectively [Shafer and Rogers 1993].

Then, to evaluate the performance of the developed matching aggregators, computational experiments are performed using variation information of a set of thirty-one image data. These simulations are conducted under various image variations of brightness (B), contrast (C), rotation (R), scaling (S), and additive white Gaussian noise (N). Furthermore, results are compared to individual matchers as well as other traditional aggregation matchers such as arithmetic mean (AM), maximum (MX), minimum (MN), and weighed arithmetic mean (WAM). Additionally, other fuzzy integral-based aggregators are considered: CI for λ-measure (CI_λ), Sugeno integral for λ-measure (SI_λ), and Sugeno integral for probability measure (SI_{pr}).

Table 2 shows the good-matching average rate for the proposed CI-aggregators. The following findings can be addressed:

- CI_{pr}-aggregator shows the same performance as WAM-aggregator for all variations of interest.
- CI_{bl}-aggregator outperforms the best individuals (i.e., SOAD-matchers) under R and S variations. It seems likely to become more complete information source. For B, C, and N variations, CI_{bl}-aggregator intelligently behaves by switching to the best individuals (i.e., SOAD-matchers). In other words, the CI_{bl} combines the behavior of the best individuals for B and C variations (i.e., MOAD-matchers) and of the best individual for N variation (i.e., SOSD-matcher). It seems to resolve a detection conflict between MOAD- and SOSD-matcher under B, C, and N variations.
- CI_{pr}-aggregator outperforms the best individuals (i.e., SOAD-matchers) under R and S variations. For B variation, the CI_{pr} is inferior to the best individual (i.e., MOAD-matcher). For C and N variations, the CI_{pr} is superior to the best individuals (i.e., MOAD-matcher for C and SOSD-matcher for N).
- CI_{pl}-aggregator outperforms the best individuals under R, S, and C variations. For B and N variations, the CI_{pl} has inferiority to the best individuals.
- Partial sources of information under R and S variations are synergistically aggregated, even from a pessimistic viewpoint. Partial sources of information under C and N variations seem to be non-interacting since the CI_{pr} overwhelms both CI_{bl} and CI_{pl} with greater accuracy. Partial sources of information under B variation are redundantly aggregated, even from a neutral viewpoint.

As a result, in a global sense, CI-based {MOAD, SOAD, SOSD}-aggregation system with respect to belief, plausibility, and probability measures, especially CI_{bl}-aggregator, seems likely to establish a powerful framework to enhance the completeness of information.

Table 2 Average rate of different aggregators

Matcher	Variation				
	R	S	B	C	N
MOAD	0.335	0.403	**0.808**	**0.885**	0.206
SOAD	**0.758**	**0.819**	0.180	0.848	0.597
SOSD	0.745	0.781	0.290	0.874	**0.603**
AM	0.806	0.823	0.786	**0.899**	0.519
MX	0.581	0.526	**0.808**	0.874	0.416
MN	0.755	0.781	0.281	0.885	0.342
WAM	0.813	0.845	0.639	0.897	0.613
CI_{λ}	**0.819**	**0.855**	0.720	0.874	**0.777**
SI_{λ}	**0.829**	**0.855**	0.673	0.859	0.671
SI_{pr}	**0.777**	**0.784**	0.571	0.883	0.339
CI_{bl}	**0.823**	**0.865**	**0.808**	0.885	0.603
CI_{pr}	**0.813**	**0.845**	0.639	**0.897**	0.613
CI_{pl}	**0.806**	**0.823**	0.659	0.888	0.561

IV. Conclusion

Based on the notion of monotone expectations, a CI-aggregator framework, (CI_{bl}, CI_{pr}, CI_{pl}), is proposed for dealing with three types of attitude toward aggregation of different sources of partial information caused by uncertain circumstances.

According to the above-mentioned findings, the proposed CI-aggregators for template matching seem globally to enhance completeness of image information and to intelligently switch to the best individual matchers under different variations.

In a near future, we would extend the present idea to the second-level aggregator that results from aggregation of three members such as {CI_{bl}, CI_{pr}, CI_{pl}}.

Acknowledgements

The first author wishes to express his thanks to Dr. H. J. Lee for priceless assistance and to Ms. M. K. Kim for partial financial support.

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