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An Approach to PWM Controller Design for the Attitude Control of Artificial Satellites

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***Abstract*—This paper concerns a design technique of pulse-width-modulated (PWM) controller via the digital redesign. The digital redesign is a converting technique a well-designed analog controller into the equivalent digital one maintaining the property of the original analog control system in the sense of state-matching. The redesigned digital controller is again converted into PWM controller using the equivalent area principle. An example–the attitude control of artificial satellites is included to show the effectiveness of the proposed method.**

I. INTRODUCTION

In general, there exist two types of digital controllers: the pulse-amplitude-modulated (PAM) controller and the pulse-width-modulated (PWM) controller. The PWM controller, which produces a series of discontinuous pulses with a fixed amplitude and variable width, has become popular in industry. Interestingly, it is observed that the significance is greatly increased for the attitude control of the artificial satellite with on-off reaction jets because jet-engine produces on-off thrust, which can be modelled as PWM signals.

It is well known that the PWM control law can be converted from PAM controller by using equivalent area principle [6] under the assumption that the sampling time is small. Recently we have developed a intelligent digital redesign technique for Takagi-Sugeno (T-S) fuzzy systems [2]–[4]. It is anticipated that the synergetic merge of these two technique can be a promising design tool of PWM controller for nonlinear control system.

This paper proposes an PWM control law design technique for T-S fuzzy systems and its application to attitude control of nonlinear artificial satellites with jet-engine using intelligent digital redesign.

Section 2 briefly reviews T-S fuzzy systems. In Section 3, some mathematical model of the artificial satellite is discussed. The intelligent digital redesign is shown in Section 4. The PWM controller design technique is shown in Section 5. Section 6 includes an computer simulation–attitude control of 3-axis artificial satellites. Conclusions are drawn in Section 7.

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II. T-S FUZZY SYSTEMS

Most physical systems are quite complex in practice and have strong nonlinearities so that it is difficult, if not impossible, to build rigorous mathematical models [7]. Fortunately, a certain class of nonlinear dynamical systems can be expressed in some forms of either a linear mathematical model locally, or an aggregation of a set of linear mathematical models.

Consider a nonlinear dynamical system of the following fuzzy IF-THEN rules:

$$\begin{aligned} R^i : &\text{ IF } z_1(t) \text{ is about } \Gamma_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is about } \Gamma_n^i \\ &\text{THEN } \dot{x}_c(t) = A_i x_c(t) + B_i u_c(t) \end{aligned} \tag{1}$$

where R^i denotes the ith fuzzy inference rule, $z_h(t)$ is the premise variable, $\Gamma_h^i, i = 1, \ldots, q, h = 1, \ldots, n$, is the fuzzy set of the hth premise variable in the ith fuzzy inference rule.

Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this T-S fuzzy system (1) is described by

$$\dot{x}_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_c(t) + B_i u_c(t)) \tag{2}$$

in which

$$\omega_i(z(t)) = \prod_{h=1}^{n} \Gamma_h^i(z_h(t)), \qquad \theta_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{q} \omega_i(z(t))}$$

and $\Gamma_h^i(z_h(t))$ is the membership value of the hth premise variable $z_h(t)$ in Γ_h^i. Some basic properties of $\theta_i(t)$ are:

$$\theta_i(z(t)) \geq 0, \quad \sum_{i=1}^{q} \theta_i(z(t)) = 1 \tag{3}$$

III. MATHEMATICAL MODELS FOR ATTITUDE CONTROL OF ARTIFICIAL SATELLITES: UNDERACTUATED CASE AND DYNAMICS ONLY

The artificial satellite in circular orbit is expected to maintain local vertical and local horizontal orientation during normal mode operation. The nonlinear equations of motion in terms of components along the body-fixed control axes can be written as follows:

$$I\dot{\omega} + \omega \times I\omega = M$$

where $\omega = [\omega_x, \omega_y, \omega_z]^T$ is the body-axis components of the absolute angular velocity of the artificial satellite, $M = [M_x, M_y, M_z]^T$ is the moment vector, I is the moment inertia matrix. Hereafter, we assume the actuator for the z-axis is faulted. Then, if the rigid body axis frame that is coincident with the principal-axis reference frame, the above dynamic equation can be written as

$$\dot{\omega}_x = \frac{(I_y - I_z)}{I_x}\omega_y\omega_z + \frac{M_x}{I_x} \tag{4}$$

$$\dot{\omega}_y = \frac{(I_z - I_x)}{I_y}\omega_z\omega_x + \frac{M_y}{I_y} \tag{5}$$

$$\dot{\omega}_z = \frac{(I_x - I_y)}{I_z}\omega_x\omega_y \tag{6}$$

First, in order to construct the T-S fuzzy system, the nonlinear term $\omega_z\omega_x$ should be expressed as a convex sum of the state as follows:

$$\omega_z\omega_x = \Gamma_1^1(\omega_x)\omega_z + \Gamma_1^2(\omega_x)\cdot\alpha_1\omega_z$$
$$1 = \Gamma_1^1(\omega_x) + \Gamma_1^2(\omega_x)$$

Solving these we have

$$\begin{cases} \Gamma_1^1(\omega_x) = \frac{\omega_x - \alpha_1}{1-\alpha_1} \\ \Gamma_1^2(\omega_x) = \frac{1-\omega_x}{1-\alpha_1} \end{cases} \tag{7}$$

Using similar procedure with the other nonlinear terms, the T-S fuzzy system of the above nonlinear equations can be obtained as follows:

R^1 : IF ω_x is about Γ_1^1, and ω_y is about Γ_2^1,
THEN $\dot{\omega} = A_1\omega + B_1 M$

R^2 : IF ω_x is about Γ_1^2, and ω_y is about Γ_2^1,
THEN $\dot{\omega} = A_2\omega + B_2 M$

R^3 : IF ω_x is about Γ_1^1, and ω_y is about Γ_2^2,
THEN $\dot{\omega} = A_3\omega + B_3 M$

R^4 : IF ω_x is about Γ_1^2, and ω_y is about Γ_2^2,
THEN $\dot{\omega} = A_4\omega + B_4 M$

where

$$A_1 = \begin{bmatrix} 0 & 0 & \frac{(I_y-I_z)}{I_x} \\ 0 & 0 & \frac{(I_z-I_x)}{I_y} \\ \frac{(I_x-I_y)}{I_z} & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & \frac{\alpha_1(I_y-I_z)}{I_x} \\ 0 & 0 & \frac{(I_z-I_x)}{I_y} \\ \frac{\alpha_1(I_x-I_y)}{I_z} & 0 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 0 & \frac{(I_y-I_z)}{I_x} \\ 0 & 0 & \frac{\alpha_2(I_z-I_x)}{I_y} \\ \frac{(I_x-I_y)}{I_z} & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & \frac{\alpha_1(I_y-I_z)}{I_x} \\ 0 & 0 & \frac{\alpha_2(I_z-I_x)}{I_y} \\ \frac{\alpha_1(I_x-I_y)}{I_z} & 0 & 0 \end{bmatrix}$$

and $B_1 = B_2 = B_3 = B_4 = \text{diag}\left\{\frac{1}{I_x}, \frac{1}{I_y}, \frac{1}{I_z}\right\}$.

IV. Intelligent Digital Redesign

Throughout this paper, a well-constructed continuous-time state-feedback fuzzy-model-based control law is assumed to be pre-designed, which will be used in redesigning the digital control law. The controller rule is of the following form:

R^i : IF $z_1(t)$ is about Γ_1^i and $\cdots$ and $z_n(t)$ is about Γ_n^i,
THEN $u_c(t) = K_c^i x_c(t)$ (8)

The defuzzified output of the controller rules is given by

$$u_c(t) = \sum_{i=1}^{q}\theta_i(z(t))K_c^i x_c(t) \tag{9}$$

The overall continuous-time closed-loop T-S fuzzy system is

$$\dot{x}_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(t))\left(A_i + B_iK_c^j\right)x_c(t) \tag{10}$$

A. Discretization of the Continuous-time T-S Fuzzy Systems

This subsection discusses the discretization of the continuous-time T-S fuzzy systems. Consider a class of T-S fuzzy systems governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q}\theta_i(z(t))(A_i x_d(t) + B_i u_d(t)) \tag{11}$$

where $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT+T)$, taking the following form:

$$u_d(t) = \sum_{i=1}^{q}\theta_i(z(kT))K_d^i x_d(kT) \tag{12}$$

where K_d^i is the digital control gain matrix to be redesigned.

The digital redesign problem is to find digital controller gains in (12) from the analog gains in (9), so that the closed-loop state $x_d(t)$ in (11) with (12) can closely match the closed-loop state $x_c(t)$ in (10) at all sampling time instants $t = kT, k = 1, 2, \ldots,$. Thus it is more convenient to convert the T-S fuzzy system into discrete-time version for derivation of the state matching condition.

Assumption 1: Assume that the firing strength of the ith rule, $\theta_i(z(t))$ is approximated by their values at time kT, that is,

$$\theta_i(z(t)) \approx \theta_i(z(kT))$$

for $t \in [kT, kT+T)$. Consequently, the nonlinear matrices $\sum_{i=1}^{q}\theta(z(t))A_i$ and $\sum_{i=1}^{q}\theta(z(t))B_i$ can be approximated as constant matrices $\sum_{i=1}^{q}\theta(z(kT))A_i$ and $\sum_{i=1}^{q}\theta(z(kT))B_i$, respectively, over any interval $[kT, kT+T)$.

Theorem 1: The pointwise dynamical behavior of the T-S fuzzy system (11) can be efficiently approximated by

$$x_d(kT+T) \approx \sum_{i=1}^{q}\theta(z(kT))(G_i x_d(kT) + H_i u_d(kT)) \tag{13}$$

where $G_i = \exp(A_iT)$ and $H_i = (G_i - I)A_i^{-1}B_i$.

Proof: See [5]. ■

The discretized version of the closed-loop system with (13) and (12) is constructed to yield

$$x_d(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))\left(G_i + H_iK_d^j\right)x_d(kT) \tag{14}$$

Corollary 1: The pointwise dynamical behavior of the continuous-time closed-loop T-S fuzzy system (10) can also be approximately discretized as

$$x_c(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))\Phi_{ij}x_c(kT) \quad (15)$$

where $\Phi_{ij} = \exp\left(\left(A_i + B_i K_c^j\right)T\right)$.

Proof: It can be straightforwardly proved by Theorem 1. ■

B. New Intelligent Digital Redesign Based on Global State Matching Concept

Our goal is to develop an intelligent digital redesign technique for T-S fuzzy systems so that the global dynamical behavior of (11) with the digitally redesigned fuzzy-model-based controller may retain that of the closed-loop T-S fuzzy system with the existing analog fuzzy-model-based controller, and the stability of the digitally controlled T-S fuzzy system is secured.

Comparing (14) with (15), to obtain $x_c(kT+T) = x_d(kT+T)$ under the assumption of $x_c(kT) = x_d(kT)$, it is necessary to determine the digital control gain matrices K_d^i such that the following matrix equality constraints should be satisfied

$$\Phi_{ij} = G_i + H_i K_d^j, \qquad i,j = 1,2,\ldots,q. \quad (16)$$

then, the state $x_d(t)$ closely matches the state $x_c(t)$ globally, provided that their initial conditions are the same, that is, $x_c(0) = x_d(0) = x_0$.

Theorem 2: If there exist symmetric positive definite matrix Q, symmetric positive semi-definite matrix O, constant matrices F_i and a possibly small positive scalar γ such that the following generalized eigenvalue problem (GEVP) has solutions

$$\underset{Q,O,F_i}{\text{Minimize}} \ \gamma \ \text{subject to}$$

$$\begin{bmatrix} -\gamma Q & \star \\ \Phi_{ij}Q - G_iQ - H_iF_j & -\gamma I \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} -Q + (q-1)O & \star \\ G_iQ + H_iF_i & -Q \end{bmatrix} < 0, \qquad i,j = 1,2,\ldots,q \quad (18)$$

$$\begin{bmatrix} -Q - O & \star \\ \frac{G_iQ+H_iF_j+G_jQ+H_jF_i}{2} & -Q \end{bmatrix} < 0, \quad (19)$$

$$i = 1,\ldots,q-1,\ j = i+1,\ldots,q.$$

then, the state $x_d(kT)$ of the discretized version (14) of the T-S fuzzy system (11) controlled via the redesigned digital fuzzy-model-based controller (12) closely matches the state $x_c(kT)$ of the discretized version of the analogously controlled T-S fuzzy system (15). Furthermore, the discretized T-S fuzzy system (14) is globally asymptotically stabilizable in the sense of Lyapunov stability criterion, where $\star$ denotes the transposed element in symmetric positions.

Proof: See [3]. ■

V. PWM Digital Controller Design

The PWM digital controller can be developed using the aforementioned digitally redesigned controller as follows.

Rewrite the digital control system in an alternative form as

$$\dot{x}_d(t) = \sum_{i=1}^{q}\theta_i(z(t))\Big(A_i x_d(t) + \sum_{h=1}^{m} B_i^{(h)} u_d^{(h)}(kT)\Big) \quad (20)$$

where $B_i^{(h)}$ is the hth column of B_i and $u_d^{(h)}(kT)$ is the hth component of the digitally redesigned control input vector $u_d(kT)$. The corresponding discrete-time model is

$$\dot{x}_d(kT+T) = \sum_{i=1}^{q}\theta_i(z(kT))\Big(G_i x_d(t) + \sum_{h=1}^{m} H_i^{(h)} u_d^{(h)}(kT)\Big) \quad (21)$$

where $H_i^{(h)} = (G_i - I)A_i^{-1}B_i^{(h)}$.

Now we consider a PWM controlled T-S fuzzy system be represented as

$$\dot{x}_d(t) = \sum_{i=1}^{q}\theta_i(z(t))\Big(A_i x_d(t) + \sum_{h=1}^{m} B_i^{(h)} u_{PWM}^{(h)}(kT)\Big) \quad (22)$$

Generally, the PWM control law is mathematically represented by

$$u_{PWM}^{(h)}(t) = \begin{cases} 0, & \text{for } t \in [kT, kT + \tau_k^{(h)}) \\ \operatorname{sgn}(u_d)u_M^{(h)}, & \text{for } t \in [kT + \tau_k^{(h)}, kT + \tau_k^{(h)} + \delta_k^{(h)} \\ 0, & \text{for } t \in [kT + \tau_k^{(h)} + \delta_k^{(h)}, kT + T) \end{cases} \quad (23)$$

where $\delta_k^{(h)}$ is the firing duration of the predetermined constant control input $u_M^{(h)}$ in the time interval $[kT, kT+T)$ and $\tau_k^{(h)}$ is the firing delay.

One easy way to design the PWM controller is to determine the firing duration $\tau_k^{(h)}$ so that the integration of control input respect to time is the same. This conversion has been widely used in industries for many years. In 1960, R. E. Andeen proved in his paper "The principle of equivalent areas" [6] the validity of this conversion under the assumption that the sampling period is suitably small.

Theorem 3: From the digitally redesigned control input $u_d(t)$, the multi input PWM control input u_{PWM} can be redesigned with the following firing duration $\delta_k^{(h)}$ and the delay $\tau_k^{(h)}$ as follows:

$$\delta_k^{(h)} = T\frac{u_d^{(h)}(kT)}{u_M^{(h)}}, \tau_k^{(h)} = \frac{1}{2}(T - \delta^{(h)})$$

Proof: The proof is omitted due to lack of space. ■

VI. Attitude Control of 3-Axis Artificial Satellites

The simulation parameters are given in Table I. The PWM control law is designed by using Theorems 2 and 3. Simulation results are shown in Figs. 2–7. As are shown in the figures, all state variables are well controlled.

TABLE I
SIMULATION PARAMETERS

parameters	value	unit
I_x	3668.0	kgm^2
I_y	970.0	kgm^2
I_z	3156.0	kgm^2

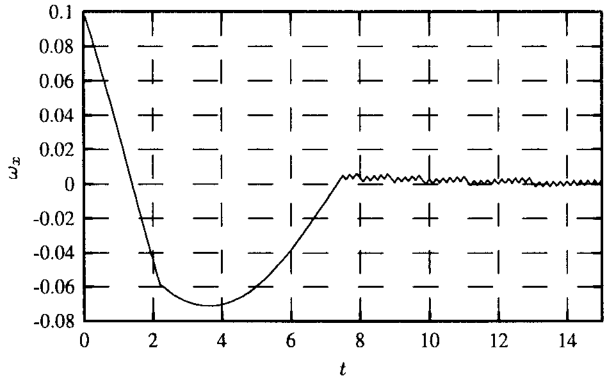

Fig. 1. x-axis angle

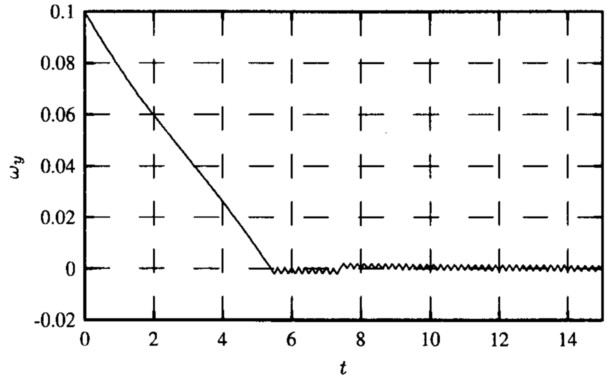

Fig. 2. y-axis angle

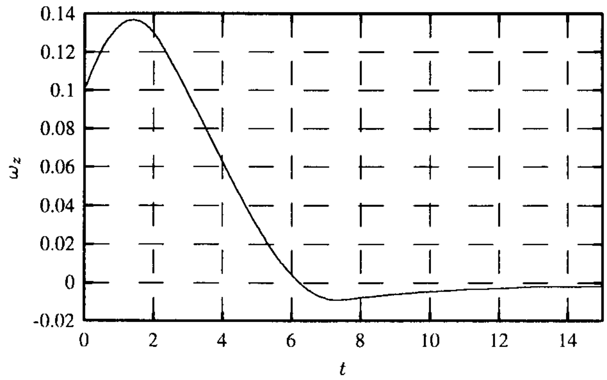

Fig. 3. z-axis angle

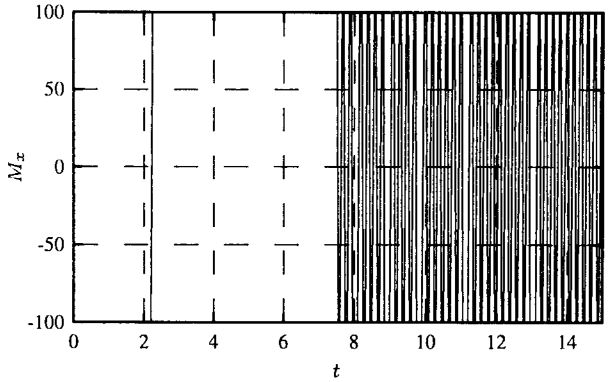

Fig. 4. x-axis torque

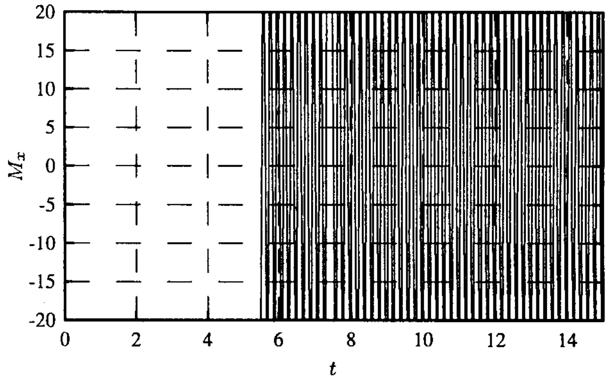

Fig. 5. y-axis torque

VII. CONCLUSIONS

This paper has discussed the PWM controller design method using intelligent digital redesign. The PWM controller is converted from the digitally redesigned controller using the equivalent area principle. An example-attitude control of artificial satellite shows the effectiveness of the proposed method.

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