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Fractal Feature Extraction

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Abtract

To achieve accurate and efficient extraction of the fractal feature, a progressive extraction method is developed. After establishing the boundaries of the targeted surface by enclosing it with internal and external covers, it determines the features of the surface by calculating the characteristics of such covers.

1. INTRODUCTION

After the pioneer work by Mandelbrot[2], fractal theory has been widely applied to many branches of science. The key is accurate fractal dimension estimation and counting scales selection. Fractal dimension estimation is the process of calculating the roughness of an object in terms of dimension values by using blanket [3] or box-counting approaches [1, 4, 5]. In this paper, we propose a new data-dependent method to analytically determine the accurate counting scale from the original data.

The remainder of this paper is organized as follows. In Section II, the proposed progressive fractal cover extraction approach is introduced. In Section III, the experimental results are given to illustrate the effectiveness of the new data-dependent approach. In the last section, the advantages of the new approach are summarized.

2 THE PROPOSED APPROACH

In [1], the fractional box-counting scheme was employed to determine fractal dimension since it separates base-measuring scale and counting scale. The base-measuring scale is the scale of minimum resolution. The counting scale is the size of the box. In this research, we focus on determining the appropriate counting scales by further exploring the idea of statistical similarity.

2.1 The Concept of Fractal Scale of Similarity

The crux of our new approach is to measure changes, then analyze the distribution of the changes. Since a fractal set is a set statistically similar to itself at different scales, such changes congregate at different values. We then set the counting scales to values between congregations of changes. The goal is to use a box [1] of a counting scale to measure the micro properties of the surface below the scale and maintain the macro property of the surface above the scale. Obviously, the macro property of the surface is statistically similar to that of its micro property. The scale is called the fractal scale of similarity.

2.2 Determination of the Scale of Similarity

The essence of fractal dimension estimation is to measure changes. To determine the Fractal Scale of Similarity, the spatial relationship of the variances on a fractal surface is first analyzed. Let surface $S \in H(X^m)$ where H is the Hausdorf space, X is the Euclidean space, and m is the dimension of X. We define ρ_0 as the base-scale and $\rho(1), \rho(2), \cdots, \rho(n)$ to be the counting-scales in increasing order. The counting-scales can be determined progressively starting from $\rho(1)$ to $\rho(n)$ in n steps.

To find out the extreme points of surface $Y=S(x_1, x_2, x_m)$, we solve the following systems of equations:

$$\frac{\partial}{\partial x_i} S(x_1, x_2, x_m)=0, \quad i=1,2,\cdots,m.$$

Let a_j denote an extreme point $(x_{j1}, x_{j2} \ldots x_{jm})$ which satisfies the above system of equations. Then, $a_j's$, $j=1,2,\cdots,n$, describe where changes would occur in S. Let a_j, $j=1,2,\cdots,n$ denote a set of the extreme points a_j, $j=1,2,\cdots,n$.

To determine the spatial relationships of the $\alpha_j's$, the distribution of the $\alpha_j's$ are analyzed. Let $\delta_j=|\alpha_j-\alpha_{j+1}|=y$, where y is a random variable in Y, for all j=1,2, ..., n-1 and plot the δ_j for all j. Pentland [6] discovered that images of natural scenes are fractal. If surface S is fractal, $\delta_j's$ will congregate at a set of values over the entire space. Figure 1 shows a continuous curve fitting the histogram of the set δ. The horizontal axis δ describes the distance between the extreme points. The vertical axis N shows the distribution of such relationship. A fractal surface S duplicates itself at different scales. At a particular scale, the local properties of a certain region on S will be "duplicated" in other regions on S. In other words, the local properties in various regions are statistically similar. At the "duplicating scale", the fractal presents further details at that scale over the entire surface S. By determining the peaks where changes occur, the locations of the changes can be computed. The actual duplicating scales for these features can be determined by analyzing the distribution of the extreme points.

Let T be a continuous curve fitting the histogram of $\delta_j's$. Then, set

$$\frac{\partial T}{\partial y}=0$$

to obtain extreme point set $\{ t_i \mid t_i \in T\}$.

Define the fractal similarity-scale β as:

$$\beta_k=\max\left\{\varepsilon * k, \left\{\frac{(t_k+t_{k+1})}{2} \mid |t_k-t_{k+1}|>\varepsilon\right\}\right\}$$

where k is a positive integer, ε is the minimum measuring unit. It is used to guarantee that the β_k increases at a minimum of ε-unit. In case when T is a flat line, β_k will have increasing value as k increases. β_k is called the similarity-scale at step-k. The counting scale can therefore be defined as:

$$\rho(k)=\beta_k, \quad k=1,2,\ldots$$

Note that $\rho(k)$ increases as the computing process goes from step k to step k+1.

2.3 The Similarity Cover Encapsulation

Let wos(β_k) be the multi-dimensional window of support with radius β_k, ε be the minimal resolution. A similarity cover is defined by covers C_1 and C_2 that enclose surface S :

$$C_1(S)=\max\{S(x_1, x_2, \cdots, x_m)+\varepsilon, \max\{S(x_1', x_2', \cdots, x_m') \mid \forall(x_1', x_2', \cdots, x_m') \in wos(\beta_k)\}\}$$

$$C_2(S)=\min\{S(x_1, x_2, \cdots, x_m)-\varepsilon, \min\{S(x_1', x_2', \cdots, x_m') \mid \forall(x_1', x_2', \cdots, x_m') \in wos(\beta_k)\}\}$$

(1)

where $S(x_1', x_2', \ldots, x_m') \in wos(\beta_k)$ and $|\max\{S(x_1', x_2', \ldots, x_m')\}-\min\{S(x_1', x_2', \ldots, x_m')\}|>\varepsilon * \beta_k$.

Figure 2 shows a surface function y=S(x) in 2-D space where C_1 and C_2 are the covers that encapsulate.

It should be pointed out that after the "encapsulation" process, those variances that are smaller than the similarity scale will be encapsulated by similarity covers while changes larger than the similarity scale will remain intact.

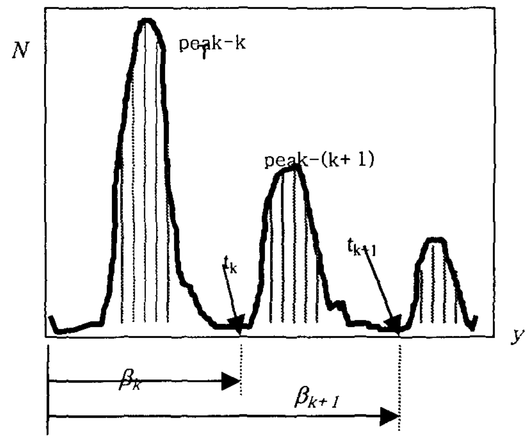

Fig 1. The histogram of δ 10wing the distribution of the extreme points on surface S

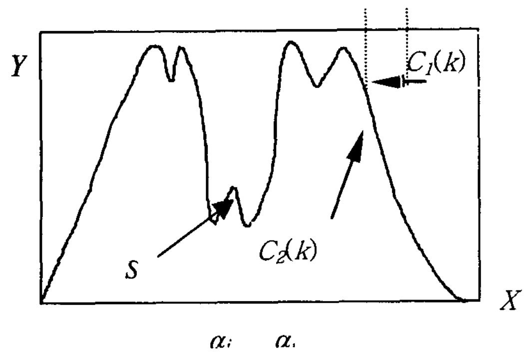

Fig 2. Similarity Cover Encapsulation

2.4 The Similarity Cover Fractal Dimension

Let $Vol_n(S)$ denote the space enclosing surface S at counting scale $\rho(n)$ where n is the n-th step in increasing the counting scale. Then $Vol_n(S)$ can be obtained by calculating the volume between the two covers.

If S is a fractal, then

$$D = \lim_{n \to \infty} \left\{ - \frac{\mathrm{Ln}(Vol_n(S))}{\mathrm{Ln}(\rho(n))} \right\}$$

exists and the D is called the Similarity Cover fractal dimension of S.

2.5 The Micro World: The Similarity Cover Feature Set

To precisely describe the micro world - an encapsulated region $R(\alpha_i)$ centered around α_i on the surface S that's smaller than similarity scale, a set of features is extracted from the region. First, we need to describe the size of such region. Let (i be the maximum distance between any two points in region $R(\alpha_i)$.

Define the Similarity Cover Fractal Feature $V_i = \{\alpha_i, \beta, \gamma_i, D_i\}$ where α_i is an extreme point in $H(X^m)$, β is the set of similarity scales with which S is measured, γ_i is the range, D_i is the fractal dimension of region $R(\alpha_i)$ on surface S. Statistically, the set $V = \{V_i\}$ describes the essential characteristics of the Similarity Cover within the micro world.

2.6 The Macro World: The Transformation of Similarity Surface

Let $C(S)$ be similarity surface denoting the interpolation of the two covers $C_1(S)$, $C_2(S)$ enclosing surface S. If S is a fractal, then $C(S)$ is also a fractal with the same features. This is obvious since each set of similarity covers only encapsulate variances smaller than the last counting scale $\rho(k)$ while keeping the variances larger than $\rho(k)$ intact.

Given a surface $S \in H(X^m)$, we can obtain the similarity surface $C(S)$ and fractal feature set $\{C(S), V\}$ for all regions of S via (1). We will show that {C(S), V} accurately describes the surface S. To show that, we re-construct S' back from C(S) and V. Define $f(\alpha_i, \beta, \gamma_i, D_i)$ to be the function generating random Brownian surface in $H(X^m)$ at point α_i within range γ_i with fractal dimension Di. Then, operation $\oplus$ is defined as:

$$C(S) \oplus V = \sum_i \{C(S_i) + f(\alpha_i, \beta, \gamma_i, D_i)\}$$

Theorem If $S' = C(S) \oplus V$, S' is statistically similar to S.

Proof:

Given S , C(S) and V, Construct similarity cover for S(with equation-1 for all regions of S, i=1,2, ..., n.

$$\begin{aligned} C_1(S') &= C_1(C(S) \oplus V) \\ &= C_1(\textstyle\sum_i (C(S_i) + f(\alpha_i, \beta, \gamma_i, D_i))) \\ &= C_1(\textstyle\sum_{i \in V} (C(S_i) + f(\alpha_i, \beta, \gamma_i, D_i))) + \textstyle\sum_{i \notin V} C_1(S_i) \\ &= \textstyle\sum_{i \in V} C_1(C(S_i) + f(\alpha_i, \beta, \gamma_i, D_i)) + \textstyle\sum_{i \notin V} C_1(S_i) \\ &= \textstyle\sum_{i \in V} [\max\{C(S_i) + f(\alpha_i, \beta, \gamma_i, D_i) + \varepsilon, \\ &\quad \max\{C(S_i') + f(\alpha_i, \beta, \gamma_i, D_i)\}\}] + \textstyle\sum_{i \notin V} C_1(S_i) \\ &= \textstyle\sum_{i \in V} [\max\{\alpha_i, (s|s \in S_i)\} + \gamma_i + \varepsilon] + \textstyle\sum_{i \notin V} C_1(S_i) \end{aligned}$$

Similarly, at step k,

$$C_2'(S') = \sum_{i \in V} [\min\{\alpha_i, (s|s \in S_i)\} - \gamma_i - \varepsilon] + C_2(S_i)$$

$$\begin{aligned} Volume' &= s * |C_1(S') - C_2(S')| \\ &= s * |\textstyle\sum_{i \in V} \gamma_i + 2 * \gamma_i| + s * \textstyle\sum_{i \notin V} |C_1(S_i) - C_2(S_i)| \\ &= 3 * s * |\textstyle\sum_{i \in V} \gamma_i| + 2 * s * \varepsilon \end{aligned}$$

On the other hand, for S,

$$\begin{aligned} Volume &= s * |C_1(S) - C_2(S)| \\ &= 3 * s * |\textstyle\sum_{i \in V} \gamma_i| + 2 * s * \varepsilon \end{aligned}$$

So, $Volume' = Volume$, or $Vol_i' = Vol_i$ for all $i = 1, 2, \ldots, n$. Since $\rho'(k) = \beta_k' = \beta_k = \rho(k)$,

$$\begin{aligned} D' &= \lim_{n \to \infty} \left\{ - \frac{\mathrm{Ln}(Vol_n'(S))}{\mathrm{Ln}(\rho'(n))} \right\} \\ &= \lim_{n \to \infty} \left\{ - \frac{\mathrm{Ln}(Vol_n(S))}{\mathrm{Ln}(\rho(n))} \right\} \\ &= D \end{aligned}$$

S′ is statistically similar to S.

3. THE EXPERIMENTAL RESULT

One of the objectives of image analysis is to identify the surface features of an image. The similarity cover theory provides a powerful tool in describing the surface properties of an image.

We can use the proposed approach to detect edges. For edge detection purposes, the window in focus should be as narrow as possible to include as few β-scales as possible. Since regression is used to estimate the similarity cover fractal dimension, the minimum number of similarity scales can not be less than two. The fewer the similarity scales, the sharper the edge. It should be pointed out that the number of similarity scale could be selected based upon different processing needs. The effectiveness of the proposed approach are shown in Figure 3 with a range image of a tool, a PET image of a head, an MRI of a stomach and a photo image. The edges are precisely defined and the surface properties well preserved.

4. CONCLUDING REMARKS

We have proposed the Progressive Similarity Cover approach for extracting fractal dimension and key features of an image with accuracy and efficiency and have presented several experimental results to corroborate the new approach. The superiority of the new proposed approach is in its data-dependent scale selection. The new method can detect the scales on which the features are statistically duplicated and is a powerful tool to extract fractal information.

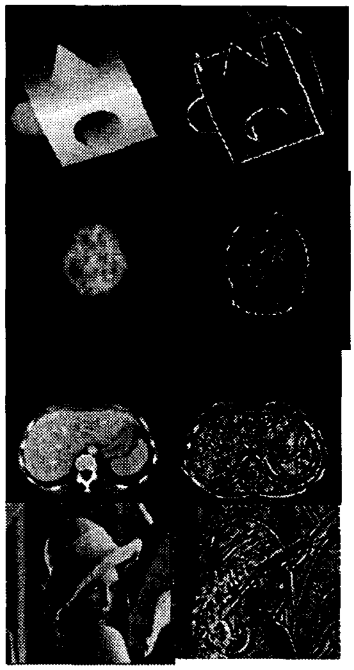

Fig 3. Edge detection

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