

## The obstacle collision avoidance methods in the chaotic mobile robots

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**Abstract**-In this paper, we propose a method to avoidance obstacle in which we assume that obstacle has an unstable limit cycle in the chaos trajectory surface. In order to avoid the obstacle, we assume that all obstacles in the chaos trajectory surface in which has an unstable limit cycle with Van der Pol equation. In this paper show also that computer simulation results are satisfy to avoid obstacle in the chaos trajectory with Chua's circuit equation of one or multi obstacle has an limit cycle with Van der Pol (VDP) equation and compare to rate of cover in one robot which have random walk and Chua's equation

### 1. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been investigate in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots their related themes [10]. Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot, where mobile robot is equipped with a mobile robot with a controller that ensures chaotic motion and the dynamics of the mobile robot is represented by Arnold equation. They applied obstacle with chaotic trajectory, but they have no mentioned about obstacle avoidance method. In this paper, we propose a method to avoidance obstacle in which we assume that obstacle has an unstable limit cycle in the chaos trajectory surface. In order to avoid obstacles, we assume that all obstacles in the chaos trajectory surface have an unstable limit cycle with Van der Pol equation. Computer simulations are also shown obstacles can be avoided with Chua's circuit equation. The rate of coverage is also compared to random walk.

### 2. Chaotic Mobile Robot embedding Chaos Equation

#### A. Mobile Robot

As the mathematical model of mobile robots, we assume a four-wheeled mobile robot as shown in Fig. 1.

Let the linear velocity of the robot  $v$  [m/s] and angular velocity  $\omega$  [rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 & 0 \\ \sin \theta & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where  $(x,y)$  is the position of the robot and  $\theta$  is the angle of the robot.

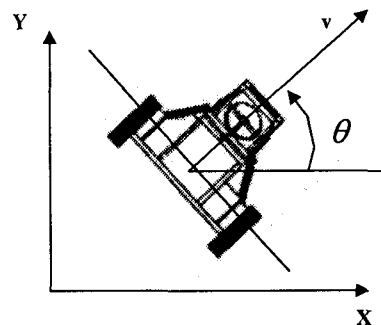


Fig. 1 Four-wheeled mobile robot

#### B. Chua's Equation (2-Double Scroll)

In order to generate chaotic motions of the mobile robot, we apply Chua's circuit equation. Chua's circuit (see Fig. 2 and 3) is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods. Since the Chua's circuit is endowed with an unusually rich repertoire of nonlinear dynamical phenomena, it has become a universal paradigm for chaos.

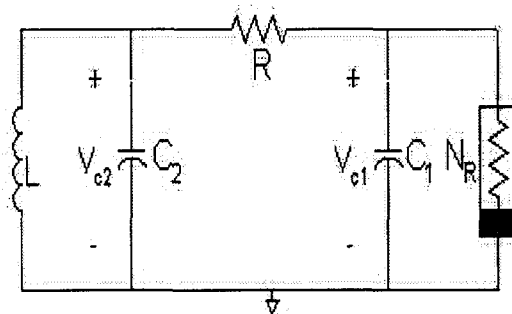


Fig. 2 Chua's circuit

From the Fig. 2, we can derive the state equation of Chua's circuit following as:

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \quad (2)$$

where  $g(x) = m_{2n-1}x + \frac{1}{2} \sum_{k=1}^{2n-1} (m_{k-1} - m_k)(|x+c_k| - |x-c_k|)$  is shown in Fig. 3

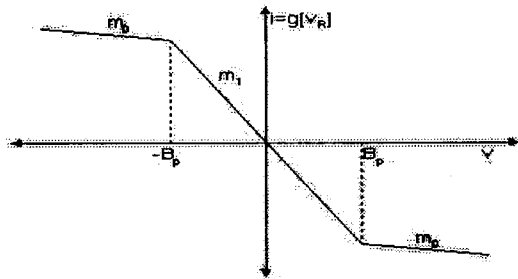


Fig. 3  $v_x - i_x$  characteristic of the NR

### C. Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the mobile robot, we define and use the following Chua's equation.

We define and use the following state variables:

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ x_3 &= \theta \end{aligned} \quad (3)$$

We now design the inputs as follows:

$$\begin{aligned} \omega &= -\beta x_2 \\ v &= \text{arbitrary constant} \end{aligned} \quad (4)$$

Finally, we can get the state equation of the Chua's circuit embedding in the mobile robot as follows:

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \quad (6)$$

Using the equation (6), we obtain the embedding chaos robot trajectories with Chua's circuit. Fig. 4.

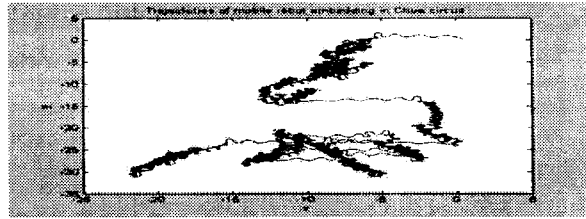


Fig. 4 Trajectories of mobile robot embedding in Chua's circuit

### D. Mirror Mapping.

Basically, equation (6) is assumed that the mobile robot moves in a smooth state space without boundary. However, real robot moves in space with boundary like walls or surfaces of obstacles. Fig. 5 show that the mirror mapping diagram. To avoid a boundary or obstacles, we consider mirror mapping when the robot approach walls or obstacles using the Eq. (7) and (8). Whenever the robot approaches a wall or obstacle, we calculated the robot new position

$$A = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \quad (7)$$

$$A = 1 / 1 + m \begin{pmatrix} 1 - m^2 & 2m \\ 2m & -1 + m^2 \end{pmatrix} \quad (8)$$

We can use equation (7) when slope is infinite such as  $\theta = 90$  and also use equation (8) when slope is not infinite.



Fig. 5 Mirror mapping

### 3. The Chaotic Behavior of Chaos Robot with obstacle

In this section, we will study the chaotic behavior of a chaos robot relay on Chua's circuit with mirror mapping.

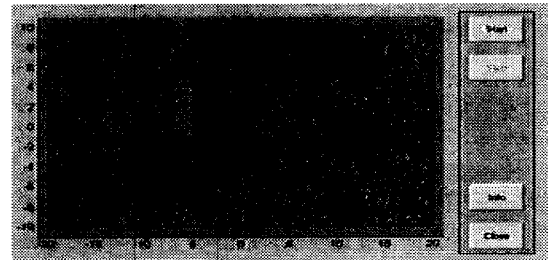


Fig.6 Trajectories of chaos robot with obstacle embedding Chua's equation

Fig. 6 is show the trajectory of chaos robot can avoid obstacle with mirror mapping Eq. (7) and (8) relaying on Chua's equation.

#### 4. The mobile robot with Van der Pol (VDP) equation representing obstacle.

In this section, we will discuss the mobile robot with VDP equation for obstacle. Since VDP equation has an unstable limit characteristics, we assume that obstacle has an unstable limit cycle with VDP equation, when mobile robots approach as obstacle with VDP equation,. As a result, the mobile robot can not move close to obstacle, and the obstacle is avoided.

##### A. VDP equation as a obstacle

In order to represent obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1 - y^2) y - x \end{aligned} \quad (9)$$

From equation (9), we can get the following limit cycle as Fig. 7

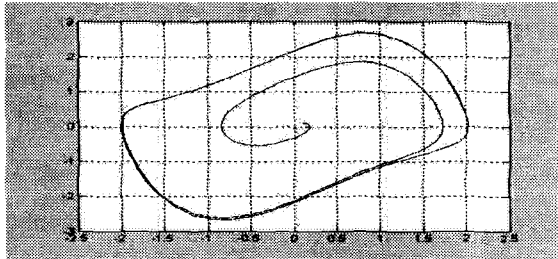


Fig. 7 Limit cycle of VDP

##### B. Using VDP equation as a obstacle in the Chua's circuit

In Fig. 8, computer simulation result show that 6 VDP obstacles and 2 robots of Chua's equation. We can see two robots are well avoided obstacle.

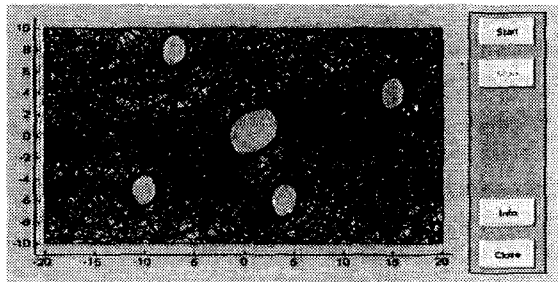
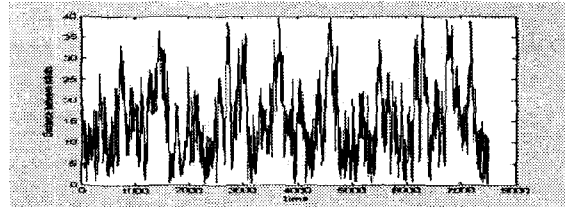
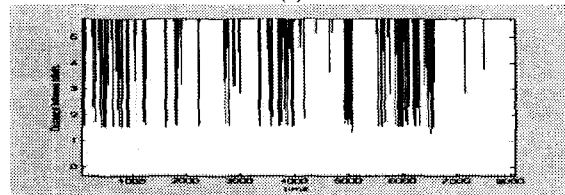


Fig. 8 computer simulation result of obstacle avoidance with 2 robots and 6 obstacles in Chua's circuit.

In Fig. 9(a), show that the distance between tow robots where we can recognize that the collision possibility of two robot is a several times such as 1600s, 2300s, 3200s, 4500s etc. In order to avoid collision between two robots, we applied VDP trajectories in each robot with Chua's circuit. In the Fig. 9(b), we can recognize that there is no collision between the two robots.



(a)



(b)

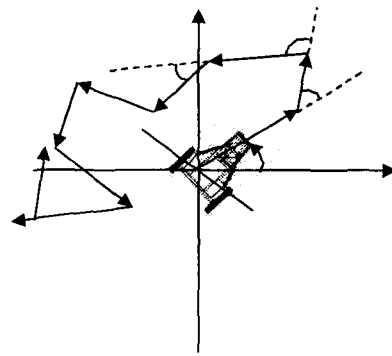
Fig.9 Distance between two robots. (a) actual distance (b) (a)'s magnify in which after applied VDP trajectories with Chua's equation.

#### 5. The calculation of rate of cover area.

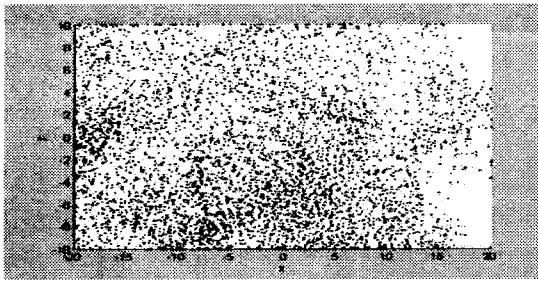
In order to calculate the rate of cover area in the surface, firstly, we employ that random work robot, Chua's circuit robot and secondly, we compare each other.

##### A. Random work robot

In Fig. 10, we can see the random robot walk and its position..



(a)



(b)  
Fig.10 Random robot walk (a) and position (b)

### B. Chua's circuit

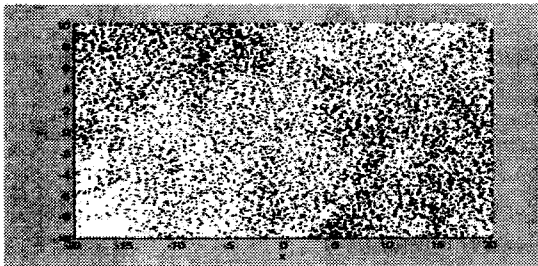


Fig. 11 Plot of robot position in Chua's circuit

Figs 10, 11 are plots of the position of the robot after every constant time during the every 7500S. It can be seen that chaotic mobile robot with Chua's circuit in Fig 11 could scan the workspace more efficiently compare to random walk robot.

### C. Comparison of cover rate in random walk and Chua's circuit

In Fig. 13, it can be seen that rate of coverage of Chua circuit (2- Double Scroll) is superior to random walk in the cover rate. Until about 2500 seconds, cover rate of Arnold equation is a slightly high but after 2500 seconds, this situation is reversed.

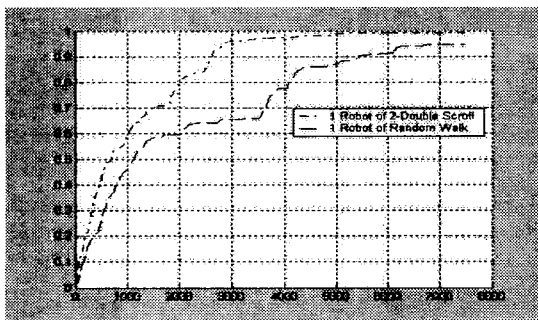


Fig. 13 Comparison of covered rate of random work, 2-Double Scroll and random walk

## 6. Conclusion

In this paper, we proposed the chaotic mobile robot, which implies a mobile robot with Chua's equation and also proposed obstacle avoidance method, which we assume that obstacle has an unstable limit cycle such as Van der Pol equation. We designed one robot or 2-robots such that the total dynamics of the mobile robots are characterized by Chua's equation and we also designed one or 2- robots such that the obstacle avoidance method of chaotic robot. By the numerical analysis, it was illustrates that avoidance obstacle methods with an unstable limit cycle such as Van der Pol equation have a good result.

In order to make a chaotic behavior in the robot system, we applied Chua's equation. As a result, we realized that the rate of coverage of Chua's circuit is superior to random walk.

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