Fuzzy Modeling and Control of Wheeled Mobile Robot

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Abstract: In this paper, the control of the differential drive wheeled mobile robot (DDWMR) is studied. Because the DDWMR have non-holonomic constraints, it cannot be stabilized by smooth feedback. The T-S fuzzy model for the DDWMR is presented and a control algorithm is developed by well known PID control and LMI based regional pole-placement.

I. INTRODUCTION

Various kinds of mobile robots have been developed and recently, many of scalars devote their efforts on these areas. Among them, the differential drive and car-like wheeled mobile robot(WMR) are most widely used in their application field. Since the WMR system is a typical non-holonomic system except the omni-directional types, the standard control laws must be developed for systems with non-holonomic constraints. Due to the fatal property that a WMR with non-holonomic constraints cannot be stabilizes by a smooth feedback, it is necessary to find more effective and advanced algorithms.[1-3]

In this paper, the control of the differential drive wheeled mobile robot(DDWMR) is studied. Because the DDWMR have non-holonomic constraints, it cannot be stabilized by smooth feedback. The T-S fuzzy model for the DDWMR is presented and a control algorithm is developed by well known PID control and LMI based regional pole-placement.

II. Modeling of Wheeled Mobile Robot

2.1. Dynamic Modeling of Wheeled Mobile Robot[1,4]

The structure of the mobile robot, considered in this paper, is shown in Fig. 1. The relation between the forward velocity and the wheel angular velocity is described by

$$\begin{bmatrix} v \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{1}$$

where, v and $\dot{\phi}$ are forward and rotation velocities of the robot, respectively, and r is the ratio of the wheel. And b is the displacement from center robot to center of wheel. The kinetic equation is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\phi} \end{bmatrix} \tag{2}$$

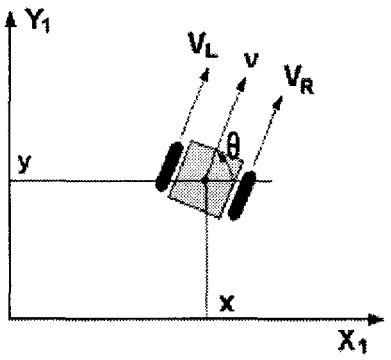

Figure 1. The mobile robot

In order to derive the dynamic equations, we now define some variables.

I_c : robot inertia except wheels and rotor

I_w : motor rotor inertia for wheels and wheel axis

I_m : motor rotor inertia for wheels and wheel diameter

m : mass of robot except wheels and motor rotor

m_c : mass of wheels and motor rotor

The dynamic equation of a of robot is described by[4,5]

$$M(q)\ddot{q} + V(q,\ \dot{q}) = E(q)\tau - \hat{A}^T(q)\lambda \tag{3}$$

where, λ is Lagrangy multiplier, τ is the torque of each wheels, and d is the displacement from the center of mass to the center of rotation, $q = \begin{bmatrix} x & y & \theta_1 & \theta_2 \end{bmatrix}^T$ and

$$M(q) = \begin{bmatrix} m & 0 & -m_c cd\sin\phi & m_c cd\sin\phi \\ 0 & m & m_c cd\cos\phi & -m_c cd\cos\phi \\ -m_c cd\sin\phi & m_c cd\cos\phi & I_c^2 + I_w & -I_c^2 \\ m_c cd\sin\phi & -m_c cd\cos\phi & -I_c^2 & I_c^2 + I_w \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} 2m_c d\dot{\phi}^2\cos\phi \\ 2m_c d\dot{\phi}^2\sin\phi \\ 0 \\ 0 \end{bmatrix}, \quad \hat{A}(q) = \begin{bmatrix} -\sin\phi & \cos\phi & 0 & 0 \\ -\cos\phi & -\sin\phi & cb & cb \end{bmatrix}$$

$$E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Eliminate the Lagrange multiplier λ, and define the state variables, input and the output as

$$x_1 \triangleq \theta_1, x_2 \triangleq \theta_2, x_3 \triangleq \dot{\theta}_1, x_4 \triangleq \dot{\theta}_2$$

$$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \qquad y = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

then, the state space representation of mobile robot is []

$$\begin{aligned} \dot{x}(t) &= A(\dot{\phi}(t))x(t) + B_0 u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

where,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_{11} & a_{12} \\ 0 & 0 & a_{21} & a_{22} \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_{11} = \frac{\left[(2mc^2b^2+I_w)cb+(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$a_{12} = \frac{\left[(2mc^2b^2+I_w)cb-(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$a_{21} = -\frac{\left[(2mc^2b^2+I_w)cb-(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$a_{22} = -\frac{\left[(2mc^2b^2+I_w)cb+(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$b_{11}(=b_{22}) = \frac{mc^2b^2+I_c^2+I_w}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$b_{12}(=b_{21}) = \frac{I_c^2-mc^2b^2}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

In the equation (7), controllability matrix $[A, B_0]$ is controllable except when the variable $\dot{\phi}(t) = 0$.

2.2. Takagi-Sugeno Fuzzy Model of wheeled Mobile Robot[5]

The fuzzy model proposed by Tagaki and Sugeno is described by IF-THEN rules which represent local linear input-output relations of a nonlinear system.[8] The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model.

For the DDWMR, the elements of matrices B_0 and C are constant and there is only one time-varying parameter $\dot{\phi}$. The fuzzy model for wheeled mobile robot described by the equation (8) becomes

If $\dot{\phi}(t) = M_i$

$$\text{THEN} \quad \begin{cases} \dot{x}(t) = A(\dot{\phi})x(t) + B_0 u(t) \\ y(t) = Cx(t), \end{cases} \quad i = 1, 2, \cdots, r \tag{5}$$

The final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))\left\{A(\dot{\phi}_i)x(t) + B_0 u(t)\right\}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\left\{A(\dot{\phi}_i)x(t) + B_0 u(t)\right\} \\ &= \sum_{i=1}^{r} h_i(z(t))A(\dot{\phi}_i)x(t) + B_0 u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{6}$$

The equation (11) is a state-space representation of wheeled mobile robot in which terms B_0 and C are which constant, and in which the only term A_i is dependent on the term $\dot{\phi}(t)$ which equal to the deference of velocities from the right wheel to left wheel.

III. Control of Mobile Robot

We are now state a controller structure presented in this paper, and a new control design algorithm for mobile robot.

3.1. Controller Structure

The most important control strategy of physical systems is reference tracking. To achieve this objective, the control structure is shown by figure 2. In figure 2, control parameters in the block are all fuzzy controller. It is shown in figure 2 that the controller has two control parameters one of which is state feedback and the other is control gain with integrator. The input signal is described by

$$u(t) = -F(\mu)x(t) + K(\mu)\int e(t)dt \tag{7}$$

where, $F(\mu)$ is a fuzzy state feedback gain matrix and $K(\mu)$ is a fuzzy integrator gain matrix. In order for obtain controller gains $F(\mu)$ and $K(\mu)$, it is needed to simplify control input or controller structure. The new state x_{n+1} can be defined at $\dot{\phi}$ in the figure 1. Then the dynamic equation becomes

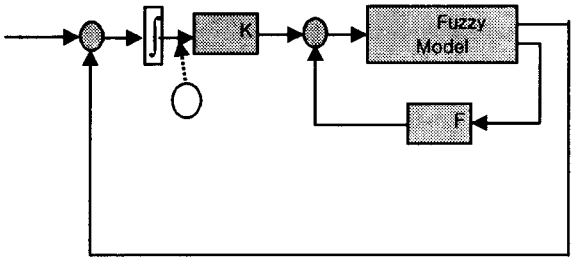

Fig. 1. Controller structure

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{n+1}(t) \end{bmatrix} = \left(\begin{bmatrix} \sum_i \mu_i A_1(\dot{\phi}_i) & 0 \\ -C & 0 \end{bmatrix} \right) \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} + \begin{bmatrix} B_0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) \quad (8.a)$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} \quad (8.b)$$

and, the control input is

$$u(t) = \begin{bmatrix} F(\mu) & K(\mu) \end{bmatrix} \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} \quad (9)$$

It is known by the equation (9) that the control input is state feedback for the system described by the equation (8).

3.2 Regional Pole Placement

The LMI region is defined following definition[6].

Definition 1. LMI regions are convex subset D of the complex plan characterized by

$$D = \left\{ z \in C : L + Mz + M^T z^* \right\} \quad (10)$$

where M and L are fixed real matrices, and z and z^* are complex valued scalar and its complex conjugate pair.

The matrix valued function

$$f_D(z) \triangleq L + Mz + M^T z^* \quad (11)$$

is called the characteristic function of the region D. We are now state a local pole placement. Let M_i be the fuzzy model obtained by substituting the $\dot{\phi}$ as i-th sample. And select a function $\mu(\rho^{i,j,\cdots l})$, a local convex function, then the following theorem states the algorithm of obtaining the controller gain matrix for M_i.

Theorem 1: For M_i, the closed loop poles lie in the LMI region D

$$D = \left\{ z \in C : L + Mz + M^T z^* \right\}$$

where,

$$L = L^T = \left[\lambda_{jk} \right]_{1 \le j,\, k \le m}, \quad M = \left[m_{jk} \right]_{1 \le j,\, k \le m}$$

if and only if there exists a symmetric matrix X satisfying following four inequalities.

$$\left[\lambda_{jk} X + m_{jk} A_i^{cl}(\dot{\phi}_i) X + m_{kj} A_i^{cl^T}(\dot{\phi}_i) \right]_{1 \le j,\, k \le m} < 0 \quad (12)$$

$$X > 0$$

proof) Proof of this theorem is simple extension Chilali and Gahinet's work [6]. QED

Theorem 1 states the local regional pole placement of the M_i. Because the equation (33) is not convex, we cannot obtain the controller gain matrix. Define $Y_i \triangleq F_i X$, then conditions of local pole placement is summarized by theorem 2.

Theorem 2: The closed loop poles lie in the LMI region D if and only if there exists a symmetric matrix X satisfying following inequalities.

$$\left[\lambda_{jk} X + m_{jk} (A(\phi_i) X + B Y_i) + m_{kj} (A(\phi_i) X + B Y_i)^T \right]_{1 \le j,\, k \le m} < 0 \quad (13)$$

$$X > 0$$

the i-th state-feedback gain matrix is

$$\begin{bmatrix} F_i & K_i \end{bmatrix} = Y_i X^{-1} \quad (14)$$

proof) The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work [6]. QED.

The theorem 1 and theorem 2 shows the local regional pole-placement condition and the way of finding local controller gains. The global pole-placement condition and global controller gain can be achieved by using approximated plant. In order for global pole-placement, the control input, made up of local controller gain, is selected by

$$u(t) = -\left[\sum_i \mu_i(\dot{\phi}_i) F_i x(t) + \sum_i \mu_i(\dot{\phi}_i) K_i \int e(t) dt \right] \quad (15)$$

By noting the equation (15), the controller gain is made up of local controller gains and which is convex combination of local controller gains between [(i,j,$\cdots$l)~(i-1,j,$\cdots$l)]. The following theorem states the global regional pole-placement.

Theorem 3. Assume that the plant model is modeled by the equation (6) and local controller gains are obtained by the equation (14) for local fuzzy model. Then the closed loop poles are lie in the desired region.

Proof). The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work. QED.

The theorem 3 states the global global-placement condition and controller design procedure is summarized as 1) sampling model 2) design local controller 3) combine it.

IV. Simulation

In simulation, the robot considered is MIROSOT soccer robot, and detailed specifications are summarized in the table 1.

Table 1. The specifications of MIROSOT robot

Size	70x70x70 mm
Wheel diameter	45 mm
Rpm	8000
Gear ratio	8:1

The mass of the robot is 0.0612 Kg m/sec^2 and the mass of wheels is 0.0051 kg m/sec^2 . And other parameters used in this paper were

$$b = 35mm, \quad c = r / 2b, d = 10mm.$$

The robot inertia except wheels and rotor is 0.05 $Kg\,cm\sec^2$ and motor rotor inertia for wheels and wheel axis is 0.0176 $Kg\,cm\sec^2$. These parameters were actually measured and computed for MIROSOT robot designed Yujin Robotics corp. In this paper, the maximum velocity of the wheel was the maximum velocity of the motor specification.

By using parameters described above, state space matrices for the mobile robot are

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 40.7370\dot{\phi} & 40.5002\dot{\phi} \\ 0 & 0 & -40.5002\dot{\phi} & 40.7370\dot{\phi} \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.1055 & -22.0917 \\ -22.0917 & 0.1055 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Membership functions of this paper are shown in the figure 4. Figure 5 and Figure 6 are simulation results for pulse reference input. The controller is the PI control loop which is designed well known LQ algorithm. In order to the diagonal gain matrix, some technical trick is used for selecting weighting matrices. It is shown by figure 1 that the velocity following error becomes zero. But the overshoot is occurred and by it, the tracking results include error.

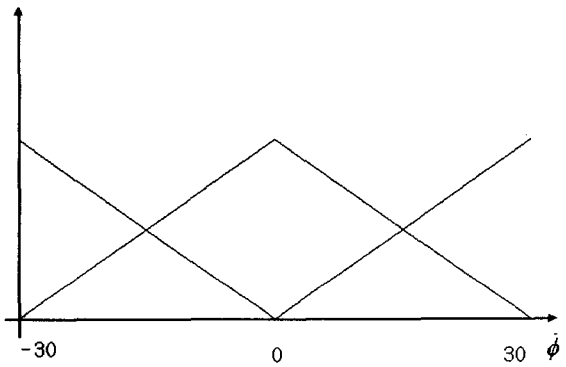

Figure 4. The membership functions

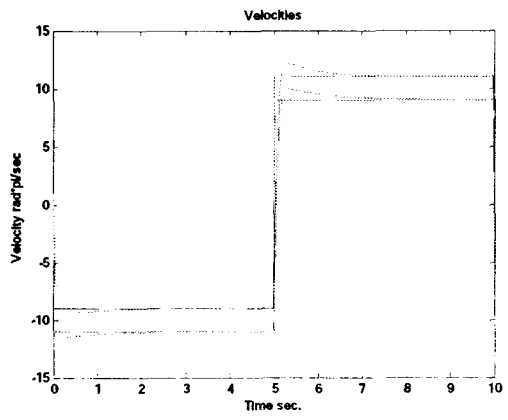

Figure 5. Velocities

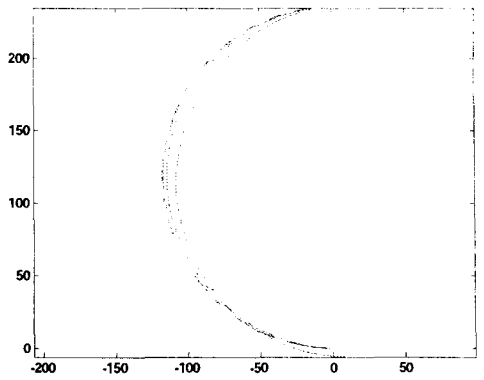

Figure 6. Tracking result

V. Conclusion

In this paper, the DDWMR is considered. The T-S fuzzy model for DDWMR is presented and control algorithm is suggested. The controller is the PI control loop which is designed well known LQ algorithm. In order to the diagonal gain matrix, some technical trick is used for selecting weighting matrices. It is shown by this paper that the presented algorithm is more easy way of control of the DDWMR and that the result of this paper can be applicable to car-like WMR.

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