

A 4-step Inference Method for Natural Language Propositions Involving Fuzzy Quantifiers and Truth Qualifiers

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Abstract—In this paper, we propose a 4-step inference method needed for constructing a natural language communication system. The method is used to obtain fuzzy quantifier Q' when QA is F is $\tau \Leftrightarrow Q'(m'A)$ is mF is m' is τ is inferred (Q, Q': quantifiers, A: fuzzy subject, m, m', m': modifiers, F: fuzzy predicate, τ : truth qualifier). We show that Q' is resolved step by step for two types of Q, including a non-increasing type (few,...) and a non-decreasing type (most,...).

I. INTRODUCTION

L.A. Zadeh researched most systematically ambiguous meanings of natural language [1-2]. He divided natural language propositions into four types: a proposition modifying a fuzzy predicate (tall, heavy,...), a proposition composed of a combination of fuzzy propositions (and, or,...), a proposition involving fuzzy quantifiers (most, few,...), and a proposition involving fuzzy qualifiers (true, possible, probable,...). And, he proposed an evaluation method based on possibility theory [1].

For these types, there are evaluation methods for propositions modifying a fuzzy predicate and ones composed of a combination of fuzzy propositions [1], but for the last two propositions, no clear evaluation algorithms have been presented, so an evaluation method that covers all four types has not been reported.

To construct a natural language communication system, we must handle fuzzy quantifiers and qualifiers, which are used in every-day conversations, so we must handle the third and fourth types. We previously reported how to transform natural language propositions involving fuzzy quantifiers and truth qualifiers by fuzzy inference [3-4]. Below, we review the relevance of our previous research to this paper. We refer to other inference methods for transforming natural language propositions involving fuzzy quantifiers and truth qualifiers.

In this paper we discuss the transformation of fuzzy subjects, fuzzy predicates, and truth qualifiers for natural

language propositions involving fuzzy quantifiers and truth qualifiers.

We followed Zadeh's notation as follows:

Q, Q': fuzzy quantifiers

A: fuzzy subject

m: modifier

F: fuzzy predicate

τ : truth qualifier

We denote a proposition as follows:

QA are F is τ [1-2].

For example, the following transformation is executed using our previous method [3-4]:

Transforming a fuzzy predicate:

Most tall men are heavy is true

\Leftrightarrow Almost all tall men are more or less heavy is true

Transforming a fuzzy subject:

Most tall men are heavy is true

\Leftrightarrow Almost all very tall men are heavy is true

In our previous method we constrained the transformation target to fuzzy predicates and subjects, and in the above examples we constrain it to a fuzzy predicate (heavy) and a fuzzy subject (tall men). But we must handle the transformation of fuzzy predicates, fuzzy subjects, and truth qualifiers simultaneously. That is what we do in this paper. In this paper, we handle the following transformation.

Few heavy men are more or less tall is more or less true

\Leftrightarrow Only a few very heavy men are tall is very true.

We review our previous method in section 2. Section 3, discusses our new 4-step inference method. Section 4, concludes with a summary of the main points.

II. OUR PREVIOUS WORK

Here, we briefly review our previous method.

Transforming a fuzzy predicate:

In this section, we show an inference method for the following truth qualified proposition [3].

QA are F is $\tau \Leftrightarrow Q'A$ are mF is τ

We divide this into four cases according to the types of quantifier Q and modifier m as follows:

$$m \begin{cases} 0 < m < 1 \text{ (dilation; more or less,...)} \\ m > 1 \text{ (concentration; very,...)} \end{cases}$$

$$Q \begin{cases} \text{monotonic non-decreasing (most, many,...)} \\ \text{monotonic non-increasing (few, little,...)} \end{cases}$$

We assume truth qualifier τ is a monotonic function (true, false,...). Next we get Q' for a given membership function of Q and value of m. First we assume N pairs of databases for the grades of fuzzy sets of A, F and we set the grade values of each pair as

$$\mu_A(i) = A_i, \mu_F(i) = F_i \quad (i=1, \dots, N)$$

In the following inference,

$$QA \text{ are } F \text{ is } \tau \Leftrightarrow Q'A \text{ are } mF \text{ is } \tau \quad (1)$$

we set

$$A = \sum_{i=1}^N A_i$$

and we obtain the inference result as follows:

$$\text{get } \mu_{Q'}(x) = \mu_Q\left(\frac{1}{A} \sum_{i=1}^N A_i F_i\right)$$

$$\text{subject to } x = \frac{1}{M} \sum_{i=1}^N A_i F_i^m, \quad 0 < r < 1, N \geq 2, m > 0$$

$$1 \geq A_1 = \dots = A_n = a \geq A_{n+1}, A_{n+2}, \dots, A_N \geq 0 \quad (2)$$

$$0 \leq F_1 = \dots = F_n = f \leq F_{n+1}, F_{n+2}, \dots, F_N \leq 1 \quad (2-1)$$

$$n/N \geq r \quad (2-2)$$

Here we set

$$1 \leq n < N.$$

For a given x satisfying $0 \leq x \leq 1$, in case of a monotonic non-decreasing truth qualifier τ , various pairs of A_i, F_i ($i=1, \dots, N$) exist under constraints (2), (2-1), and (2-2). So various Q's exist and the membership function of Q' has not been determined uniquely. We proposed the extreme value as an inference result Q' [3] for the following reasons:

(1) corresponding to human intuition

(2) getting only one inference result

The mathematic expressions of Q' are all given as $\mu_Q(\sqrt[m]{x})$

Transforming a fuzzy subject:

Next we consider an inference method of obtaining Q' for a given membership function of Q and value of m [4].

We show the inference method in the following truth qualified proposition.

$$QA \text{ are } F \text{ is } \tau \Leftrightarrow Q'(mA) \text{ are } F \text{ is } \tau \quad (3)$$

We assume N pairs of database for the grade of fuzzy set A, F (for $i=1, \dots, N$, we denote grade values of fuzzy set A, F as A_i, F_i). We set

$$A = \sum_{i=1}^N A_i, M = \sum_{i=1}^N A_i^m$$

and obtain an inference result Q' in the following expression.

$$\text{get } \mu_{Q'}(x) = y = \mu_Q\left(\frac{1}{A} \sum_{i=1}^N A_i F_i\right)$$

$$\text{subject to } x = \frac{1}{M} \sum_{i=1}^N A_i^m F_i, \quad 0 < r < 1, N \geq 2, m > 0$$

$$1 \geq A_1 = \dots = A_n = a \geq A_{n+1}, A_{n+2}, \dots, A_N \geq 0 \quad (4)$$

$$0 \leq F_1 = \dots = F_n = f \leq F_{n+1}, F_{n+2}, \dots, F_N \leq 1 \quad (4-1)$$

$$n/N \geq r \quad (4-2)$$

For a given x ($0 \leq x \leq 1$), various pairs of A_i, F_i ($i=1, \dots, N$) satisfy the above constraints (4), (4-1), and (4-2). So various Q's exist and the inference result is not determined uniquely. So we take the extreme value for the same reasons as in the fuzzy predicate transformation. In the followings, we assume τ is a non-decreasing and injective truth qualifier.

If Q is monotonic and non-increasing and $m > 1$, we get the membership function of Q' as follows [4]:

$$\mu_{Q'}(x) = \text{MIN}[\mu_Q(y)] \\ = \mu_Q[\text{MAX}(y)]$$

We set $G(x) = \text{MAX}(y)$, $m=2$.

For quantization factor $R=1$, we get the inference result by G(x).

$$G(x) = \begin{cases} \frac{x^{1/2}}{x^{1/2} + (1-x)^{1/2}} & \text{for } 0 \leq x \leq B \\ 1 + 2(\sqrt{2}-1)(x-1) & \text{for } B \leq x \leq 1 \end{cases}$$

We used the following notation.

$$B = \frac{2 - \sqrt{2}}{4} \approx 0.15$$

III. THE 4-STEP INFERENCE METHOD

Using the method in section 2, we handle a 4-step inference referring to simultaneous transformations of fuzzy predicate, fuzzy subject, and truth qualifier.

Now we handle the following transformation.

$$QA \text{ are } F \text{ is } \tau \Leftrightarrow Q'(m'A) \text{ are } mF \text{ is } m'' \tau$$

We divide this inference into the following four steps.

STEP1

$$QA \text{ are } F \text{ is } \tau \Leftrightarrow Q'(m'A) \text{ are } F \text{ is } \tau$$

STEP2

$$Q'(m'A) \text{ are } F \text{ is } \tau \Leftrightarrow Q''(m'A) \text{ are } mF \text{ is } \tau$$

STEP3

$$Q''(m'A) \text{ are } mF \text{ is } \tau \Leftrightarrow Q(m'A) \text{ are } mF \text{ is } TT$$

IV. CONCLUSION

We obtained the inference result as an fuzzy quantifier after transforming a fuzzy predicate, a fuzzy subject, and a truth qualifier for natural language propositions involving fuzzy quantifiers and truth qualifiers. These result correspond to human intuition.

STEP4

$Q(m'A) \text{ are } mF \text{ is } TT \Leftrightarrow Q'''(m'A) \text{ are } mF \text{ is } m'' \tau$

We can execute STEP1 and STEP2 using the method given in section 2. We can easily execute STEP3 and STEP4 based on truth qualification and inverse truth qualification. We explain this using a concrete example as follows:

Knowledge

Few heavy men are more or less tall is more or less true

Question:

How few very heavy men are tall is very true?

Now we infer by the four steps as follows:

STEP1

Few heavy men are more or less tall is more or less true.

$\Leftrightarrow Q$ very heavy men are more or less tall is more or less true.

STEP2

Q very heavy men are more or less tall is more or less true.

$\Leftrightarrow QQ$ very heavy men are tall is more or less true.

STEP3

QQ very heavy men are more or less tall is more or less true.

\Leftrightarrow Few very heavy men are tall is TT .

STEP4

Few very heavy men are more or less tall is TT .

$\Leftrightarrow QQQ$ (only a few) very heavy men are tall is very true.

This 4-step inference is shown in Fig. 1. In STEP1 we get the transformed fuzzy quantifier by $\text{few} \rightarrow Q$ after transforming the fuzzy subject heavy men \rightarrow very heavy men.

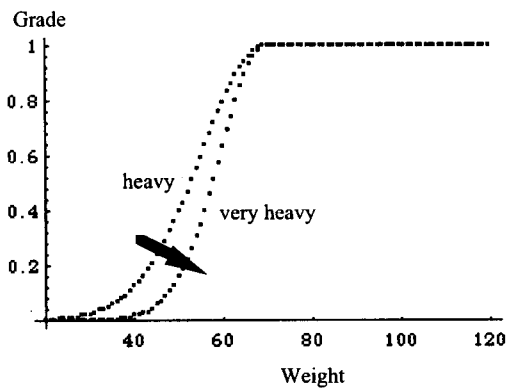
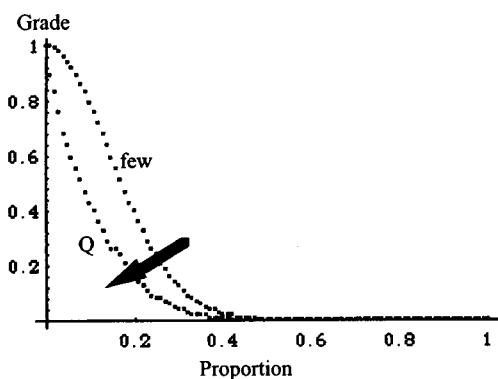
In STEP2 we get the transformed fuzzy quantifier by $Q \rightarrow QQ$ after transforming the fuzzy predicate heavy more or less tall \rightarrow tall.

In STEP3 we get the transformed fuzzy quantifier by $QQ \rightarrow \text{few}$ after transforming the truth qualifier more or less true $\rightarrow TT$. In STEP4 we get the transformed fuzzy quantifier by $\text{few} \rightarrow QQQ$ (only a few) after transforming the truth qualifier $TT \rightarrow$ very true.

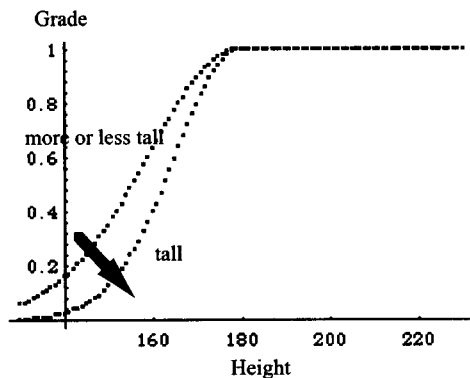
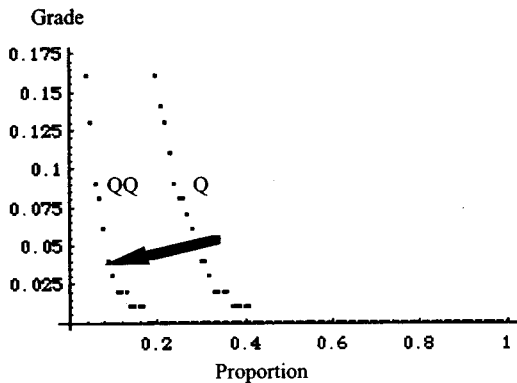
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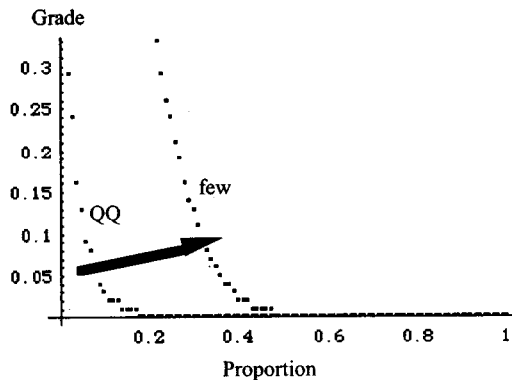
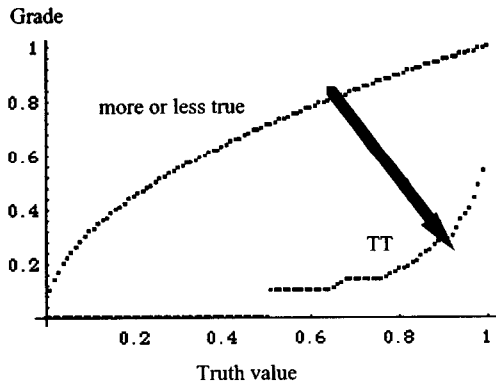
STEP1



STEP2



STEP3



STEP4

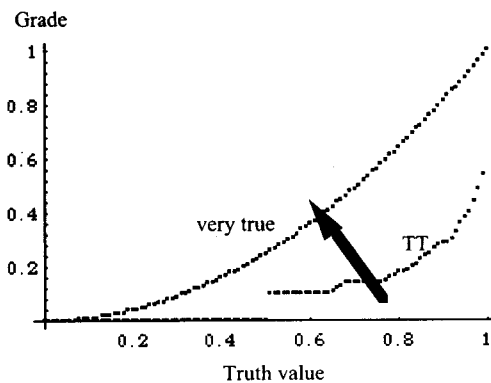
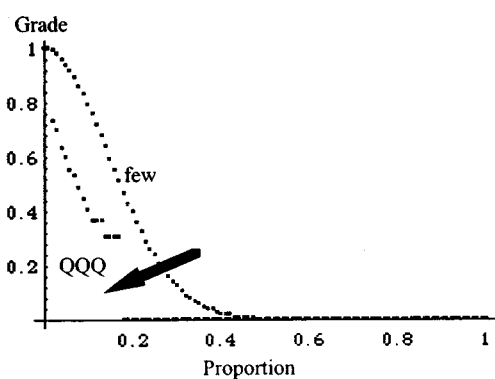


Fig.1. Step by step for 4-step inference