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The Synchronization in Hyper-Chaos

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Abstract- In this paper, we introduce a new hyper-chaos synchronization method called embedding synchronization using hyper-chaos consist of State-Controlled Cellular Neural Network (SC-CNN). We make a hyper-chaos circuit using SC-CNN with the n-double scroll. A hyper-chaos circuit is created by applying identical n-double scroll with weak coupled method to each cell. Hyper-chaos synchronization was achieved using embedding synchronization between the transmitter and receiver about each state variable in the SC-CNN.

1. Introduction

Recently, there has been interest in studying the behavior of chaotic dynamics. Chaotic systems are characterized by sensitive dependence on initial conditions, making long term prediction impossible, self-similarity, and a continuous broad-band power spectrum, etc. Chaotic systems have a variety of applications, including chaos synchronization and chaos secure communication [1-6]. Chaos synchronization and secure communication has been a topic of intense research in the past decade. However, secure communication or cryptographic using chaos has several problems [7]. First, almost all chaos-based secure communication or cryptographic algorithms use dynamical systems defined on the set of real number, and therefore are difficult for practical realization and circuit implementation. Second, security and performance of almost all proposed chaos-based methods are not analyzed in terms of the techniques developed in cryptography. Moreover, most of the proposed methods generate cryptographically weak and slow algorithms.

To address these problems, we need a hyper-chaos circuit to increase the complexity in secure communication or cryptographic communication. In this paper, we introduce a new hyper-chaos synchronization method called embedding synchronization using State-Controlled Cellular Neural Network (SC-CNN) as a hyper-chaos circuit. We make a hyper-chaos circuit using SC-CNN with the n-double scroll [8].

In order to make a hyper-chaos circuit, we use identical n-double scroll with weak coupled method to each cell. Then we accomplish a hyper-chaos synchronization using embedding synchronization method between the transmitter and receiver about each state in the SC-CNN.

2. Hyper-chaos circuit

To create a hyper-chaos circuit, we use the n-double scroll using the weak coupling method [8].

2.1 n-Double scroll circuit

In order to synthesize a hyper-chaos circuit, we first consider Chua's circuit modified to an n-double scroll attractor. The electrical circuit for obtaining n-double scroll, according to the implementation of Arena et al. [12] is given by

$$\begin{aligned} \dot{x} &= \alpha[y - h(x)] \\ \dot{y} &= x - y - z \\ \dot{z} &= -\beta y \end{aligned} \tag{1}$$

with a piecewise linear characteristic

$$h(x) = m_{2n-1}x + \frac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1} - m_i)(|x + c_i| - |x - c_i|) \tag{2}$$

consisting of 2(2n-1) breakpoints, where n is a natural number. In order to generate n double scrolls one takes $\alpha = 9$ and $\beta = 14.286$. Some special cases are:

1-double scroll

$$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, c_1 = 1$$

2-double scroll

$$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, m_2 = -\frac{4}{7}, m_3 = m_1, c_1 = 1, c_2 = 2.15, c_3 = 3.6$$

3-double scroll

$$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, m_2 = -\frac{4}{7}, m_3 = m_1, m_4 = m_2, m_5 = m_3,$$
$$c_1 = 1, c_2 = 2.15, c_3 = 3.6, c_4 = 8.2, c_5 = 13$$

The 2-double scroll attractor is shown in Fig.1

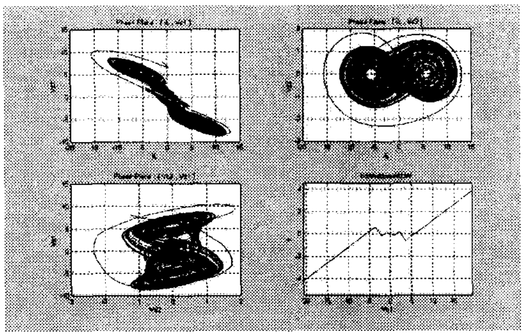


Fig. 1 2- double scroll attractor

2.2 Hyper-chaos circuit

To synthesize a hyper-chaos circuit, we second consider one-dimension cellular neural network (CNN) with n-double scroll cell [8]. The following equations describe a one-dimensional CNN consisting of identical n-double cell with diffusive coupling as

$$\begin{aligned} \dot{x}^{(j)} &= \alpha\left[y^{(j)} - h(x^{(j)})\right] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} - z^{(j)} \\ \dot{z}^{(j)} &= -\beta\, y^{(j)} \quad j = 1,2,\ldots,L \end{aligned} \tag{3}$$

or

$$\begin{aligned} \dot{x}^{(j)} &= \alpha\left[y^{(j)} - h(x^{(j)})\right] \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} - z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{z}^{(j)} &= -\beta\, y^{(j)} \quad j = 1,2,\ldots,L \end{aligned} \tag{4}$$

where L denotes the number of cells. We impose the condition that $x^{(0)} = x^{(L)}, x^{(L+1)} = x^{(1)}$ for equation (3) and (4).
For the coupling constants, $K_0 = 0, K_j = K(j = 1,\ldots,L-1)$ and positive diffusion coefficients D_x, D_y are chosen base on stability theory.

Hardware implementation for hyper-chaos circuit with a CNN using n-double scroll and hyper-chaos attractor are shown in Fig. 2 and 3 respectively.

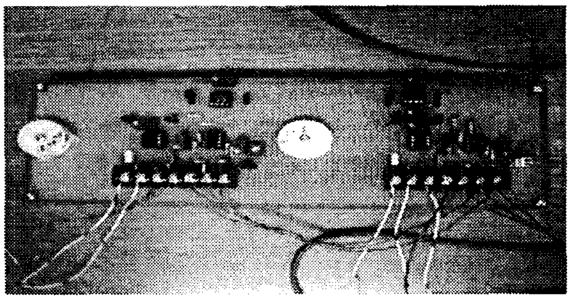


Fig. 2 Hardware implementation for hyper-chaos circuit

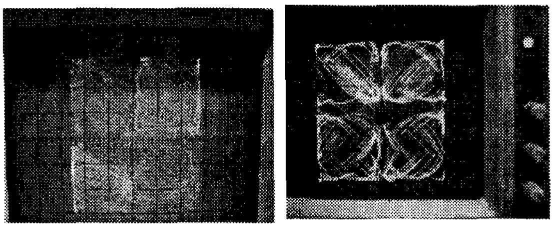


Fig. 3 Hyper-chaos attractor

2.3 C- CNN model [12,13]

In [12, 13], the follow generalized cell was introduced:

$$\dot{x}_j = x_j + a_j y_j + G_o + G_s + i_j \tag{5}$$

where j is the cell index, x_j the state variable, y_j the cell output given as

$$y_j = 0.5(|x_j + 1| - |x_j - 1|) \tag{6}$$

where, a_j a constant parameter and i_j a threshold value. In equation (5), G_o is linear combination of the outputs and G_s is state variable of the connected cells.

Generalizing the output nonlinearity (6), the following new output PWL equation is considered

$$y_j = \frac{1}{2}\sum_{k=1}^{2n-1} n_k(|x + b_k| - |x - b_k|) \tag{7}$$

where b_k is the break point and the coefficients n_k is related to the slopes of segments.

SC-CNN cells required to generate the n-double scroll in accordance with the state equation (5) and output equation (7) are given by

$$\dot{x}_1 = -x_1 + a_1 y_1 + a_{12} y_2 + a_{13} y_3 + \sum_{k=1}^{3} s_{1k} x_k + i_1$$
$$\dot{x}_2 = -x_2 + a_{21} y_1 + a_2 y_2 + a_{23} y_3 + \sum_{k=1}^{3} s_{2k} x_k + i_2 \qquad (8)$$
$$\dot{x}_3 = -x_3 + a_{31} y_1 + a_{32} y_2 + a_3 y_3 + \sum_{k=1}^{3} s_{3k} x_k + i_3$$

where x_1, x_2, x_3 are state variables and y_1, y_2, y_3 are corresponding outputs. More details about the SC-CNN are given in reference [12,13]

3. The synchronization of hyper-chaos circuit

The state equation of dimensionless type of SC-CNN is written as follows:

The state equation of transmitter

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_1 + \alpha(x_2 - g_1) \\ \dot{x}_2 &= -x_2 + x_1 + x_3 \\ x_3 &= -x_3 - \beta x_2 + x_3 \\ g_1 &= m_3 x_i + 1/2 \sum_{k=0}^{2} (m_k + m_{k+1})(|x_1 + c_k| - |x_1 - c_k|) \end{aligned} \qquad (9)$$

The state equation of receiver

$$\begin{aligned} \dot{x}_4 &= -x_1 + x_1 + \alpha(x_2 - g_2) \\ \dot{x}_5 &= -x_5 + x_4 + x_6 \\ x_6 &= -x_6 - \beta x_5 + x_6 \\ g_2 &= m_3 x_i + 1/2 \sum_{k=0}^{2} (m_k + m_{k+1})(|x_1 + c_k| - |x_1 - c_k|) \end{aligned} \qquad (10)$$

In terms x_1, x_4 of equations (9) and (10), we can find that there are contained x_2 term each other. Using the x_2 term, we propose the new synchronization method which is accomplish only embedding of a part of term in the differential equations of transmitter and receiver with Sc-CNN. We called this synchronization method is embedding synchronization.

The block diagram for embedding synchronization of hyper-chaos in Fig. 4., and the result of embedding synchronization is shown in Fig. 5.

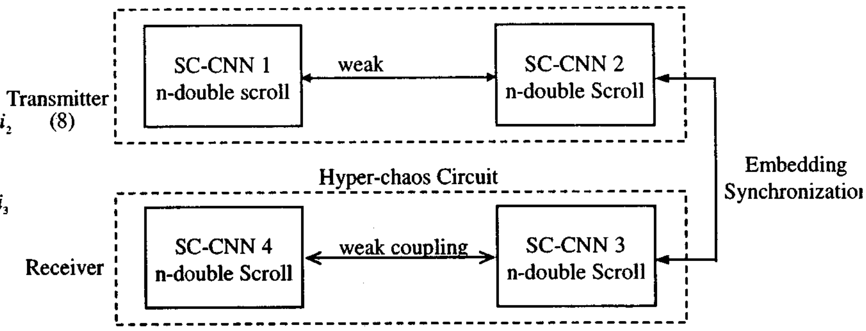
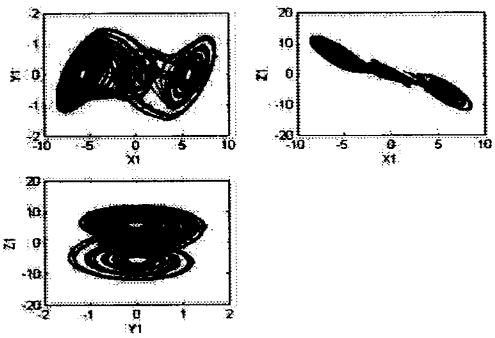
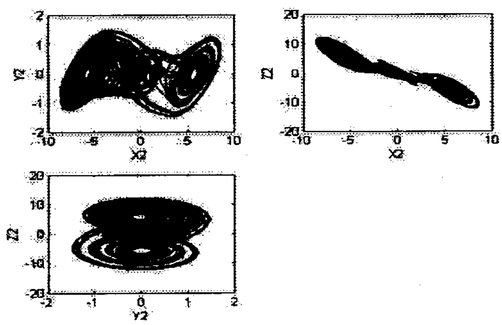


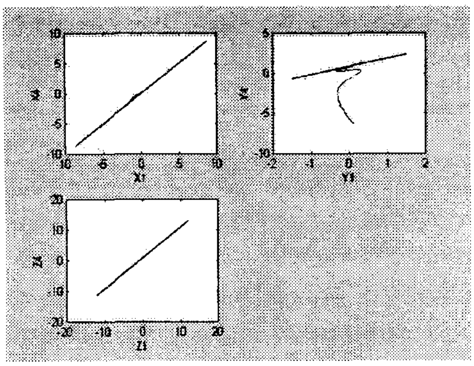
Fig. 4 The Block diagram of hyper-chaos embedding synchronization.



(a)



(b)



(c)

Fig. 5 Synchronization results. (a) Phase plane in the transm itter after synchronization, (b) Phase plane in the receiver after synchronization, (c) phase unite diagram

Im the Fig. 5, We confirms effective synchronization result between the transmitter and receiver in the SC-CNN.

5. Concluding Remarks

In this paper, we introduced a new hyper-chaos synchronization method which is called embedding synchronization using SC-Cellular Neural Network (SC-CNN) as a hyper-chaos. As a computer simulation result, we confirm embedding synchronization method by compare to phase plane in the transmitter and receiver about each other.

Acknowledgement

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