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A Motion Compression Method by Min S-norm Composite Fuzzy Relational Equations

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Abstract— A motion compression method by min s-norm composite fuzzy relational equations (dual-MCF) is proposed, where a motion sequence is divided into intra-pictures (I-pictures) and predictive-pictures (P-pictures). The I-pictures and the P-pictures are compressed by using uniform coders and non-uniform coders, respectively. A design method of non-uniform coders is proposed to perform an efficient compression and reconstruction of the P-pictures, based on the dual overlap level of fuzzy sets and a fuzzy equalization. An experiment using 10 P-pictures confirms that the root means square errors of the proposed method is decreased to 82.9% of that of the uniform coders, under the condition that the compression rate is 0.0055. An experiment of motion compression and reconstruction is also presented to confirm the effectiveness of the dual-MCF based on the non-uniform coders.

1 Introduction

Fuzzy relational calculus can be applied to image processing schemes by normalizing the brightness level of original images into $[0, 1]$ [1]. A motion compression method based on max t-norm composite fuzzy relational equations (MCF) has been proposed in [2]. In this paper, a dual-MCF is proposed by replacing the max t-norm composite fuzzy relational equations of MCF with the dual ones, i.e., min s-norm composite fuzzy relational equations. A motion sequence is decomposed into intra-pictures (I-pictures) and predictive-pictures (P-pictures) in the dual-MCF. The I-picture corresponds to the original frame of the motion, and the P-picture is obtained by subtracting the reconstructed images of I-picture and the original frame of the motion. The I-picture and P-picture are compressed by using uniform coders and non-uniform coders, respectively. In order to perform an efficient compression and reconstruction of P-pictures, a non-uniform coders design method is proposed. It is based on the dual overlap level of fuzzy sets and a fuzzy equalization.

Section 2 shows a formalization of the dual-MCF. A design method of non-uniform coders for the compression of P-pictures is presented in Sec. 3. In Sec. 4, an experiment using 10 P-pictures shows the effectiveness of the proposed method. Furthermore, an experiment using tennis motion (extracted form standard motion imagery [3]) is also presented to confirm the effectiveness of the dual-MCF based on the non-uniform coders.

2 Formalization of Dual-MCF

2.1 An Overview of Dual-MCF

A motion compression method by min s-norm composite fuzzy relational equations (dual-MCF) treats motion (an image sequence with size $M \times N$) as fuzzy relations $\mathbf{R} = \{R_k | k = 1, \ldots, S\} \subset F(\mathbf{X} \times \mathbf{Y})$, $\mathbf{X} = \{x_1, \ldots, x_M\}$, $\mathbf{Y} = \{y_1, \ldots, y_N\}$ by normalizing the intensity range of each pixel into $[0, 1]$, where, k indicates the index of time sequence. In the dual-MCF, the motion is divided into intra pictures (I-pictures) and predictive pictures (P-pictures) shown in Fig. 1. The I-picture corresponds to the original frame of the motion, and the P-picture is obtained by subtracting the reconstructed images of I-picture and the original frame of the motion. Figure 2 shows an example of an I-picture and a P-picture of tennis motion [3]. The motion is compressed by the dual-MCF as shown in Fig. 3, i.e., I-pictures are compressed by U-ICF (Image Compression method based on Fuzzy relational equations, Uniform coders type [1]), while P-pictures are compressed by N-ICF (ICF, Non-uniform coders type).

2.2 U-ICF

The compression of U-ICF is performed by the min s-norm composite fuzzy relational equations. The U-ICF compresses the I-picture $R_k^{(I)} \in F(\mathbf{X} \times \mathbf{Y})$ of

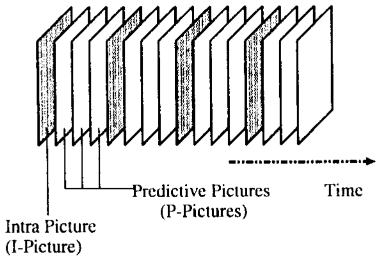

Figure 1: A sequence of images in dual-MCF

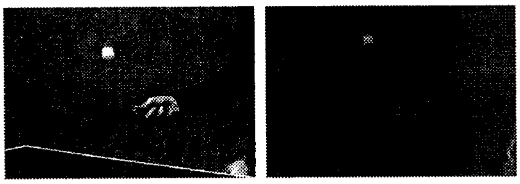

Figure 2: An example of I-picture (left) and P-picture (right)

a motion $\mathbf{R}$ only, where $R_k^{(I)}$ is defined by

$$R_k^{(I)} = R_k. \tag{1}$$

The I-picture is compressed into $G_k^{(I)} \in F(\mathbf{I} \times \mathbf{J})$ by

$$G_k^{(I)}(i,j) = \min_{y \in \mathbf{Y}} \left\{ B_j(y) \ s \ \min_{x \in \mathbf{X}} \left\{ A_i(x) \ s \ R_k^{(I)}(x,y) \right\} \right\}, \tag{2}$$

where s indicates a continuous s-norm, while $A_i \in \mathbf{A} \subset F(\mathbf{X})$ and $B_j \in \mathbf{B} \subset F(\mathbf{Y})$ stand for coders. The coders $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \{A_1, A_2, \ldots, A_I\}, \tag{3}$$

$$A_i(x_m) = 1.0 - \exp\left(-Sh\left(\frac{iM}{I} - m\right)^2\right), \quad (m = 1, 2, \ldots, M), \tag{4}$$

and

$$\mathbf{B} = \{B_1, B_2, \ldots, B_J\}, \tag{5}$$

$$B_j(y_n) = 1.0 - \exp\left(-Sh\left(\frac{jN}{J} - n\right)^2\right), \quad (n = 1, 2, \ldots, N), \tag{6}$$

where Sh denotes the sharpness of fuzzy sets A_i and B_j. The distribution of coders is uniform, therefore, the image compression process is called U-ICF (image compression and reconstruction based on fuzzy relational equations, Uniform coders). The compression rate can be adjusted by the number of fuzzy sets $I \times J$ contained in coders $\mathbf{A}$ and $\mathbf{B}$. Afterwords, as can be seen from Fig. 3, the compressed image obtained by U-ICF or N-ICF is compressed by Lempel-Ziv coding method. The compression rate is defined as

$$\rho = \frac{\text{File size of compressed image}}{\text{File size of original image}}. \tag{7}$$

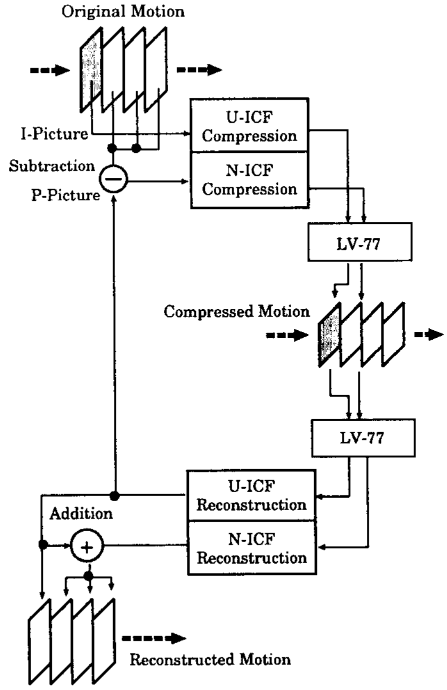

Figure 3: Overview of MCF

The image reconstruction corresponds to an inverse problem under the condition that the compressed image $G_k^{(I)}$ and the coders $\mathbf{A}$ and $\mathbf{B}$ are given. A reconstructed image $\tilde{R}_k^{(I)} \in F(\mathbf{X} \times \mathbf{Y})$ is given by

$$\tilde{R}_k^{(I)}(x,y) = \max_{j \in \mathbf{J}} \left\{ B_j(y) \beta_{s'} \min_{i \in \mathbf{I}} \left\{ A_i(x) \beta_{s'} G_k^{(I)}(i,j) \right\} \right\}, \tag{8}$$

where $\beta_{s'}$ denotes the s-relative pseudo-complement defined by

$$a \beta_{s'} b = \inf\{c \in [0,1] | a s' c \geq b\}, \tag{9}$$

and a s-norm s' should be selected such that

$$a, b \in [0,1], \quad a s b \geq a s' b. \tag{10}$$

2.3 N-ICF

The P-picture $R_k^{(P)} (\in F(\mathbf{X} \times \mathbf{Y}))$ is compressed and reconstructed by N-ICF unit shown in Fig. 3, where the P-picture $R_k^{(P)}$ is obtained by

$$R_k^{(p)}(x,y) = \left\{ \left(R_k(x,y) - \tilde{R}_{k-1}^{(I)}(x,y) \right) + 1.0 \right\} / 2.0, \quad (x,y) \in \mathbf{X} \times \mathbf{Y}. \tag{11}$$

Equation (11) means the subtraction and the bias. In the case of fuzzy relational equation theory, the universe of discourse corresponds to closed unit interval $[0,1]$, i.e., minus real numbers cannot be treated. Therefore, the bias of Eq.(11) should be performed. In the case of N-ICF, the compression and reconstruction are also done by Eqs. (2), (8)-(10), respectively. However, the distribution of coders **A** and **B** of N-ICF is different from that of U-ICF. The N-ICF can focus different parts of the image by changing allocation of the fuzzy sets of coders, non-uniformly. Therefore, the N-ICF is useful for the compression/reconstruction of the line image that is often observed in P-pictures shown in Fig. 2 (right).

3 Non-uniform Coders Design for N-ICF

In the N-ICF, non-symmetric Gaussian coders are defined by

$$\mathbf{A} = \{A_1, A_2, \ldots, A_I\}, \quad (12)$$

$$A_i(x_m) = \begin{cases} 1.0 - \exp\left(-Sh_{left}^{(A_i)}(A_i^c - m)^2\right), & if \quad m \le A_i^c, \\ 1.0 - \exp\left(-Sh_{right}^{(A_i)}(A_i^c - m)^2\right), & if \quad m > A_i^c, \end{cases} \quad (13)$$

$$(m = 1, 2, \ldots, M),$$

where A_i^c denotes the center point of the fuzzy set A_i, while, $Sh_{left}^{(A_i)}$ and $Sh_{right}^{(A_i)}$ stand for the shape of the fuzzy sets A_i. The distribution of the coder (expressed by $Sh_{left}^{(A_i)}$ $Sh_{right}^{(A_i)}$, and A_i^c) is adjusted by the following steps. For simplicity, the coder **A** is only considered.

1. Calculate the standard deviation $\sigma(x_m)$ given by
$$\sigma(x_m) = \frac{1}{N}\{\sum_{y \in \mathbf{Y}}(R_k^{(P)}(x_m, y) - E(x_m))^2\}, \quad (14)$$
$$E(x_m) = \frac{1}{N}\sum_{y \in \mathbf{Y}} R_k^{(P)}(x_m, y), \quad (15)$$
$$(\mathrm{m} = 1, \ldots, \mathrm{M}).$$
2. Calculate the equalization index σ_T/I defined by
$$\sigma_T/I = \frac{1}{I}\sum_{x \in \mathbf{X}} \sigma(x). \quad (16)$$
3. Start from the lower bound of **X**, i.e., x_1. Proceed toward higher values of **X** computing the moving value of the summation,
$$\sum_{x \in \mathbf{X}} \sigma(x). \quad (17)$$
Stop once the value of this summation has reached the value of σ_T/I and record the corresponding value of the argument as π_i (region of fuzzy set $A_i \in F(\mathbf{X})$, see Fig. 4 (left)). Determine the center point A_i^c of π_i (see Fig. 4) (right)).
4. Initialize the coder **A** defined by Eqs. (12) - (13).
5. For the $A_{i-1}, A_i \in \mathbf{A}$, the dual overlap level $\alpha(A_{i-1}, A_i)$ of A_{i-1} and A_i,
$$\alpha(A_{i-1}, A_i) = \max_{x \in \mathbf{X}_{(i-1)}^{(i)}} \{\min(A_{i-1}(x), A_i(x))\}, \quad (18)$$
where $\mathbf{X}_{(i-1)}^{(i)} = \{x_m \in \mathbf{X} | A_{i-1}^c \le x_m \le A_i^c\}$ is computed shown in Fig. (4) (right).
5-A) If $\alpha(A_{i-1}, A_i) \ge \alpha_{opt} + \epsilon$, then $Sh_{right}^{(A_i)}$ of A_{i-1} and $Sh_{left}^{(A_i)}$ of A_i are decreased, where α_{opt} denotes the optimal overlap level 0.15 and ϵ stands for a tolerance which is set at 0.05.
5-B) If $\alpha(A_{i-1}, A_i) \le \alpha_{opt} - \epsilon$, then $Sh_{right}^{(A_i)}$ of A_{i-1} and $Sh_{left}^{(A_i)}$ of A_i are increased.
5-C) If $\alpha_{opt} - \epsilon \le \alpha(A_{i-1}, A_i) \le \alpha_{opt} + \epsilon$, then the adjustment process of A_{i-1} and A_i is stopped.
6. Repeat step 5 for the successive fuzzy sets in **A**.

In this method, the first and the last fuzzy sets in **A** are defined by trapezoidal membership functions shown in Fig. 4 (right)).

The coder **B** is also designed according to the above process.

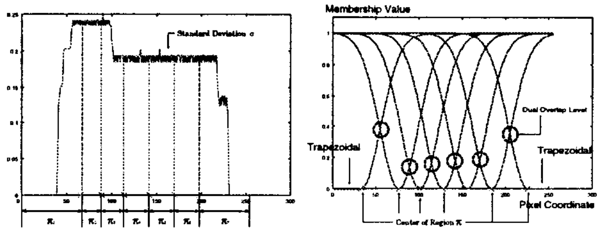

Figure 4: Standard deviation $\sigma(x)$ of the proposed method (left) and an example of a non-uniform coder **A** (right)

4 Experimental Comparisons

4.1 A Comparison of Non-uniform Coders with Uniform Coders

In order to show the effectiveness of the non-uniform coders, an experiment of compression and reconstruction of P-pictures is performed. In this experiment, a comparison of the non-uniform coders obtained by the proposed method with uniform ones (Eqs. (3)-(6)) is presented. Ten P-pictures (e.g.,

Table 1: RMSE comparison

	Non-uniform	Uniform
RMSE	7.247	8.741

Fig. 2 (right)) of size $M \times N = 384 \times 240$ are compressed into size of $I \times J = 88 \times 60$ by the non-uniform coders and the uniform coders, respectively. The compression rate ρ is 0.0055 (with entropy coding). The average of RMSE (Root Mean Square Errors) of the reconstructed images obtained by the non-uniform and uniform coders is shown in Table 1, where the RMSE is defined as

$$RMSE = \sqrt{\frac{\sum_{(x,y)\in \mathbf{X}\times\mathbf{Y}} \left(R(x,y) - \tilde{R}(x,y)\right)^2}{|\mathbf{X}\times\mathbf{Y}|}}. \quad (19)$$

4.2 Motion Compression and Reconstruction

An example of motion compression/reconstruction by the dual-MCF is presented, where the original motion (the size of each frame $M \times N = 384 \times 240$) is shown in Figs. 5, here, the original frame 1 corresponds to the I-pictures, while, the original frame 2,3, and 4 correspond to the P-pictures. Therefore, the original frame 1 is compressed and reconstructed by U-ICF, under the condition that the number of fuzzy sets in coders $I \times J$ is 172×120. The frame 2,3, and 4 are compressed and reconstructed by N-ICF under the condition that the number of fuzzy sets in coders $I \times J$ is 88×60.

Figures 6 show the reconstructed images.

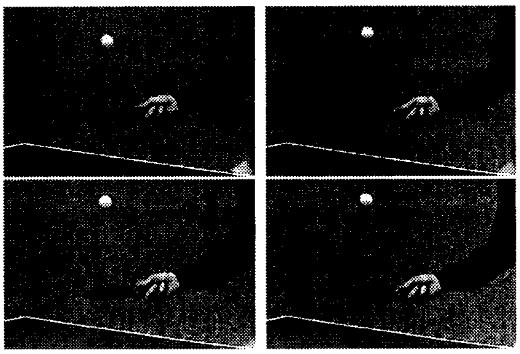

Figure 5: Original frames

5 Conclusions

A motion compression method by min s-norm composite fuzzy relational equations (dual-MCF) has been presented by replacing the max t-norm composite fuzzy relational equations of MCF [2] with the dual ones, i.e., min s-norm composite fuzzy relational equations. In the case of dual-MCF, a motion sequence is divided into intra-pictures (I-pictures) and predictive-pictures (P-pictures). The I-picture and P-picture are compressed by using uniform coders and non-uniform coders, respectively. In order to perform an efficient compression and reconstruction of P-pictures, a design method of non-uniform coders has been proposed. The proposed design method is based on the dual overlap level of fuzzy sets and a fuzzy equalization. Through an experiment using 10 P-pictures, it has been shown that the root means square errors of the proposed method is decreased to 82.9% of the uniform coders, under the condition that the compression rate is 0.0055. Furthermore, an experiments of compression and reconstruction of tennis motion by MCF based on the non-uniform coders has been performed. It was confirmed that the quality of reconstructed motion is acceptable to recognize the human behavior of the tennis motion.

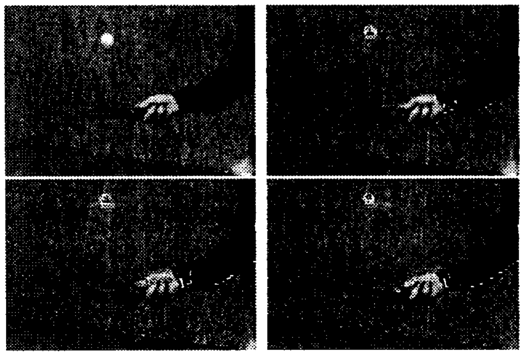

Figure 6: Reconstructed frames

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[2] H. Nobuhara, and K. Hirota, "Non-uniform Coders Design for Motion Compression Method by Fuzzy Relational Equation," International Fuzzy System Association World Congress, pp. 428-435 (Lecture Notes in Artificial Intelligence No. 2715), Istanbul, Turkey, June 29 - July 2, 2003.

[3] http://sampl.eng.ohio-state.edu/ sampl/database.htm