최영미 이형천
Ajou University

AM-10

Analysis and Approximation of Linear feedback control problems for the Boussinesq equations

In this work we consider the mathematical formulation and numerical resolution of the linear feedback control problem for Boussinesq equations. The controlled Boussinesq equations is given by

$$\frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla\mathbf{u} + \nabla p = \beta\theta\mathbf{g} + \mathbf{f} + \mathbf{F} \qquad \text{in } (0,T)\times\Omega,$$

$$\nabla\cdot\mathbf{u} = 0 \qquad \text{in } (0,T)\times\Omega,$$

$$\mathbf{u}|_{\partial\Omega} = 0, \quad \mathbf{u}(0,\mathbf{x}) = \mathbf{u}_0(\mathbf{x})$$

$$\frac{\partial\theta}{\partial t} - \kappa\Delta\theta + (\mathbf{u}\cdot\nabla)\theta = \tau + T, \qquad \text{in}(0,T)\times\Omega$$

$$\theta|_{\partial\Omega} = 0, \qquad \theta(0,\mathbf{x}) = \theta_0(\mathbf{x}),$$

where Ω is a bounded open set in $\mathbb{R}^n$, $n=2$ or 3 with a C^∞ boundary $\partial\Omega$.

The control is achieved by means of a *linear feedback law* relating the body forces to the velocity and temperature field, i.e.,

$$\mathbf{f} = -\gamma_1(\mathbf{u}-\mathrm{U}), \qquad \tau = -\gamma_2(\theta-\Theta)$$

where (U,Θ) are target velocity and temperature. We show that the unsteady solutions to Boussinesq equations are stabilizable by internal controllers with exponential decaying property. In order to compute (approximations to) solution, semi discrete-in-time and full space-time discrete approximations are also studied. We prove that the difference between the solution of the discrete problem and the target solution decay to zero exponentially for sufficiently small time step.