# 유변학적인 분체유동해석의 연속체적인 접근

박민수, 주상우
영남대학교 기계공학부

# RHEOLOGICAL STUDY OF FRICTIONAL-COLLISIONAL BEHAVIOUR FOR GRANULAR FLOW : A CONTINUUM APPROACH

Min Soo Park, Sang Woo Joo
Department of Mechanical Engineering Yeungnam University

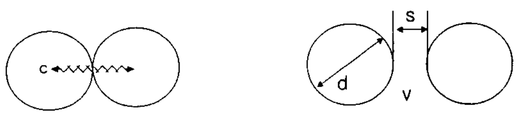
## 1. Abstract

The frictional-collisional behaviour of granular materials in motion is for a granular flow due to investigated by a continuum theory. There are three basic mechanical phenomena, which are kinetic, collision, and friction phenomenon. Using a binary collision model, the flying time, collision time and friction time indicate the time for kinetic motion, for the collision motion and for friction motion respectively. A dynamic constitutive equation is then postulated with four state variables, dispersive pressure, viscosity, thermal diffusivity and energy sink. The conservation laws for mass, momentum and energy are posed along with the constitutive model to study, as an example, the granular flow on an inclined and vertical plate due to gravity.

## 2. Introduction

The thesis is that, depending upon the agitation of the particle, their mean free path may vary so that the bulk material under the rapid motion of the particles is not density preserving. We close this system of equations by phenomenology relationships for pressure, viscosity of the "viscous" stress, the diffusivity of the flux of fluctuation energy and the annihilation rate of the fluctuation energy. these qualities are coupled with the other field variables, primarily the fluctuation energy. This model was applied to the bulk density to the general density. We still restrict considerations to binary collisions but define the time of an encounter between two particles to consist of the small but finite contact time plus the time of free flight prior to the next collision. With this contact time being included in the formal definition of the duration of an encounter the proposed model is able to avoid the ofter mentioned in the transport coefficients.

## 3. Parameterization of three phenomenon




$$t_c = \alpha \frac{d}{c}, \ t_f = \beta \frac{d}{f} \qquad t_k = \frac{s}{v}$$

Figure 1. Two identical spheres microscopic model.

More explicit descriptions are shown to the physical effects that is correspond to the two particles sketched. The two different kinds of times in the contact motion and the other time for friction motion are prepared, where $s$ is the mean separation distance of particles, $v$ is approaching speed, $d$ is granular diameter, $c=\sqrt{(E/\rho)}$ is the elastic speed of granular media and $f=\sqrt{(\tau/\rho)}$ is the frictional speed of granular media. in Fig 1. $\alpha$ and $\beta$ are dimensional numbers of order unity that can be treated as an adjustable parameter of this theory.

## 4. The equation of motion

We should ponder our granular system to be describable by the three governing equations of classical continuum as in fluid mechanics to

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}(\rho u_i)=0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u_i)=-\frac{\partial}{\partial x_k}[P\delta_{ik}+\rho u_i u_k-\eta(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i})]+\rho g_i \tag{2}$$

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho u^2+\frac{1}{2}\rho\,\bar{v}^2)=-\frac{\partial}{\partial x_k}[\rho x_k(\frac{P}{\rho}+\frac{1}{2}u^2+\frac{1}{2}\bar{v}^2)-u_i\eta(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}) -K\frac{\partial}{\partial x_k}(\frac{1}{2}\rho v^2)]+\rho u_i g_i-I \tag{3}$$

in which $u_i$ is the macroscopic velocity of grain system, $v$ fluctuation velocity and $K$ functions like the thermal diffusivity. $K$ functions like a thermal diffusivity. The property of $I$ gives the rate at which energy is lost from the system.

## 5. The Microscopic Model

### 5.1 The state of dispersive pressure

Dispersive pressure is established for one effect dimensionally, it is defined " force per are" or "mass times acceleration per area". The mass of a cell is separable in case of kinetic time and collision & friction time.

$$P=t\left(\rho(d+s)\frac{v}{t_k}+\rho d\frac{v}{t_c}+\rho d\frac{v}{t_f}\right)=t\rho\left((d+s)\frac{v^2}{s}+\frac{vc}{\alpha}+\frac{vf}{\beta}\right) \tag{4}$$

### 5.2 The state of viscosity

When the independent motions, the dynamics viscosity is dimensionally given by " density times area divided by time". If the three independent quantities are taken to be $\rho$, area and time respectively in discussing above, then

$$\sigma=q\rho\left(\frac{(d+s)^2}{t_k}+\frac{d(d+s)}{t_c}+\frac{d(d+s)}{t_f}\right)\frac{du}{dy} \tag{5}$$

### 5.3 The state of thermal diffusivity

It might be resulted in the dimensional similarity with the kinetic viscosity, so that simply postulating the dimension of the thermal diffusivity is shown $K\sim\eta/\rho$ .

$$K=r\left(\frac{(d+s)^2}{t_k}+\frac{d(d+s)}{t_c}+\frac{d(d+s)}{t_f}\right) \tag{6}$$

### 5.4 The collisional energy sink

As $\triangle E\sim(1-e^2)mv^2/2$ where $e$ is the coefficient of restitution and $m$ is the mass of the colliding particles. Multiplying by the collision rate $v/s$ and the number density yield, where $\gamma$ is a dimensionless factor proportional to $1-e^2$.

$$I=\gamma\rho\left(\frac{v^3}{s}+\frac{v^2c}{\alpha d}+\frac{v^2f}{\beta d}\right) \tag{7}$$

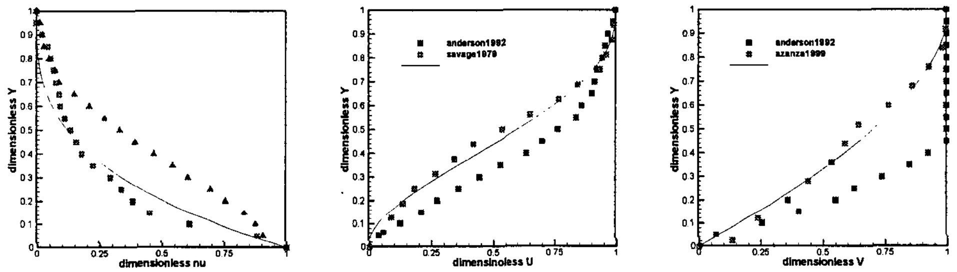
## 6. Some application of the equation

### 6.1 Steady state Granular flow in inclined plate

Consider the granular flow in inclined plate with assumptions, fully developed flow, steady state, uniform depth and the variables being function of only y-direction as

$$u_{bottom}=0,\quad v_{bottom}=0,\quad \left.\frac{dv}{dy}\right|_{top}=0 \tag{8}$$

Before solving the granular flow inclined plate constitutive equation for volume fraction from the Bagnold(1954)'s experiment was applied.




(a) (b) (c)

Figure 2. Comparison between present and other experimental and theoretical results for volume fraction, velocity and thermal velocity in inclined plate flow.

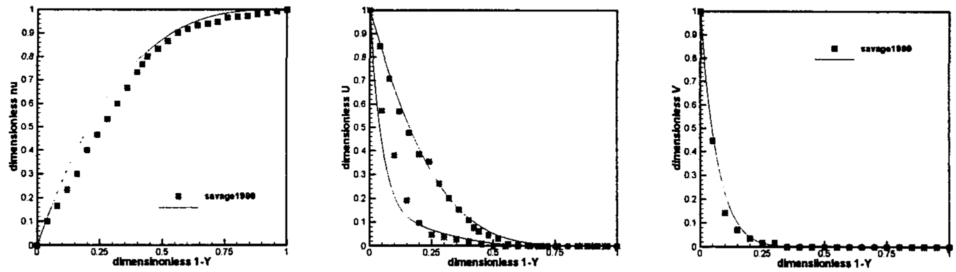
Fig.2(a) shows the comparison between this paper and other experimental and theoretical result for volume fraction of granular flow inclined plate. It is remarkable that my work and one of his work for considering the frictional-collisional granular flow theory down a rough plane can derive the similar physical meanings to understand the granular behaviour as flow. In Fig.2(b), my work show the corresponding to experimental results rather than theoretical results, and the characteristic of thermal velocity profile in Fig.2(c) by Anderson(1992) comparing with experimental result by Azanza(1999) and the theoretical result by Anderson(1992) shows the corresponding to experimental results rather, so we can confirm the characteristic of the flow velocity, the thermal velocity and the volume fraction again comparing with other works by dimensionless of variables

### 6.2 Steady state granular flow in vertical channel

Consider the granular flow in vertical channel with assumptions as same as the granular flow in inclined plate flow. The boundary conditions being a little different with inclined problem take no flow velocity and the thermal velocity at the wall for comparing with the pervious works in the following.

$$u_{wall}=0,\quad v_{wall}=0,\quad \left.\frac{dv}{dy}\right|_{center}=0 \tag{9}$$

Mass conservation equation is satisfied automatically, the momentum equation for y-direction and x-direction can be obtained respectively. The Fig.3(a) shows the comparison between my work and the theoretical result by Savage(1999) for the volume fraction. It seems not to be fair to compare each other though they have a few different view point, so the modified equation was used for comparison. Fig. 3(b). shows that plug flow region of the theoretical result by Savage(1999) seems to be correspond well for my work while the detective area is getting closed to the wall cause of the simple boundary analysis contributed. Fig. 3(c) shows the comparison of thermal velocity with theoretical result by Savage(1999) for $w$=80 which seems to be corresponding to the center rather than the wall.




(a) (b) (c)

Figure 3. Comparison between my proposal and theoretical results for volume fraction, velocity and thermal velocity in vertical flow.

## 7. Conclusion

We consider the frictional collisional behaviour of granular materials in flow motion investigated by a continuum theory due to the gravity, proposed the three phenomenon, kinetic, collision and friction, made result and compared the volume fraction, the thermal velocity and the flow velocity with previous work. For the further work, we have to investigate the importance of $\alpha$ and $\beta$, which is too complicated to measure formally, and many experimental skills were expected to be developed for improvement of granular physics. some of them will be dealt with in an upcoming paper

## 8. Acknowledgment

This work was supported by Grant No. R05-2000-000-00315-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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