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Phase inversion of seismic data

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Abstract: Waveform inversion requires extracting a reliable low frequency content of seismic data for estimating of the low wave number velocity model. The low frequency content of the seismic data is usually discarded or neglected because of the band-limited response of the source and the receivers.

In this study, however small the spectral of the low frequency seismic data is, we assume that it is possible to extract a reliable phase information of the low frequency from the seismic data and use it in waveform inversion. To this end, we exploit the frequency domain finite element modeling and source-receiver reciprocity to calculate the Frechèt derivative of the phase of the seismic data with respect to the earth model parameter such as velocity, and then apply a damped least squares method to invert the phase of the seismic data. Through numerical example, we will attempt to demonstrate the feasibility of our method in estimating the correct velocity model for prestack depth migration.

1. Introduction

In prestack depth migration, an accurate velocity model is crucial for obtaining high fidelity image of the subsurface. Geophysicists are continuously developing an efficient velocity estimation tool to be used in conventional velocity analysis including the Migration Velocity Analysis to the reflection tomography.

In this study, we present a new algorithm of phase inversion that provides an accurate velocity model comparable to those obtained by the waveform inversion technique. Our method calculates the wavefield in the frequency domain using the existing finite element or finite difference modeling technique. By solving the wave equation in frequency domain, we can express a wavefield in the frequency domain as $d = Ae^{i\omega t}$, where, A is the amplitude, ω is the angular frequency and t is the phase. In conventional seismic inverse problems, we choose the misfit as a function of both the amplitude and the phase of the wavefield. By assuming that the amplitude of both the measured data and the modeled data is fixed and the phase term only varies, we construct the objective function that can only be the function of the phase term of the wavefield. From a practical point of view, the use of a low frequency of seismic data in the waveform inversion cannot be regarded as a realistic option due to the band-limited response of the source and the receivers. In our study, however small the spectral of the low frequency component of the seismic data is, we assume that the phase information of the seismic data at low frequency band is available and usable for the waveform inversion to estimate the low wave number velocity profile.

We demonstrate the feasibility of our method by showing the inversion example of the Marmousi model. Finally we generate the most energetic Kirchhoff migration image by using an inverted Marmousi model as a smooth velocity model for the prestack depth migration.

2. Theory

1) Objective function of the phase of the wavefield

The measured wavefield in frequency domain can be given in the polar form as

$$d_{i,j} = A_{i,j}(\omega)e^{i\omega t^1_{i,j}(\omega)},\ i = 1,...,N_r, j = 1,...,N_s \tag{1}$$

where, $A_{i,j}(\omega)$ is the amplitude, ω is the angular frequency, $t^1_{i,j}(\omega)$ is the dimensionless phase time, N_s is the number of shots, N_r is the number of receivers and i and j denote the receiver and shot number, respectively. The modeled wavefield via the frequency domain modeling technique can be expressed as

$$u_{i,j} = B_{i,j}(\omega)e^{i\omega t^2_{i,j}(\omega)},\ i = 1,...,N_r, j = 1,...,N_s \tag{2}$$

where, $B_{i,j}(\omega)$ is the amplitude of the modeled data at each frequency and $t^2_{i,j}(\omega)$ is the phase of the modeled data at each frequency. The l_2 norm of the phase error can be defined as

$$E=\sum_{\omega}\sum_{i=1}^{N_r}\sum_{j=1}^{N_s}\left[t_{i,j}^{1}(\omega)-t_{i,j}^{2}(\omega)\right]^{2} \quad (3)$$

where E is the l_2 norm of the phase residual error, $t_{i,j}^{2}(\omega)$ is the phase of the modeled wavefield, $t_{i,j}^{1}(\omega)$ is the phase of the measured data, subscript i denotes the receiver number and subscript j denotes the shot number. In applying a Gauss-Newton method to the seismic inverse problem, computation of the Frechèt derivative of the phase of the wavefield is necessary (Shin et al., 2001). Following Shin et al. (2001)'s approach, we parameterize our subsurface by $N=N_x\times N_z$ elements where N_x and N_z are the number of elements in x-direction and z-direction, respectively. At each element, we identify a velocity v_i. In this manner, we define our unknown model parameter vector $\mathbf{p}$ to be

$$\mathbf{p}=(v_1,v_2,\ldots\ldots,v_N) \quad (4)$$

Taking derivative of equation (2) with respect to the parameter gives

$$\frac{\partial u_{i,j}(\omega)}{\partial p_l}=\frac{\partial B_{i,j}(\omega)}{\partial p_l}e^{i\omega t_{i,j}^{2}(\omega)}+iB_{i,j}(\omega)\omega\frac{\partial t_{i,j}^{2}(\omega)}{\partial p_l}e^{i\omega t_{i,j}^{2}(\omega)},\ l=1,..,N,i=1,..,N_r,j=1,..,N_s \quad (5)$$

By dividing both sides of equation (5) by the wavefield of equation (2), we obtain

$$\frac{1}{u_{i,j}(\omega)}\frac{\partial u_{i,j}(\omega)}{\partial p_l}=\frac{1}{B_{i,j}(\omega)}\frac{\partial B_{i,j}(\omega)}{\partial p_l}+i\omega\frac{\partial t_{i,j}^{2}(\omega)}{\partial p_l},\ l=1,\ldots,N,i=1,\ldots,N_r,j=1,\ldots,N_s \quad (6)$$

From equation (6), we note that the Jacobian $\partial t_{i,j}^{2}(\omega)/\partial p_l$ is in the imaginary part of equation (6). Thus the Frechèt derivative of the phase is given as

$$\frac{\partial t_{i,j}^{2}(\omega)}{\partial p_l}=\frac{1}{\omega}\mathrm{Im}\left[\frac{1}{u_{i,j}(\omega)}\frac{\partial u_{i,j}(\omega)}{\partial p_l}\right],\quad l=1,\ldots,N,i=1,\ldots,N_r,j=1,\ldots,N_s \quad (7)$$

We have explained our choice for computing the Frechèt derivative of the wavefield with respect to the each model parameter $p_{l(l=1,\ldots,N)}$.

For the computation of Frechèt derivative of the wavefield, we invoke the reciprocity theorem combined with the frequency domain finite element modeling technique as Shin et al.(2001) did for seismic migration and inversion. Next we will explain how to define the residual error of the phase between the modeled data and the measured data. Dividing equation (1) by equation (2) and taking logarithm gives

$$\log\left(\frac{d_{i,j}(\omega)}{u_{i,j}(\omega)}\right)=\log\left(\frac{A_{i,j}(\omega)}{B_{i,j}(\omega)}\right)+i\omega(t_{i,j}^{1}(\omega)-t_{i,j}^{2}(\omega)),i=1,\ldots,N_r,j=1,\ldots,N_s \quad (8)$$

Extracting the imaginary part of equation (8) and dividing it by angular frequency gives a phase residual error as

$$t_{i,j}^{1}-t_{i,j}^{2}=\frac{1}{\omega}Im\left[\log\left(\frac{d_{i,j}(\omega)}{u_{i,j}(\omega)}\right)\right],\ i=1,\ldots,N_r,j=1,\ldots,N_s \quad (9)$$

2) Gauss-Newton method of inversion

Since we computed the Jacobian matrix required for the application of Newton type technique to the seismic inverse problem, we have two options for inverting the phase data. The first one is the classical steepest descent method and the second one is the Gauss-Newton method (Shin et al., 2001). The Gauss-Newton formula can be given as

$$\mathbf{J}\Delta\mathbf{p} = \mathbf{r} \tag{10}$$

Here, $\mathbf{J}$ is a Jacobian matrix whose size is $(N_s \times N_r) \times N$, $\Delta\mathbf{p}$ is the model parameter vector and $\mathbf{r}$ is the steepest descent vector (gradient vector). Bearing in mind the possibility of mode and measurement errors, equation (9) can be best solved by the standard least squares method, resulting in what we refer to as the normal equation.

$$\mathbf{J}^T\mathbf{J}\Delta\mathbf{p} = \mathbf{J}^T\mathbf{r} \tag{11}$$

Here, $\mathbf{J}^T$ is the transpose matrix of Jacobian matrix. In Gauss-Newton method, we obtain a step length by inverting the approximate Hessian and multiplying the steepest descent vector $\mathbf{r}$ by the inverse of the approximate Hessian. However, computation of the huge approximate Hessian is a formidable task to even the most high-performance computer if the number of unknown increases. Computation time of a row or a column of the approximate Hessian corresponds to that of the single arrival Kirchhoff migration (Ha and Shin, 2002). Furthermore, computing an inverse of the huge Hessian is also extremely expensive. Hence, we regularize the steepest descent direction by diagonal approximation of the approximate Hessian (Shin et al., 2001), which can be expressed as

$$\Delta\mathbf{p}^k = \frac{\alpha^k}{\left[diag(\mathbf{J}^T\mathbf{J}) + \lambda\mathbf{I}\right]}\mathbf{J}^T\mathbf{r} \tag{12}$$

where k is an iteration number, α^k is the scaling constant at each iteration, λ is a Lagrange multiplier and $\mathbf{I}$ is an identity matrix. When solving equation (12) to update the parameter, the normal equations (12) were solved at each frequency and averaged.

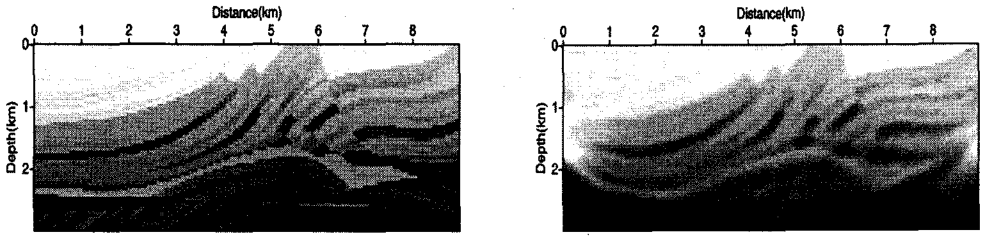
$$\Delta\tilde{\mathbf{p}} = \frac{1}{N_f}\sum_{\omega}\Delta\mathbf{p}(\omega) \tag{13}$$

where N_f is the total number of frequencies, $\Delta\tilde{\mathbf{p}}$ is the averaged step length vector and $\Delta\mathbf{p}(\omega)$ is the step length vector obtained at each frequency.

3. Numerical examples

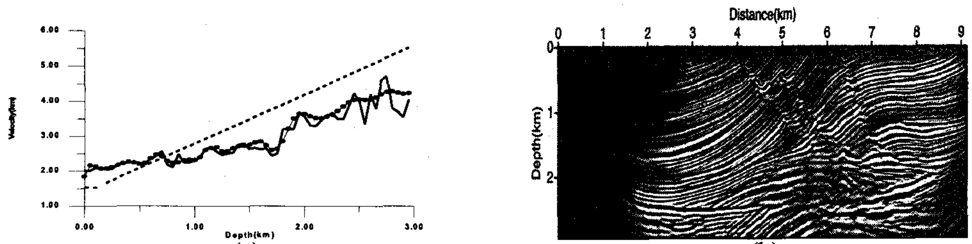
We tested our inversion to the Marmousi model shown in Figure 1(a). It is widely known that the Marmousi model is one of the notorious velocity models to be estimated from its seismic data. We modeled 30 discrete frequencies ranging from 0 to 8hz. And we chose a horizontally stratified model consisting of sixty flat layers as the initial model with which we could start off. Figure 1(b) shows the updated model at 600th iteration. Many of the features of the Marmousi model can be observed with the model stabilizing towards the true model. To take a closer look at the velocity profile in depth, we displayed the velocity profiles at 5.5km of the true model, the initial model and the inverted model in Figure 2(a). We note that the inverted velocity profile, on the whole, approximates to that of the true model.

Having completed the phase inversion of Marmousi data, we proceeded on to the Kirchhoff prestack depth migration using the inverted velocity model as an initial model. Figure 2(b) shows the prestack depth migration image of the Marmousi data by using the most energetic traveltime, as suggested by Shin et al.(2002).



(a) (b)

Fig. 1. (a)The Marmousi model and (b) the inverted velocity model.



(a) (b)

Fig. 2. (a)The velocity profiles below the surface at 5.5km. The dotted line represents the inverted velocity profile, the solid line represents the velocity profile of the true model, while the dashed line represents the velocity profile of the initial model. (b)The prestack depth migration image using the inverted velocity model for migration.

4. Conclusion

Our initial assumption is that it is acceptable to discard the amplitude of the low frequency seismic data but we can use the phase information of the low frequency for the inversion of the seismic data. This assumption, therefore enables us to use the low frequency seismic data for the better estimation of the low wave number velocity profile for the waveform inversion.

To this end, we proposed a new algorithm of calculating the Frechèt derivative of the wavefield for the application of Gauss-Newton method to the phase inversion of the seismic data by exploiting the modern sparse matrix technology and the principle of the source-receiver reciprocity of the wave equation. In defining the residual error of the phase, we have taken the logarithm of the measured data divided by the modeled data in the frequency domain. Our algorithm allows us to build the approximate Hessian as well as the steepest descent direction. In order to regularize the steepest descent direction, we used only the diagonal approximation of the approximate Hessian at the cost of computing the steepest descent direction. In obtaining the step length at each iteration, we averaged the step length calculated at all frequencies used.

By assuming that we use the phase information of the low frequency seismic data for the estimation of the low wave number velocity profile, we claim that the phase inversion of the seismic data can be one of the choices to be used in inverting the seismic data so as to obtain a better estimation of the initial velocity model, which in turn will be applied in prestack depth migration. One of the important technical problems associated with seismic inversion is the unknown source wavelet. For this study, we assume that the exact source signature is known, despite the fact that it is not difficult to parameterize common wavelet, as suggested by Pratt (1999).

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