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3-D elastoplastic finite element analysis of umbrella arch reinforcement system for tunnelling

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Abstract: In this paper, a mathematical framework based on a homogenisation technique to simulate "umbrella arch reinforcement system" (UARS) and its implementation into a 3D Finite Element program that can consider stage construction situations are presented. The constitutive model developed allows considering the main design parameters of the problem and only requires geometrical and mechanical properties of the constituents. Additionally, the use of a homogenisation approach implies that the generation of the Finite Element mesh can be easily produced and that re-meshing is not required as basic geometrical parameters such as the orientation of the pipes are changed. The model developed is used to simulate tunnelling with the UARS. From the analyses, the effects of the main design parameters on the elastic and the elastoplastic analyses are considered.

1. Introduction

The UARS for ground reinforcement has been commonly applied in tunnelling under difficult ground conditions, with satisfactory results. Previous experience on the use of the technique has shown effective reductions on potentially large tunnel convergence, ground settlement and water inflow (KICT, 1986; Societa Italiana Gallerie, 1991; Pelizza and Pelia, 1993; Pelia, 1994; Bae et al., 1997). However, due to the geometrical and mechanical complexity of the problem, there have been very few attempts to analyse this problem and no adequate method for determining the design parameters of the reinforcement system has been developed.

In this study, a mathematical framework based on a homogenisation technique similar to the one used by Lee et al. (2000), but with a different geometrical definition of the reinforced ground is adopted to derive an anisotropic composite material representing the elastoplastic macroscopic behaviour of UARS. A new algorithm for nonlinear stress integration is also proposed. The constitutive model developed is then incorporated into a FE program, capable of considering stage construction, to produce a tool that allows considering the effect of the main design parameters of UARS.

2. Homogenisation technique for UARS

The general idealised geometry of the system to simulate is shown in Figure 1. As shown, two sets of axes are defined, a global axis X-Y-Z associated with the geometry of the tunnel and a local axis x-y-z associated with the geometry of the reinforcement system. The latter coincides with the axis of orthotropy of the reinforcement system z being parallel to the pipe and y perpendicular to the arch formed by the set of pipes. As shown in Figure 1, it is assumed that the injected grout produces a continuum in the x direction but a finite depth band in the y direction. The use of the local axes allows defining the properties of the reinforced ground in its axis of orthotropy regardless of the orientation of the pipes with relation to the tunnel. The orientation of the reinforcement system at each

[redacted]

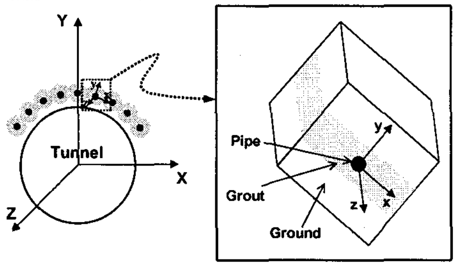


Figure 1. Geometrical description of UARS model.

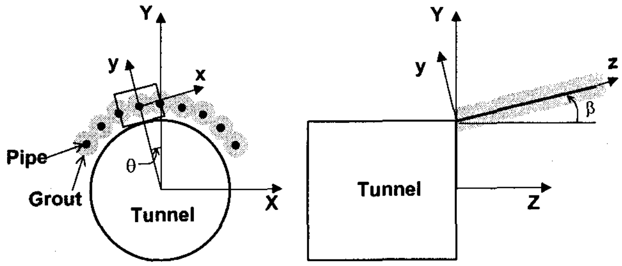


Figure 2. Typical arrangement of pipes.

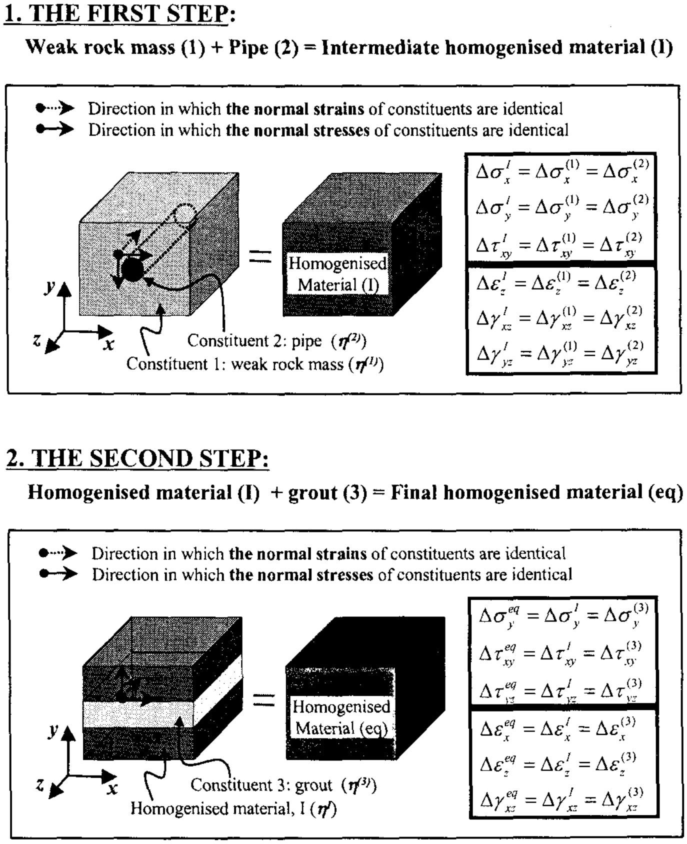


Figure 3. Schematic diagram of dual homogenisation procedure.

location is included in the model by converting the properties of the reinforced ground into the global axes. For this, transformation matrices are composed by introducing angles θ and β, shown in Figure 2. This procedure allows defining the FE mesh independently from the orientation of each constituent of the UARS.

Then, a two-stage homogenisation procedure as shown in Figure 3 is employed to define the composite UARS, which contains three basic isotropic constituents, i.e. ground, pipes and grout (see Figure 1). The mathematical formation of the procedure in details can be found in a reference (Bae et al., 2003). The basic mathematical framework is similar to that given by Pietruszczak and Niu (1992) for the simulation of structural masonry. In the local axes of the UARS, the properties of the homogenised material at the end of each step, depends only on the volume fraction of each constituent and its constitutive law. The stress-strain relationship for each constituent can be described as shown in Equation (1), where [D] indicates either an isotropic elastic or an elastoplastic constitutive matrix. The same equation can be used to describe the constitutive behaviour of the stage 1 and stage 2 homogenised equivalent composite materials. In the first stage, the ground (constituent 1) and the pipes (constituent 2) are homogenised, leading to a homogenised material (composite I). In the second stage, composite I is homogenised with the grout (constituent 3) to obtain the final equivalent composite material (composite eq).

3. Elastoplastic simulation in homogenisation framework

The nonlinear behaviour of the homogenised umbrella arch reinforced ground (UARG) material is considered at a constituent level. As constituents undertake elastoplastic straining, their constitutive matrices [D] are modified according to the classic elastoplastic theory (Zienkiewicz and Taylor, 2000). A detail of the nonlinear stress integration algorithm developed can be found in a reference (Bae et al, 2003). At the end of each convergence iteration, within each load increment iteration, the basic homogenisation procedure described in the previous section is repeated, using the constituent elastoplastic matrix for the constituents that have yielded. From this procedure, the matrix for the homogenised material and the elastic or elastoplastic structural matrices for the constituents are obtained. The structural matrices and the stiffness matrix for the homogenised material used for the i[th] iteration are shown as follows.

$$\begin{aligned}\left[S_{k,ep}^{i-1}\right] &= S_{k,ep}^{i-1}\left(v_1, v_2, v_3, \left[D_{1,ep}^{i-1}\right], \left[D_{2,ep}^{i-1}\right], \left[D_{3,ep}^{i-1}\right]\right)\\ \left[D_{h,ep}^{i-1}\right] &= D_{h,ep}^{i-1}\left(v_1, v_2, v_3, \left[D_{1,ep}^{i-1}\right], \left[D_{2,ep}^{i-1}\right], \left[D_{3,ep}^{i-1}\right]\right)\end{aligned} \tag{1}$$

At each iteration, the updated elastoplastic structural matrices are used to obtain the strain increment in each constituent from the increment in strain in the homogenised material (Equation (2)). Following the classic elastoplastic algorithm, the admissible stress increment in each constituent is computed. The admissible stress increment in the homogenised material is then obtained using the averaging rule and added to the stress state at the end of the previous iteration (Equation (3)). Following standard elastoplastic theory, the equivalent nodal forces are computed from the stresses in the equivalent homogenised material and compared with applied load increment to check convergence.

$$\Delta\varepsilon_k^i = \left[S_{k,ep}^{i-1}\right]\cdot\Delta\varepsilon_h^i \tag{2}$$

$$\sigma_h^i = \sigma_h^{i-1} + \sum_{k=1}^{3} v_k \Delta\sigma_k^i \tag{3}$$

In the current version of the code, each constituent can be considered using either Mohr-Coulomb, Drucker-Prager, Tresca or Tension cut-off as a yield criterion, combined with either a linear or an exponential softening/hardening rule (Zienkiewicz and Taylor, 2000). The use of the elastoplastic structural matrices instead of the elastic ones ensures that the kinematic and equilibrium conditions of the homogenisation formulation are satisfied incrementally and therefore at all stages. As shown in Equation (3), the averaging rule is applied incrementally instead of being applied in total stresses due to the conditions of the construction procedure. Nevertheless, a certain approximation is introduced, since the elastoplastic structural matrices employed correspond to the previous iteration and, therefore, do not consider the changes in the [D] matrices of the constituents that will take place in the current iteration. This approximation induces an accumulation of error in the results. However, if the load increments considered are sufficiently small the errors accumulated are not significant. In order to avoid this accumulation of errors, the stresses and strains of constituents and homogenised material could be adjusted to satisfy fully the homogenisation conditions. However, this adjustment would be arbitrary and would not provide any benefit to the formulation.

A method of stress adjustment was used by Lee et al. (2000). In that approach, instead of updating the structural matrices and equivalent constitutive matrix, the stresses were adjusted. Although this approach had a much lower computational cost, the accuracy of the results was severely affected on the areas where yielding took place.

Implementation in staged construction

For simulating the staged construction, nodal forces are calculated at all the nodes on the excavated surfaces at every excavation stage, from the initial stresses around the tunnel. These initial stresses have been generated by considering the depth of the points under the ground surface and the ground specific weight. Similarly, as the UARG is applied, a series of elements within the FE model change from ground elements into homogenised UARG elements. In order to maintain global equilibrium within the model, the stress state in the ground element is transferred into the homogenised material. Internally, all the stress is transferred into the ground constituent (Equation (4)). Stress is only absorbed by the grout and the pipes as more loads are introduced in system.

$$\begin{aligned}\sigma_h &= \sigma_{gound}\\ \sigma_{ground\ constituent} &= \sigma_{ground}\\ \sigma_{pipe\ constituent} &= 0\\ \sigma_{grout\ constituent} &= 0\end{aligned} \tag{4}$$

As a result of this, however, the kinematic constraints and equilibrium conditions to be satisfied are only valid incrementally. Similarly, the averaging rule is also only valid incrementally.

4. Study on the reinforcing effect of UARG

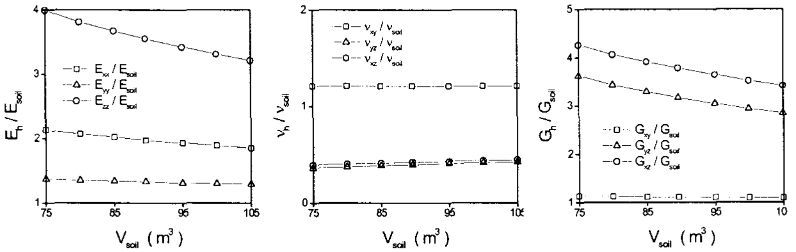
Orthotropic elastic characteristics of UARG

In this section, a series of parametric studies on the effects of the main design variables of the problem on the orthotropic elastic properties are presented. The isotropic elastic material properties and volume contributions of the constituents assumed in these studies are given in Table 1.

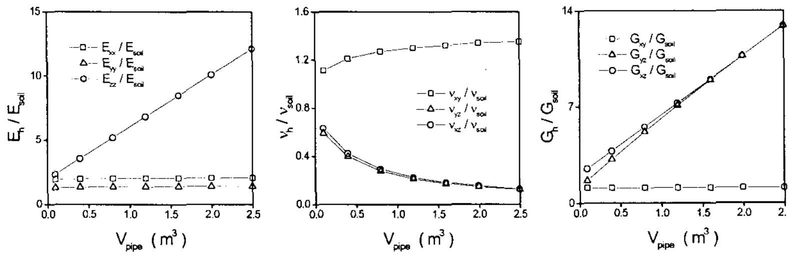
Table 1. Elastic material properties of constituents of UARG.

Constituents	Young's modulus (MPa)	Poisson's ratio	Volume contribution (m^3)
Ground material	500	0.33	89.6
Grout material	5,000	0.17	10.0
Pipe	200,000	0.25	0.4

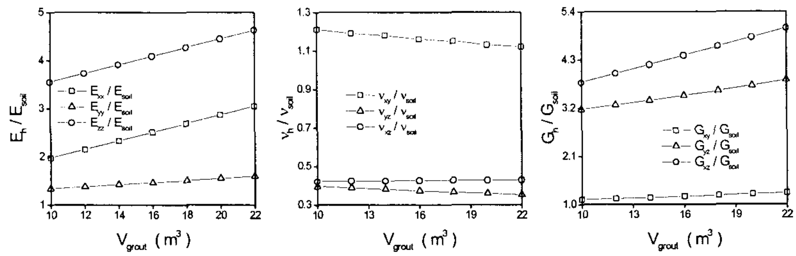
When considering the effects of modifying the volume fractions, the volume of the constituent considered is modified while the volume of the other two constituents is left unchanged. Since this results in a change on the total volume, the volume fraction of all constituents changes. The volumes of the constituents not modified are shown on Table 1.



(a) For volume changes of soil mass



(b) For volume changes of a set of pipes



(C) For volume changes of grout material

Figure 4. Evolution of orthotropic elastic constants.

Figure 4(a), (b) and (c) show the effects of changes in the volume of ground, pipe and grout, respectively, on the nine orthotropic elastic constants of UARG. The results have been normalised by the properties of plain ground material. Figure 4(a) represents the effect of increasing the zone affected by the reinforcement, while maintaining the level of reinforcement introduced. As expected, the Young's and shear moduli decrease as the volume of ground material increases, due to the lower stiffness of the ground material. The effects are most significant in the z direction, i.e. parallel to the pipe, as a result of the kinematic conditions imposed. The effects of varying the degree of reinforcement are shown on Figure 4(b). The results show a very significant effect on all the properties, but particularly in the z direction. This suggests that the elastic characteristics of UARG are mainly governed by the contribution of pipes. Figure 4(c) illustrates the effect of modifying the extent of grouting on the properties of UARG. As a result of the geometric arrangement, i.e. grout is injected through the set of pipes, and the modelling assumptions, i.e. the grout is assumed to form a continuum in between the pipes, the most affected properties are the Young's moduli in x and z directions. Conversely, the Young's modulus in y direction is only slightly affected. This result is in disagreement with the prediction by Lee et al. (2000), as a result of the different geometrical hypotheses. The results obtained suggest that grouting enhances the arching effect in the circumferential direction but provides a small reinforcing effect on the ground stiffness in the radial direction.

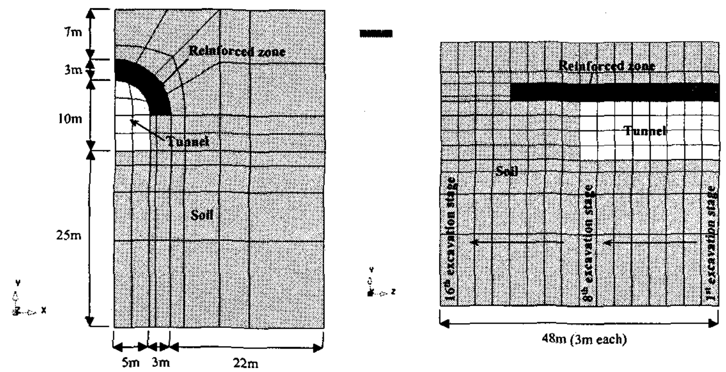


Figure 5. FE model used in parametric study.

Reinforcing effect of UARG on tunnel excavation

The proposed constitutive model for UARG allows considering the main design variables required for the design of a tunnelling procedure using grouted pipe-roofing reinforcement. Additionally, as a result of the macroscopic nature of the model, the effects of these variables on the excavation of the tunnel can be considered without the complex and time-consuming modification of the meshes. Using these advantages, a series of parametric studies were undertaken to investigate the effects of some of the main design variables on the behaviour of a tunnel excavated applying grouted pipe roofing reinforcement. The following parameters have been considered:

- The angle of pipe installation (β = 0°, 10°, 20°, 30°)
- The length of pipe overlapping (0m, 3m, 6m)
- The lengths of pipes (L = 6m, 12m, 18m)

The underlined parameters are the default values used while the other parameters are changed. The tunnel considered has a horseshoe-shaped cross section and is excavated in a homogeneous isotropic layer of soil, with a cover at the crown of 10 m. The total length of the tunnel is 48m, excavated in 16 stages of 3 m. Details of the tunnel analysed are shown in Figure 5. The stresses in the soil before the excavation were generated using a specific weight of 22000 kN/m^3 and a k_0 of 0.7. The effect of the grouted pipe-roofing reinforcement is modelled by assigning the properties of the UARG to a 3 m thick layer around the top half of the tunnel. In order to consider the effect of pipe overlapping, two UARG materials are defined, one with double pipe volume than the other, but same grout volume and same constituent material properties (see Table 2). The volume fractions of the constituents have been calculated from the volume of the elements to be reinforced, the centre to centre (CTC) distance between pipes, the pipe dimensions and the amount of grout injected. A 3-D FE mesh was generated using 1152 20-noded isoparametric hexahedral elements with a total of 5,657 nodes. The refinement of the mesh was minimised to reduce the computational time.

Table 2. Material properties of constituents of GMPRs used in FE analyses.

Material		Young's modulus (MPa)	Poisson's ratio	Volume fraction (%)	Cohesion (kPa)	Internal friction angle (°)
Soil		200	0.33	-	100	35
UARG I	Soil	200	0.33	89.6	100	35
	Pipe	200,000	0.25	0.4	-	-
	Grout	4,900	0.17	10	1500	30
UARG II	Soil	200	0.33	89.2	100	35
	Pipe	200,000	0.25	0.8	-	-
	Grout	4,900	0.17	10	1500	35

Angle of pipe installation (β)

The β angle (see Figure 2) is taken into account in the UARG formulation in the definition of the local axes and, therefore, on the transformation of the equivalent material properties into the global axes. Four β angles were considered in the parametric study.

Table 3 shows the tunnel convergences at the cross section at Z = 24 m, for various β angles. The results indicate that the introduction of the UARG results in a significant reduction on the displacement. The most effective reduction was achieved with the inclined pipes. And the effects of the introduction of the UARG on the horizontal displacements at the tunnel face and the ground settlement under the 8th excavation stage are shown. The horizontal displacement at the crown of the tunnel face is reversed for β angles bigger than 20°. This change in the direction of the displacements at the crown results in the distortion of the tunnel in the axial direction and, therefore, could reduce its stability. For bigger β angles, slightly bigger ground settlement reductions are obtained, but the curvature of the gradient settlement curve also increases. This could have more negative consequences on the structures affected by the ground settlement.

Length of pipe overlapping

The material properties of UARG I and UARG II are applied to reinforced ground areas with a single set of pipes and to reinforced ground areas with overlapping pipes, respectively. Three cases of overlapping configurations are considered. When the pipes are installed every two 3m-long excavation stages, 6 of the 12m of the pipes overlap with the previous set of pipes, when the pipes are installed every 3 stages, only 3 of the 12 m overlap and when the pipes are installed every 4 stages no overlapping takes place.

Table 3 shows the effect of the overlapping length on the tunnel convergence. The results indicate that although the overlapping reduces the displacements, increasing the overlapping length over 3 m does not enhance the results. On the face displacement and ground settlement, the effect of overlapping is not significant.

Table 3. Reinforcing effects (%) with respect to a number of geometric design variables.

Installation angle (β)	For vertical displacement	For hogizontal disp. on lateral surface	For hogizontal disp. at top of tunnel face	For ground settlement
0°	39.4	18.4	78.1	46.4
10°	40.6	19.2	94.4	49.9
20°	43.5	22.1	116	52.1
30°	47.3	26.5	124.5	52.9
Overlapping length				
0m	40.6	19.2	94.4	49.9
3m	43.4	21.9	83.5	51.3
6m	44.1	21.9	114.9	52.1
Pipe length				
6m	39.2	19.2	70.8	45.3
12m	41.4	21.4	71.2	48.3
18m	43.5	21.9	68.8	51.5

Length of pipes

The effect of the length of the pipes was considered in the case of no pipe overlapping, and, therefore, as the length of the pipes was modified, the stages at which the pipes were installed were also modified. The three cases considered are 6 m long pipes installed every 2 excavation stages, 12 m pipes every 4 stages and 18 m pipes every 6 stages.

The results in Table 3 indicate that the reduction of the vertical displacement at the crown is proportional to the increase on the length of the pipe over 6 m. Conversely, the horizontal movement at the wall is almost not affected by increasing the pipe length over 12 m. The length of the pipes has a limited influence on the horizontal displacements at the crown of the tunnel face. For the pipe lengths considered, a reversal of the horizontal movements is not obtained. The increase of the pipe lengths also reduces the curvature of the settlement curves.

5. Direct tension test with elastoplastic medium of UARS

In order to verify the results of the elastoplastic model proposed, a simple arrangement was first considered. The arrangement consists of a single UARG under direct tension at an angle with the reinforcement varying between 0 and 90°, as shown in Figure 6. The results are also compared with the case of plain ground material. In this model, the constituents have the material properties given in Table 2, a Mohr-Coulomb yield criterion with associated flow rule and perfect plasticity. The volume fractions considered are 63, 2 and 35% for the soil, pipe and grout, respectively.

As expected, the stiffness and yielding stress increase as the direction of loading becomes more parallel to the direction of the pipes. For the case of $\theta = 90°$, an elastic response of obtained due to the very high stiffness of the pipes in their axial direction.

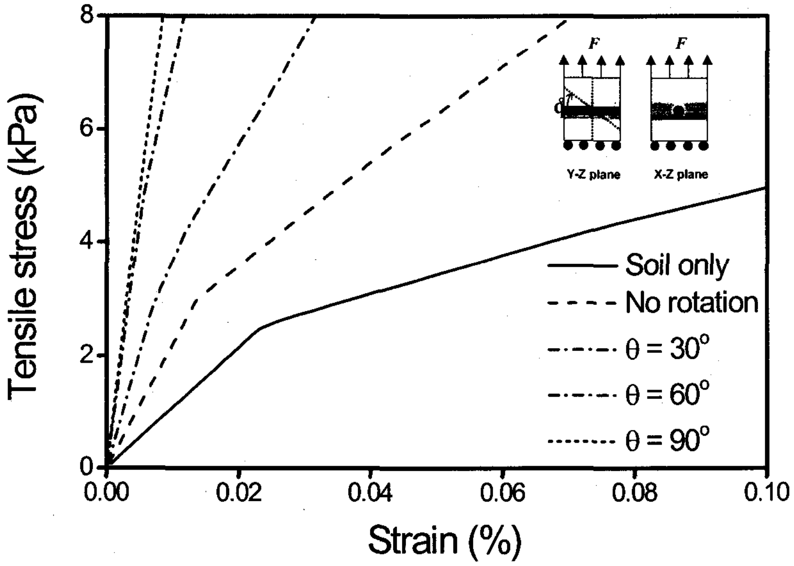


Figure 6. Elastoplastic response of UARG.

6. Conclusions

In this study, a new mathematical framework for modelling the ground reinforcement system known as UARS has been proposed using a homogenisation technique. This tunnelling method has been previously applied successfully in the construction of tunnels in weak ground conditions.

The framework developed provides the 9 independent orthotropic elastic constants of the reinforced ground material and the structural matrices that allow deriving the stresses/strains in each constituent from the stresses/strains in the homogenised ground material. The constitutive matrix of the equivalent material and the structural matrices are obtained as functions of the mechanical properties, volume contribution and internal geometrical arrangement of the three constituents of the reinforcement system, i.e. ground, steel pipes and grout.

This homogenisation framework is capable of considering the effect of the main parameters of the problem, i.e. amount of reinforcement and grout, the geometry of the pipe installation, the length of pipes, the extent of the grouted area, the length of excavation prior to the installation of additional reinforcement and the length of pipe overlapping. Additionally, the use of a homogenisation approach implies that the generation of the Finite Element mesh for the complex geometry of the problem can be simulated easily and that re-meshing is not required as basic geometrical parameters such as the orientation of the pipes are changed.

The use of a homogenisation approach also allows considering the elastoplastic behaviour of the composite material at a constituent level, using well-known constitutive models. This approach, therefore, only requires constituent parameters, instead of composite parameters, which would be very difficult to define and obtain. In order to satisfy the kinematic and the equilibrium constraints of the two stages of the homogenisation procedure during the non-linear computations, the homogenisation is applied incrementally. Then, the homogenisation procedure proposed has been implemented in a Finite Element model capable of considering stage construction situations. The tool developed allows the effective simulation of tunnelling operations using the UARS and flexibility in the design stages.

The mathematical formulation proposed has been used to study the effect of the main design parameters on the elastic response of a tunnel. The results obtained suggest that the model represents adequately the problem analysed. However, experimental data are required to fully validate the model proposed. A number of investigations are currently being undertaken in the authors' research groups to further understand the validity, range of applications and effectiveness of this homogenisation technique for tunnelling. Results of these studies will be reported shortly. The model proposed has also been used to undertake the elastoplastic analysis of a tunnelling operation. The results indicate that, in the example considered, the nonlinear effects have a limited effect on the results. It is expected, however, that the nonlinear effects will have a greater influence in weaker grounds.

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