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An investigation into the application of fractals for rock roughness estimation

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Abstract: Profiles of naturally fractured surfaces of three sedimentary rock samples were plotted from the measured data using a mechanical profilometer. Fractal dimension of these profiles were computed and statistical F-test indicates that fractal dimension (FD) values can be used as a parameter for distinguishing the rock types. The comparison between FD values and a commonly used profile-roughness parameter called the Mayer's Z_2 parameter shows the superiority of the FD values as roughness estimator.

Two-dimensional fractal roughness parameters of the same naturally fractured rock surfaces were also studied from their scanning electron microscopic (SEM) images at various magnification levels. The most suitable level of magnification of the SEM images for the study of the 2-D fractal roughness parameter was identified. The values of 2-D fractal roughness parameter for three different rocks were also compared using different methods of fractal dimensioning.

1. Introduction

Rock mass is less like a homogenous material to be studied with the use of the conventional methods of analysing the rock properties. Characterizing the rock deformations is of utmost importance for the prediction of different behaviours of jointed rock mass. Owing to the non-homogeneity present in the rocks, the characterization of rock surfaces is carried out on a subjective basis. There is lack of existence of any non-subjective method for characterization of the rock mass. This is mainly due to the complex nature of the rock constituents in terms of their geometry, formation details, material properties etc.

It is difficult to apply a simple mathematical relationship to describe roughness of rock joints. Several attempts have been made to characterize irregular rock surface by applying statistical analysis (Wu & Ali, 1978; Reeves 1985). The statistical models take time to apply and are often based on assumptions of certain well known statistical distribution which may not be applicable to rock joint types (Turk et. al. 1987). Rock roughness or deformations can be characterized mathematically either in a single or in two-dimensions. Rock characterization in single dimension is carried out using the surface profiles measured along the surface of the rock and analyzing those using suitable mathematical techniques. Whereas the two dimensional characterization is based on the analysis of a 2-D data of the surface roughness. Recent modifications in the field of applied mathematics have been investigated to analyze the rock deformation in terms of the roughness present in rocks. The fractal geometric concept, proposed here, for a non-subjective and uniform measure of the rock deformation may be an appropriate approach. The validity of the fractal parameters is compared with Z_2, the traditional parameter for roughness characterization.

Till date the determination of JRC values of the rock joint surfaces is totally subjective in nature, the geometry and shear strength of the rock surfaces are the most important factors in the stability analysis (Krahn & Morgenstern, 1979). The shear strength of rock is controlled by many factors, such as rock-rock frictional resistance, cohesion and rock mass density, joint continuity and surface roughness etc.

A well-known parameter known as Z_2 has been widely used to designate the rock surface characterization. In the computation of Z_2 the RMS value of the first derivative of the profile is used as:

$$Z_2 = (1/L)\sum(dx/dy)^2$$

Out of the several numerical methods (Barton & Chaubey 1977) for the surface characterization of rock it has been observed various researchers (Barton & Chaubey 1977; Chakravarty, D. 2001) that the parameter Z_2 may be used as an unbiased and consistent estimate for surface roughness of rocks. Because of very specific and complex properties of naturally broken rock surfaces, fractal geometry has been found to be effectively used for obtaining accurate descriptions regarding surface roughness.

2. Fractal Methods followed

The complexity observed in naturally occurring objects, such as, the rock surface is more accurately depicted by the concepts of fractal geometry, which means that the complexity or the surface roughness remains equally irregular however closely it is examined, thus revealing the limitations of Euclidean geometry. In a naturally broken rock surface as the magnification increases more and more irregularity becomes visible, theoretically even at infinite magnifications irregular lines still appear. Mandelbrot (1967), indicates that most of the lines and shapes are in scales.

Fractals are characterized by their particular property called the dimension; D of any set. The smaller the fractal dimension the smoother the curve is, whereas, high values of fractal dimensions represent rugged nature of the surface under inspection. The concept of the fractal dimension of any set has been derived from the concept of space filling by the object. There are several methods existing for the calculation of the fractal dimension of any give set.

Out of the several methods for the determination of fractal dimension of any given set, in the present investigation the following three methods have been considered for the calculation of fractal dimension for one dimensional profilometric data:

I. Box counting method,

This method is, again, implemented in two ways:

(a) By manual counting of boxes,
(b) By the use the computer for box counting purpose.

II. Structured walk method,

III. Hybrid method.

The Box Counting Method:

In this method boxes of varying sizes, d, are used to cover the curves of the surface profiles. The number of boxes, N(d), required to cover the curve completely is counted and it is found that this has a power law relationship with the size of the box, d.

$$N(d) = N_0 d^{-D}$$

where D is the fractal dimension the profile and is defined as

$$-D = \frac{\log N(d)}{\log d}$$

In the present study five profiles at 36 degrees to each other on the surface of naturally broken rock in case of three different sedimentary rock types, e.g., shale, sandstone and high grade limestone have been considered for analysis rock cases of six analysis of roughness. All the five profiles of having 200 data points each, the surface profiles of sand stone are shown in Fig.1.

The Conventional Measure of the Surface Parameter by the Z_2 measure

The profile data were fed to the computer program, which reads the data as a sequence of x values with the corresponding elevation data, the y values. The results from the Z_2 calculations of the different profiles are displayed in Table 1.

Table 1. Z_2 values obtained from the surface profiles.

Rock Type	Shale	Sandstone	High Grade limestone
Line 1	0.3334	0.5001	0.7692
Line 2	0.1099	0.7236	0.6987
Line 3	0.1731	0.5984	0.7503
Line 4	0.1408	0.6893	0.6490
Line 5	0.2807	0.5983	0.6098
Mean Value	0.2076	0.6219	0.6954
SD	0.0500	0.0878	0.0670

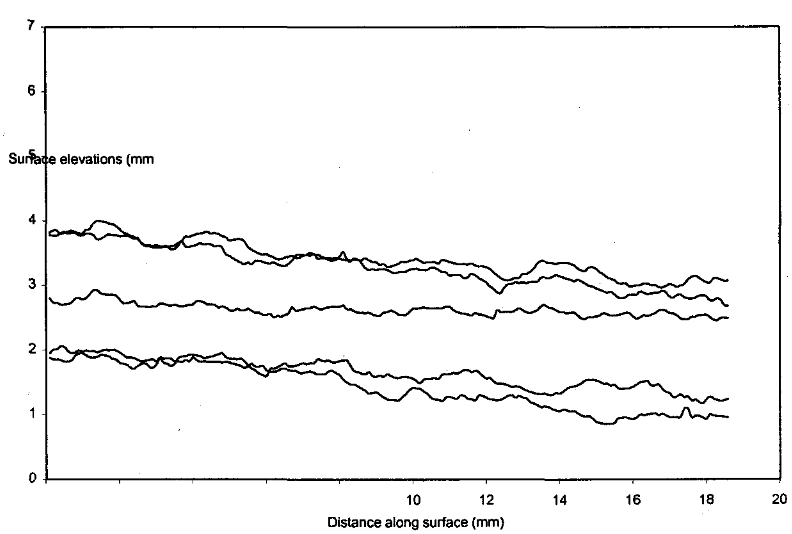

Fig. 1. Plots of the five surface profiles of sandstone.

Investigated fractal methods for digitized images of rocks

Scanned images of the rock thin-sections were used as input for this part of the study. The pixel values in gray level images were analysed using the three different methods for the determination of fractal dimension, namely, the box-counting method, the prism method and the differential box counting method. The images were obtained at three different magnifications under the scanning electron microscope, namely, 100x, 900x and 1760x. Several trials were carried out for the choice of the best level of magnifications used for this study. One of the sand stone images obtained at a magnification of 900x is shown in Fig. 2.

Fig. 2. Digital image of the SEM surface of limestone at 900x magnification.

3. One Dimensional Fractal Analysis

It has been observed that the surfaces of the naturally fractured rock samples are having very complex geometries, which cannot be explained by the application of the Euclidian geometric model of straight lines and curves etc. (Chakravarty, D., 2001). Thus in this study a fractal model of the rock surface was tried was tried out using the above mentioned three methods on the profiles lying on the fractured surface.

The calculated values of the fractal parameters for each of the rock profiles were carried out by plotting the log of number boxes required to completely fill the profile with the log of the box size. The negative gradient of best fit straight line for these points gives the fractal dimension of the surface profile. The fractal dimensions for all the five profiles of each of the three different rock types were calculated and shown in Tables 2 & 3. On critical examination of the results it was noted that, in general, FD values are quite consistent within the same rock type, but at the same time their mean values are distinctly different for different rock types. This proves that the fractal dimension can be an effective measure of roughness of rocks. The coefficient of variation of the FD values was also calculated and it was seen that the fractal dimensions are consistent representative parameters of the surface un-evenness that has been measured in the surface profiles using the mechanical profilometer.

Table 2. Fractal dimensions of different rocks using Manual Box Counting Method.

Rock type	Profile No.					Mean	SD
	1	2	3	4	5		
Shale	1.1045	1.0838	1.1012	1.1011	1.1024	1.1086	0.0075
Sandstone	1.1874	1.1900	1.188	1.1897	1.1887	1.1889	0.0009
High grade Limestone	1.208	1.2071	1.2078	1.2019	1.1998	1.205	0.003

Table 3. Fractal dimensions of different rocks using Computerised Box Counting Method.

Rock type	Profile No.					Mean	SD
	1	2	3	4	5		
Shale	1.1104	1.1052	1.109	1.1075	1.1101	1.1084	0.0021
Sandstone	1.1754	1.1810	1.1795	1.1797	1.1767	1.1785	0.0023
High grade Limestone	1.198	1.193	1.1958	1.189	1.187	1.1927	0.0044

Application of the fractal dimension for rock property determination

After the determination of fractal dimension of the rock samples, correlation analysis was carried out with the intension to find the best method for the fractal analysis for the rock types used; the correlation coefficients between the fractal dimensions of the rock types calculated by the different methods with each other as well as with the Z_2 values were found out and presented in Table 4. From this table, it can be observed that all these studied parameters consistently characterize the roughness property of the surface profiles and this table shows that these parameters are positively correlated amongst themselves. It can also be seen that the correlation coefficient is maximum for fractal dimensions calculated using structured walk method and manual box counting method.

The roughness representation capacity of the fractal dimensions compared to the Z_2 values was established using the statistical F-Test. In this test the F-values, i.e., the ratio of variance between samples to variance within samples, are computed for Z_2 and all the four different fractal dimension values. It was observed that the minimum F-value was obtained for Z_2 parameter and the highest F-value was obtained for the hybrid-method of fractal dimensioning. However, in all the cases the F-values exceeded the minimum tabulated F-valued at 99% confidence interval.

Table 4. Correlation matrices for the different one dimensional fractal parameters.

Rock type		Z_2	D_M	D_B	D_s	D_H
Shale	Z_2	1.00				
	D_M	0.68322	1.00			
	D_B	0.64280	0.88023	1.00		
	D_s	0.71523	0.95778	0.60115	1.00	
	D_H	0.66862	0.73998	0.56823	0.61888	1.000

D_M = Manual box counting method, D_B = Computerized box counting method, D_s = Structured walk method, D_H = Hybrid method

Correlation of FD with some rock properties

The correlation results of JRC values with the different FD values are given below,

It may however be noted that the FD values are not very widely scattered indicating higher reliability of the present methods of fractal dimensioning of JRC profiles.

$JRC = 62.2 \times FD\ (man) - 59.6$ (coefficient of determination = 0.962) (1)

$JRC = 1700 \times D_H - 1699$ (coefficient of determination = 0.9345)

$JRC = 1700 \times (D_H - 1)$ (coefficient of determination = 0.8876) (2)

These equations appear to be similar to that proposed by Smith et. al. (Chakravarty, D., 2001).

The possible dependence of the rock parameters on the fractal dimension was also investigated. Two fractal information, namely: the fractal dimension and the y-axis intercept or the amplitude part, were found to be promising. For each fractal dimensioning method, both the parameters namely, the FD and the Y-axis intercept were determined, which were used to determine the relationship with some of the rock properties, such as, cohesion c, the angle of internal friction ϕ, porosity n and shear strength s. The relationship between c, and s have been obtained by the linear regression analysis were found to have highly coefficient of correlation.

4. Two Dimensional Fractal Analysis

The digital images of naturally occurring objects have been found to be examples of fractals. The digital images obtained from the natural planes of weakness of the rock are observed to be representing fractal characteristics. Table 5, shows the values of the obtained fractal dimension for the three rock types studied. Similar to the case of one-dimensional fractal parameters, the two dimensional parameters were also correlated with the rock properties. In the study for the choice of the best magnification at which the fractal parameters can be easily studied, it was established that a magnification of 900x is the best for our case. A small test was carried out to establish the best method for the calculation of the fractal parameters for the different rock type under consideration, and it was seen that the differential box-counting method for the determination of the fractal parameter performs the best.

Table 5. The 2 D Fractal dimensions of rocks images using Differential Box Counting Method at 900 x magnification.

Rock type	Image Number					Mean	SD
	1	2	3	4	5		
Shale	2.318	2.316	2.295	2.345	2.323	2.3194	0.0178689
Sandstone	2.430	2.421	2.432	2.443	2.261	2.3974	0.0766505
High grade Limestone	2.570	2.55	2.531	2.546	2.512	2.5418	0.0217071

5. Discussion and Conclusion

There has been a vast amount of work carried on the subjective nature of the rock roughness characterization. Till date the research works to characterize the rock surface roughness has mostly been carried out by the means of the standard statistical tools. The most commonly used conventional measure of the surface roughness called as the Mayer's Z_2 value is also error prone and has some basic disadvantages as discussed earlier. The limitations of the Euclidian geometric models to the field of naturally occurring objects were also overlooked. The results obtained from the Euclidian geometric models did use to deviate from the actual results but did not pose problems because of the fact that the accuracy needed was not of that high level, moreover so close viewing of the natural objects was never felt necessary. But as we go deep into the problem, trying to locate the actual cause of the natural phenomena we find that the Euclidian Geometry is not sufficient for the modelling of natural objects because of the fact that the complexity involved in the natural objects specially their non-smoothening phenomena is not tackled properly by the Euclidian geometric model, which assumes that every function or curve smoothens out on a closer view. Whereas, the reverse thing happens in the case of naturally occurring objects as we get closer and closer to the object the nature of the complexities become clearer and clearer thus at no magnification do they become smooth, thus making the applicability of the Euclidian model impossible for rock like material.

The JRC values, which were previously subjective, can now be designated with the help of a non-subjective parameter called the fractal dimension. Thus, once the relationship between the JRC values and the fractal dimensions is established we can replace the JRC by the corresponding fractal dimension values.

As the application of the fractal dimension the various rock properties were correlated with those rock properties, which were supposed to be dependent on the rock surface properties. The high value of the coefficient of determination shows clearly that the relations found are good. It is observed that the knowledge of fractal dimension does not give us any idea of the rock properties directly rather we have to find the empirical properties first by the use of the previous data for one rock and then only this type of empirical relations can be used safely.

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