2003년도 대한전기학회 하계학술대회 논문집 2003.7.21-23

Direct Adaptive Control of Chaotic Systems Using a Wavelet Neural Network

Jong Tae Choi*, Yoon Ho Choi** and Jin Bae Park*
*School of Electrical and Electronic Engineering, Yonsei University, Seodaemunku, Seoul 120-749, Korea
**School of Electronic Engineering, Kyonggi University, Suwon, Kyonggi 442-760, Korea

Abstract - This paper presents a design method of the wavelet neural network(WNN) controller based on a direct adaptive control scheme for the intelligent control of chaotic systems. The conventional control methods such as optimal control, adaptive control and robust control may not be feasible when an explicit, faithful mathematical model cannot be constructed. Therefore, an intelligent control system that is an on-line trained WNN controller based on a direct adaptive control method is proposed to control chaotic systems whose mathematical models are not available. The gradient-descent method is used for training a wavelet neural network controller. Finally, the effectiveness and feasibility of the proposed control method is demonstrated with applications to the chaotic system.

1. Introduction

In the last few decades, chaos has received increasing attention in various areas such as mathematics, engineering, physics, biology, and economics. One attractive topic concerning chaos is chaos control, which is needed to prevent a chaotic system from becoming unstable or being trapped in performance- degraded situations due to the unpredictability and irregularity of chaos.

The conventional control techniques such as feedback control, optimal control, and robust control were introduced to control the chaotic nonlinear systems, and these kinds of techniques confirmed the effectiveness of chaos control[1-3]. But most of these techniques can be applied to control chaotic systems when the exact or at least the approximate mathematical model for chaotic systems is available. To overcome this shortage of them, the direct/indirect adaptive control methods which may be considered as a kind of adaptive control strategy, can be used for controlling chaotic systems[4].

On the other hand, the intelligent control techniques based on neural networks and fuzzy logic are developed to control chaotic nonlinear systems[5]. Even though these intelligent control strategies have shown the effectiveness especially for unknown chaotic system, they have some drawbacks which come from their own inherent characteristics. Therefore, the intelligent control techniques using the wavelet transform(WT), which has a excellent analysis ability, are introduced[6].

In this paper, we propose the design method of a wavelet neural network(WNN) controller based on a direct adaptive control technique for controlling chaotic systems. Finally, in order to evaluate the performance of our controller, we apply the proposed method to the continuous- time chaotic nonlinear system.

2. Wavelet Neural Networks

The theory of wavelets was first proposed by Mallat in the field of multi-resolution analysis (MRA)[7]. A family of wavelets is constructed by translations and dilations performed on a single fixed function called the mother wavelet. A wavelet ϕ_j is derived from its mother wavelet ϕ by:

$$\phi_j(z) = \phi\left(\frac{x-m_j}{d_j}\right) \tag{1}$$

where, its translation factor m_j and its dilation factor d_j are real numbers ($d_j > 0$). And, we choose the first derivative of a Gaussian function as a mother wavelet.

$$\phi(x) = -x\exp\left(-\frac{1}{2}x^2\right) \tag{2}$$

2.1 Configuration of a WNN

Fig. 1 shows the configuration of a WNN, which has N_i inputs, one output, and N_w wavelets.

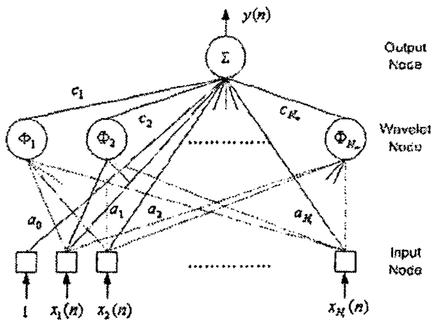

Fig. 1 Configuration of a WNN

Each wavelet of wavelet nodes is given by the product of the mother wavelets as follows:

$$\Phi_j(\mathrm{x}) = \prod_{k=1}^{N_i}\phi(z_{jk}),\ \ with\ z_{jk} = \frac{x_k - m_{jk}}{d_{jk}} \tag{3}$$

where, $k = 1, \cdots, N_i,\ j = 1, \cdots, N_w$.

In Fig. 1, the output of a WNN is composed by each wavelet and parameters as follows:

$$y = \Psi(\mathrm{x},\theta) = \sum_{j=1}^{N_w} c_j\Phi_j(\mathrm{x}) + a_0 + \sum_{k=1}^{N_i} a_k x_k \tag{4}$$

where, a_0 and a_k are connection weight between input nodes and output nodes. c_j is connection weight between wavelet nodes and output nodes, and θ is the set of adjustable parameters.

$$\theta = \{a_0, a_k, c_j, m_{jk}, d_{jk}\} \tag{5}$$

2.2 Training method of a WNN

As usual, the training is based on the minimization of the following quadratic cost function:

$$J(\theta(n)) = \frac{1}{2}(y_r(n) - y(n))^2 = \frac{1}{2}e^2(n) \tag{6}$$

where, $y(n)$ is the output value of n-th WNN and $y_r(n)$ is desired output value.

The minimization is performed by the following iterative gradient-descent method:

$$\begin{aligned}\theta(n+1) &= \theta(n) - \Delta\theta(n) \\ &= \theta(n) - \eta\frac{\partial J(\theta(n))}{\partial\theta(n)}\end{aligned} \tag{7}$$

where, η is the learning rate of a WNN.

The partial derivative of the cost function with respect to $\theta(n)$ is:

$$\frac{\partial J(\theta(n))}{\partial\theta(n)} = -e(n)\frac{\partial y(n)}{\partial\theta(n)} \tag{8}$$

where, $\frac{\partial y(n)}{\partial\theta(n)}$ is the gradient of the plant output, $y(n)$, with respect to parameters set, $\theta(n)$, and the components of this vector are:

- *parameter* a_0

$$\frac{\partial y(n)}{\partial a_0} = 1 \tag{9}$$

- *direct connection parameters* a_k

$$\frac{\partial y(n)}{\partial a_k} = x_k \tag{10}$$

- *weights* c_j

$$\frac{\partial y(n)}{\partial c_j} = \Phi_j(\mathrm{X}) \tag{11}$$

- *translations* m_{jk}

$$\frac{\partial y(n)}{\partial m_{jk}} = -\frac{c_j}{d_{jk}}\frac{\partial\Phi_j(\mathrm{X})}{\partial z_{jk}} \tag{12}$$

where, $\frac{\partial\Phi_j(\mathrm{X})}{\partial z_{jk}} = \phi(z_{j1})\phi(z_{j2})\cdots\dot{\phi}(z_{jk})\cdots\phi(z_{jN_i})$,

$$\dot{\phi}(z_{jk}) = \frac{d\phi(z_{jk})}{dz_{jk}} = (z_{jk}^2 - 1)\exp\left(-\frac{1}{2}z_{jk}^2\right)$$

- *dilations* d_{jk}

$$\frac{\partial y(n)}{\partial d_{jk}} = -\frac{c_j}{d_{jk}}z_{jk}\frac{\partial\Phi_j(\mathrm{X})}{\partial z_{jk}} \tag{13}$$

3. Direct Adaptive Control

In this section, we describe the design method of a WNN controller based on direct adaptive control technique.

3.1 Structure of the direct adaptive control

Assume the following time-varying dynamic system:

$$\begin{aligned}\dot{x} &= f(x, u, t) \\ y &= h(x, t)\end{aligned} \tag{14}$$

where, x is the state variable, u is the control input, and y is the system output.

If the control input u is $\psi(x,t)$, the dynamic of closed-loop system is $\dot{x} = f(x, \psi(x,t), t)$.

The purpose of the on-line trained controller is to find u such that the error value $e(t)$ between the desired output $y_r(t)$ and the real system output $y(t)$ is minimized asymptotically. Therefore, WNN controller based on a direct adaptive control technique can find control input u by mapping the nonlinear function $\psi(x,t)$ appropriately.

The overall configuration of the direct adaptive control system is shown in Fig. 2.

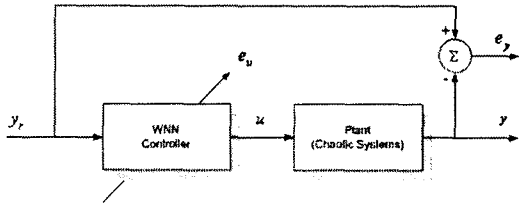

Fig. 2 Configuration of the direct adaptive control system

A WNN controller finds the control input u by training the inverse dynamics of plant iteratively. But, because the output error e_u of the WNN controller for updating the parameters of Eqn. (5) can't be found directly, the proposed method trains the parameters of a WNN controller through the transformation of the output error e_y of plant.

3.2 Training method of direct adaptive control

Define the following cost function so as to train the WNN controller based on direct adaptive control technique.

$$E = \frac{1}{2}(y_r - y)^2 \tag{15}$$

where, $e_y = y_r - y$.

The partial derivative of the cost function with respect to the parameter set of a WNN controller, θ, is calculated by:

$$\frac{\partial E}{\partial\theta} = -e_y\frac{\partial y}{\partial\theta} = -e_y\frac{\partial y}{\partial u}\frac{\partial u}{\partial\theta} = -e_y J(u)\frac{\partial u}{\partial\theta} \tag{16}$$

where, $J(u) = \frac{\partial y}{\partial u}$ is the feedforward Jacobian of plant.

Jacobian $J(u)$ can be calculated by using the variation rate $\frac{\Delta y}{\Delta u}$ of plant output y and control input u within the time-interval. Therefore, we can transform the output error of plant into the output error of a WNN controller easily.

Also, the partial derivative $\frac{\partial u}{\partial\theta}$ of the control input u with respect to the parameters of a WNN controller θ can be calculated by using the equations from Eqn. (9) to Eqn. (13).

4. Simulation Results

In this section, we present some simulation results to validate the control performance of proposed controller for the continuous-time chaotic system. We consider the Duffing system as the controlled chaotic system. The state equation of the system is as follows:

$$\begin{bmatrix}\dot{x}(t)\\ \dot{y}(t)\end{bmatrix}=\begin{bmatrix} y(t) \\ -p_1x(t)-x^3(t)-py(t)+qcos(wt)\end{bmatrix} \tag{17}$$

where, the parameter set is as follows:

$$\{p, p_1, q, w\} = \{0.4, -1.1, 1.8, 1.8\}$$

The control objective for Duffing system is to follow the periodic solution of the Duffing system. In tracking the Duffing, we define the initial system state as $(1,0)$ and the reference signal as the periodic solution in case of $q=2.3$ of Eqn. (17).

The parameters used this simulation and simulation results are shown in Table 1. Fig. 3 and 4 show the tracking control results of state x and y for Duffing system, respectively. Also, Fig. 5 shows the control input signal for Duffing system, and Fig. 6 shows the feedforward Jacobian of Duffing system.

Table 1 Parameters and simulation results

Number of wavelet function	12
Sampling time	0.01
Learning rate	0.0001
Control result of state x (MSE)	0.1518
Control result of state y (MSE)	0.5483

5. Conclusion

In this paper, we have presented the design method of a WNN controller based on direct adaptive control technique for the intelligent control of chaotic system whose mathematical models are unknown. In our design method, the parameters of a WNN controller was tuned by a gradient-descent method. Finally, in order to evaluate the performance of our controller, the proposed method was applied to the Duffing system, which is the representative continuous-time chaotic system. The simulation results has shown that the proposed WNN controller has the faster convergence property and more accurate control performance than those obtained by some conventional neural network control schemes.

This work is supported by KOSEF R01-2001-000-00316

[Reference]

[1] E. Ott, C. Grebogi and J. A. Yorke, "Controlling Chaos", Phys. Rev. Lett., Vol. 64, No. 11, pp. 1196-1199, 1990.

[2] E. A. Jackson and A. W. Hübler, "Periodic Entrainment of Chaotic Logistic Map Dynamics", Physica D, Vol. 44. pp. 407-420, 1990.

[3] G. Chen and X. Dong, "On Feedback Control of Chaotic Continuous-Time Systems", IEEE Trans. on Circuits and Systems, Vol. 40, No. 9, pp. 591-601, 1993.

[4] K. S. Park, J. B. Park, Y. H. Choi, T. S. Yoon and G. Chen, "Generalized Predictive Control of Discrete-Time Chaotic Systems", Int. Jour. of Bifurcation and Chaos, Vol. 8, No. 7, pp. 1591-1597, 1998.

[5] K. B. Kim, J. B. Park, Y. H. Choi and G. Chen, "Control of Chaotic Dynamical Systems Using Radial Basis Function Network Approximators", Information Sciences, Vol. 130, pp. 165-183, 2000.

[6] Y. Oussar, I. Rivals, L. Personnaz and G. Dreyfus, "Training Wavelet Networks for Nonlinear Dynamic Input-Output Modeling", Neurocomputing, Vol. 20. pp. 173-188, 1998.

[7] S. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Transform", IEEE Trans. on Pattern Anal. Machine Intell., Vol. 7, No. 7, pp. 674-693, 1989.

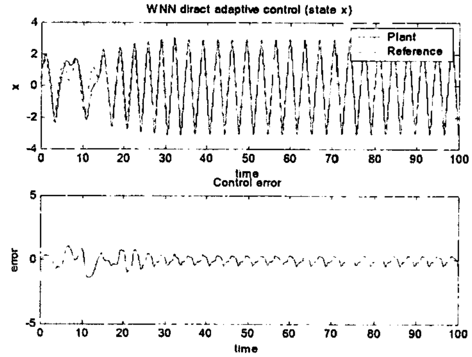

Fig. 3 Control result for Duffing system (state x)

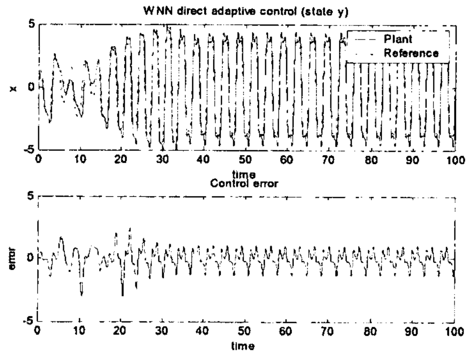

Fig. 4 Control result for Duffing system (state y)

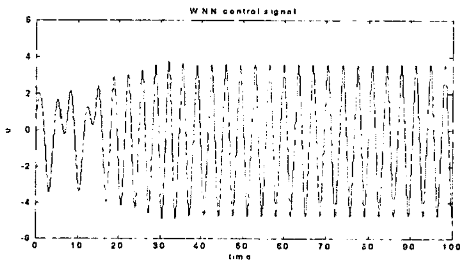

Fig. Control input for Duffing system

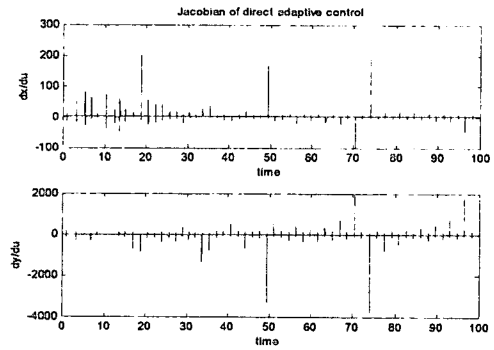

Fig. 6 Feedforward Jacobian for Duffing system