미끄럼 유동을 고려한 초소형 공기 베어링의 정특성

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Static Characteristics of Micro Gas-Lubricated Journal Bearings with a Slip Flow

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Abstract – The fluid mechanics and operating conditions of gas-lubricated journal bearings in micro rotating machinery such as micro polarization modulator and micro gas turbine are different from their larger size ones. Due to non-continuum effects, there is a slip of gas at the walls. Thus in this paper, the slip flow effect is considered to estimate the pressure distribution and load-carrying capacity of micro gas-lubricated journal bearings as the local Knudsen number at the minimum film thickness is greater than 0.01. Based on the compressible Reynolds equation with slip flow, the static characteristics of micro gas-lubricated journal bearings are obtained. Numerical predictions compare the pressure distribution and load capacity considering slip flow with the performance of micro journal bearings without slip flow for a range of bearing numbers and eccentricities. The results clearly show that the slip flow effect on the static characteristics is considerable and becomes more significant as temperature increases.

Keywords – Micro Gas-Lubricated Bearings, Slip Flow, Knudsen Number, Load-Carrying Capacity

1. INTRODUCTION

In recent years, micro rotating machinery has made their emergence and there are expectations that they can be economically manufactured and broadly used in the future. Many of these machines use rotating parts and other moving parts that carry a load and need bearing elements for support. However, it is difficult to use a typical bearing, such as rolling element or oil lubricated bearing, for these machines due to its small scale. Naturally, a need of adequate bearing has been grown and one of possible candidate solutions is a gas-lubricated bearing.

In fact, a gas-lubricated bearing has been used widely in conventional rotating systems due to their distinct advantages compared to rolling element or oil lubricated bearings as follows; no contamination due to leakage of lubricant, relatively compact structure and low friction loss. With these advantages, the studies to develop a gas-lubricated bearing that could be used in micro rotating machinery are performing widely. E.S. Piekos and K.S. Breuer [1] studied the stability of a gas-lubricated journal bearing in micro gas turbine using pseudo-spectral orbit simulation. Similarly, D.J. Orr [2] studied the stability of a gas-lubricated journal bearing for micro gas turbine in both numerical and experimental method. He showed that there was not only whirl but also radial instability in extremely short gas-lubricated journal bearing for micro gas turbine. Some study focuses on structural aspect. Lee *et al.* [3] studied the coupled boundary effect of gas-lubricated journal and thrust bearings and suggested that coupling of journal and thrust bearings could be very effective on increasing a load-carrying capacity.

However in complete adaptation of gas-lubricated bearings to micro rotating machinery, some crucial point arises. The fluid mechanics of these bearings are very different compared to their conventional scale ones. Their study falls in the area of microfluid mechanics, stimulated by its applications to micro machines.

Concerning a gas-lubricated bearing that is served for micro rotating machinery, a minimum film thickness is considerable. If the film thickness of bearing were small enough to reach an arbitrary ratio compared to a molecular mean free path of gas, gas starts to slip on the bearing surface, i.e. slip flow.(4) At slip flow region, surface velocity and average fluid velocity on the surface are different and the compressible Reynolds equation forfeits its validity as a governing equation.

To explain the slip flow, the Knudsen number (Kn), defined as the ratio of the molecular mean free path (λ) to the film thickness (h), is one important parameter:

$$Kn = \frac{\lambda}{h} \quad [1]$$

If Kn <0.01, the fluid is considered a continuum, and the compressible Reynolds equation is used to describe the gas flow. If $0.01 < Kn < 10$, the fluid is considered a rarified gas, and the Reynolds equation with rarefaction coefficients should be used. If $Kn>10$, the fluid is considered a free molecular flow.(5) The rarefaction effect on the gas-lubricated bearings have been generally ignored because the ratio of the mean free path to the clearance is greater than 0.01. However concerning the gas-lubricated bearings for micro rotating machinery, the local Knudsen number at the minimum film thickness may be greater than 0.01 because they should operate at high eccentricity ratio to maintain a stability.(2) In addition, some micro rotating machinery such as micro gas turbine operates at very high temperature condition and it is expected that the slip flow effect can be especially significant because the molecular mean free path increases with temperature.(5)

Thus in this paper, the slip flow effect is considered to estimate the pressure distribution and load-carrying capacity of the gas-lubricated journal bearings for micro gas turbine (see Fig.1). The modified compressible Reynolds equation with rarefaction effect is utilized as a governing equation and solved numerically. The static characteristics of gas-lubricated journal bearing at high temperature are compared with those at room temperature with or without rarefaction effect.

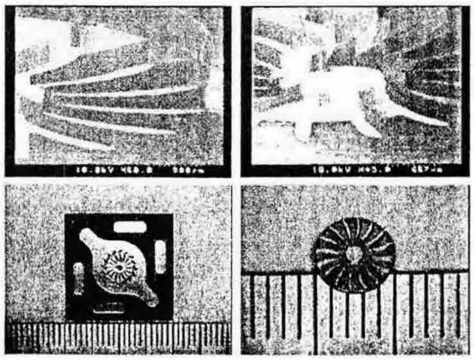

Fig. 1 Micro Gas Turbine manufactured by KIST

2. ANALYSIS

2.1. FIRST ORDER SLIP FLOW MODEL

In analysis procedure, the lubricant is modeled as an isothermal perfect gas. The coordinates are given in Fig. 2 and the pressure distribution within the clearance of gas-lubricated bearing is governed by the modified Reynolds equation as follows;

$$\nabla \cdot \left(-\frac{1}{12\mu} \varphi^{P} p h^{3} \nabla p + \frac{\overset{\omega}{U}}{2} p h \right) + \frac{\partial}{\partial t}(ph) = 0 \quad [2]$$

Variables are defined in the Nomenclature. $\varphi^{P}(p,h)$ denotes the molecular rarefaction coefficient in the following form.

$$\varphi^{P}(p,h) = \sum_{k=0}^{3} \left\{ c_k(p) h^{-k} \right\} \quad [3]$$

The rarefaction coefficient, Equation [3] is valid for high Knudsen numbers using power series expressions in terms of inverse Knudsen number.[5] A first order slip flow boundary condition[6] is one of the possible candidate approximated equations for thin gas film lubrication and is widely used in the case that the film thickness is greater than 0.1 μm.[7] This approximation can be selected without loss of validity because it is known that gas-lubricated journal bearing in micro gas turbine can't operate at extremely high eccentricity ratio where the film thickness is to be 0.1 μm due to its inherent instability.[2]

For the first order slip flow approximation, the coefficients c_k can be chosen as $c_0 = 1$, $c_1 = 6a\lambda$, $c_2 = c_3 = 0$. The rarefaction coefficient, Equation [3] is maintained throughout the theoretical formulation presented here, although it will be set to the first order slip boundary condition in results presented later. For the 1st order slip approximation, the molecular mean free path in the coefficient c_1 is calculated according to the reference[8] with accommodation coefficient set to 1.5

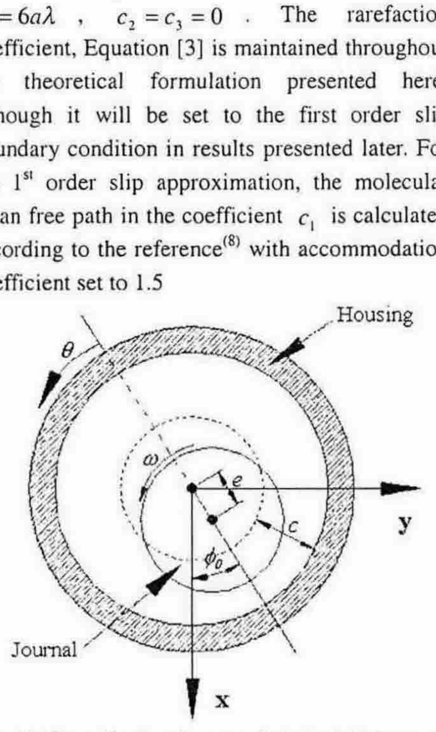

Fig. 2 Coordinates for gas-lubricated journal bearing

2.2. SOLUTION PROCEDURE

Assuming a steady-state condition of gas-lubricated journal bearing, the modified Reynolds equation [2] can be reduced as follows;

$$\nabla \cdot \left(-\frac{1}{12\mu} \varphi^{p} p h^{3} \nabla p + \frac{\overset{\omega}{U}}{2} p h \right) = 0 \qquad [4]$$

To solve a pressure distribution in the bearing clearance, this modified Reynolds equation is discretized by the control-volume formulation. The finite difference scheme, the successive over-relaxation method[9], is applied to solve the steady-state pressure distribution numerically. The accuracy of result is checked by doubling the mesh density. Convergence is achieved until 10^{-9} of $\sum \left| \left(p_{i,j}^{n+1} - p_{i,j}^{n} \right) / p_{i,j}^{n} \right|$ is reached. The relaxation factor ranging from 0.1 to 1.0 is used to accelerate the convergence of the solution. In the analysis, the following boundary conditions are used.

$$\overline{p} = 1 \quad at \quad z = \pm L/2 \qquad [5]$$

$$\overline{p} = 1 \quad at \quad \theta = \theta_1,\ \theta_1 + 2\pi \qquad [6]$$

After the pressure distribution in the bearing clearance is found with the boundary conditions of [5] and [6], the fluid film forces on the bearing surface can be readily calculated as follows;

$$\begin{Bmatrix} \overline{F}_x \\ \overline{F}_y \end{Bmatrix} = \frac{1}{p_a R^2} \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \int_{-L/D}^{L/D} \int_0^{2\pi} (\overline{p} - 1) \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} d\theta d\overline{z} \qquad [7]$$

The dimensionless load-carrying capacity is then given by

$$\overline{W} = \frac{W}{p_a R^2} = \sqrt{\overline{F}_x^2 + \overline{F}_y^2} \qquad [8]$$

2.3. ANALYSIS MODEL

Generally, micro rotating machinery is made from micro fabrication technology. The micro fabrication technology differs from conventional machining because it consists of a continuous process of chemical reaction in addition, subtraction, modification, patterning and fabrication of materials. Although this technology yields various advantages to micro rotating machinery, there is a shortcoming; the limitation of aspect ratio of microstructure. This leads a limitation of bearing geometry. Consequently, the L/D (length to diameter) ratio of journal bearing in micro gas turbine is extremely low value compared to their larger cousins. In this manner, the analysis model of this work has very low L/D ratio, which has the same value of D.J. Orr's work.[2] The purpose of this accordance is a yielding of limitation of micro fabrication technology as well as an easy comparison of results with his work. The properties of analysis model are given in Table 1.

Table 1 Properties of analysis model

L/D ratio	0.075
Eccentricity	0.1 - 0.9
Bearing number	0.1 - 20
Temperature, K	300 and 1600
Nominal clearance, μm	12

3. RESULTS AND DISCUSSION

Because the bearing under consideration is so short, one would expect that the film might have difficulty developing the pressure. In other words, the lubricant would likely be expelled out the ends of the short bearing rather than remain in the bearing interior and develop the pressure. In fact, this statement is not true.

Fig. 3 gives the comparison of pressure distribution without slip and with slip effect at the axial mid-plane of bearing. In the axial direction, it is evident that the pressure extremes are located at the bearing mid-span because a tilting or misalignment of bearing is not considered. As one proceeds along the circumferential direction from left to right, a large pressure rise is observed due to fluid being forced into the squeeze gap resulting from the eccentricity ratio of the bearing. As the lubricant moves into the divergent portion of the squeeze gap downstream of the point minimum film thickness, a sub-ambient region is encountered before recovery to nominally ambient conditions. The figure shows that the solution of modified Reynolds equation without slip is virtually identical to the D.J. Orr's solution, which implies that the numerical analysis in this work is accurate. However, Fig. 3 indicates the significant differences of pressure distribution between slip and no-slip condition, and implies that predicted bearing characteristics with no-slip condition would be invalid. Although the pressure distribution at the relative large film thickness is identical between slip and no-slip condition, large

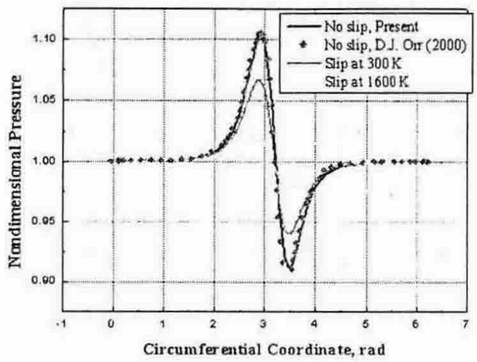

Fig. 3 Comparison of pressure distribution between slip and no-slip condition at the axial mid-plane at $\Lambda = 1.0, \varepsilon = 0.85$

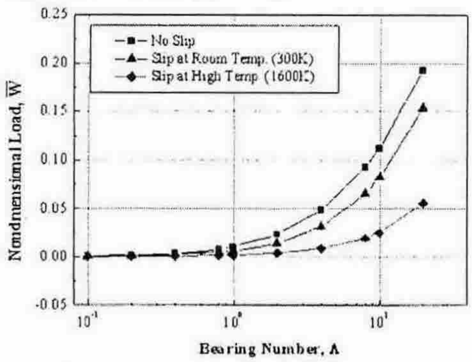

Fig. 4 Comparison of nondimensional load-carrying capacity between slip and no-slip condition in a range of bearing number at $\varepsilon = 0.9$

differences of peak pressure are observed between slip and no-slip condition. The predicted results herein show that the pressure near the minimum film thickness decreases when the slip condition is concerned, and accelerates as the temperature increases. This phenomenon means that there would be a possibility of overestimation of the load-carrying capacity because a large portion of load-carrying capacity of the bearing leans on the generated pressure near minimum film thickness.

Fig. 4 provides the comparison of load-carrying capacity between slip and no-slip condition in a range of bearing number from 0.1 to 20. It is evident that load-carrying capacity decreases under the slip condition. In other words, the slip flow effect could not be ignored in the estimation of performance of gas-lubricated bearings in micro rotating machinery such as micro gas turbine. In the figure, the absolute differences of load-carrying capacity between slip and no-slip condition increase with the bearing number. However in the numerical analysis, we examined the absolute differences of load-carrying capacity at more higher bearing number and found that these absolute differences disappear when the bearing number reaches to extremely large value. Noting that the Couette flow term is related with the rotational velocity of journal and the Poiseuille flow term is related with the rarefaction coefficient, this phenomenon could be explained that the Couette flow term takes a large portion compared to the Poiseuille flow term in the governing equation at extremely high bearing number. However, an increment of temperature also increases the slip flow effect on load-carrying capacity because the molecular mean free path is proportional to temperature.

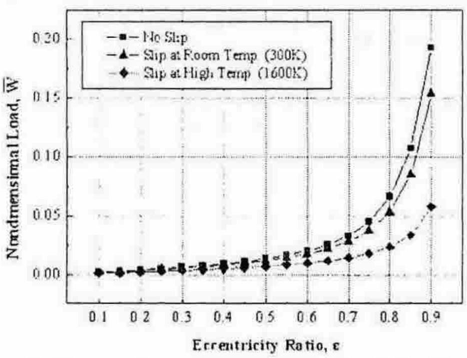

Fig. 5 Comparison of nondimensional load-carrying capacity between slip and no-slip condition in a range of eccentricity ratio at $\Lambda = 20.0$

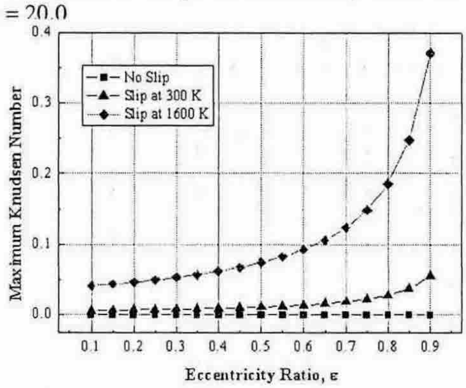

Fig. 6 Maximum Knudsen number with eccentricity ratio

Fig. 5 demonstrates the comparison of load-carrying capacity between slip and no-slip condition in a range of eccentricity ratio from 0.1 to 0.9. Because a large portion of load-carrying capacity leans on the generated pressure in converging wedge as well as the slip flow effect becomes more significant as the minimum film thickness decreases, it is expected that the differences of load-carrying capacity between slip and no-slip condition would increase in proportion to eccentricity ratio. Fig. 4 strongly supports this synthesis. The absolute value of difference of load-carrying capacity increases with increasing the eccentricity ratio. Fig. 6 clarifies a causation of this phenomenon. As the eccentricity ratio increases, the minimum film thickness of bearing decreases. Consequently, the maximum local Knudsen number increases rapidly, and as a result, the slip flow effect becomes more significant at large eccentricity ratio. In the standpoint of temperature, the results are consistent with the previous results of Fig. 4.

4. CONCLUSION

The slip flow effect is considered to estimate the pressure distribution and load-carrying capacity of micro gas-lubricated journal bearings. Based on the modified compressible Reynolds equation with 1st order slip flow approximation, the pressure distribution and load-carrying capacity of micro gas-lubricated journal bearings are demonstrated. The pressure distribution shows a difference scheme between slip and no-slip condition. In addition, load-carrying capacity for bearing number and eccentricity ratio also shows discrepancies between slip and no-slip condition. In conclusion, these results strongly support that the slip flow has substantial influence on the bearing characteristics and should not be ignored in estimation of performance of micro gas-lubricated bearings.

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APPENDIX

NOMENCLATURE

a	= accommodation coefficient
c	= radial clearance
c_k	= rarefaction coefficient
D	= bearing diameter
e	= eccentricity
h, $\bar{h}$	= film thickness; $\bar{h} = h/c$
h_n	= nominal film thickness
F_x, F_y	= resultant forces in x and y direction
Kn	= Knudsen number
Kn_n	= nominal Knudsen number
L	= bearing length
p, $\bar{p}$	= pressure; $\bar{p} = p / p_a$
p_a	= ambient pressure
R	= bearing radius
t	= time variable
$\vec{U}$	= velocity of journal surface
W	= load capacity
z	= axial coordinate
ε	= eccentricity ratio
ϕ_0	= attitude angle
γ	= whirl frequency ratio
φ^P	= rarefaction coefficient
λ	= molecular mean free path
θ	= circumferential coordinate
Λ, $\vec{\Lambda}$	= bearing number; $\Lambda = (6\mu\omega / p_a)(R/c)^2$, $\vec{\Lambda} = \Lambda\hat{i}$
μ	= fluid viscosity
ω	= journal angular velocity
$\hat{i}$	= unit vector in the θ direction

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