

Thermo-mechanical Contact Analysis on Disk Brakes by Using Simplex Algorithm

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A numerical procedure for analyzing thermo-elastic contact applied to an automotive disk brake and calculating subsurface stress distribution has been developed. The proposed procedure takes the advantage of the simplex algorithm to save computing time. Flamant's solution and Boussinesq's solution are adopted as Green function in analysis. Comparing the numerical results with the exact solutions has proved the validity of this procedure.

Keywords: Contact Mechanics, Thermo-elasticity, Simplex Algorithm, Brake

1. INTRODUCTION

It would be beneficial to know the contact pressure distribution a priori for analyzing the frictional sliding and wear in thermo-elastic contact phenomena [1-2] such as automotive disk brakes. On the other hand most of contacts include extremely small portion of a whole surface and thus Flamant's and Boussinesq's classic solutions are generally used for relating issues [3-7] even though some difficulties are unavoidable in calculation.

Since the brilliant advent of computers, the evolution on numerical techniques in engineering analyses has been accelerated [8]. Especially the finite element analysis (FEA) and boundary element method (BEM) gain a preferred acceptance to investigate contact mechanism. However, these methods must process huge matrices in practical contact problems and call on inherent inefficiency to waste computational resources.

For this study we develop a Simplex design algorithm program for obtaining the contact pressure in thermo-elastic contact problems and apply to an automotive disk brake system. In the past, Seireg and Rodrigues [9] presented the design of elastic bodies in contact by using a Simplex-type algorithm proposed by Conry and Seireg [10]. In their study [10], they proved a successful and far efficient method to analyze elastic contact problems by applying the well-known Simplex-type design algorithm [11]. In this presentation we try to extend their methodology to thermo-elastically coupled contact problems; for obtaining pressure distribution in the zone of contact for thermo-elastic contact problem, a numerical procedure that considers both mechanical and thermal load is developed.

Flamant's solution and Boussinesq's solution are adopted as response functions for the respective two-dimensional and three-dimensional contact analyses. Influence coefficient for two- and three-dimensional contact problem is determined by using these solutions. Constitutive equations for contact problem are described by using the condition of the compatibility of deformation and the condition of equilibrium. Pressure distribution in the zone of contact, solution of constitutive equations, is obtained by using Simplex algorithm. Simplex algorithm has an advantage to achieve effective computing because this algorithm considers only nodes in the

zone of contact. The validity of this procedure was proved by comparing the numerical results with the exact solutions.

Contact coefficient that influences on heat dissipation in the gap between two bodies is assumed in two ways. In the first assumption, the contact coefficient is constant. In the second, contact coefficient is proportional to a pressure in the zone of contact. A temperature distribution of two bodies is calculated by the contact coefficient and the finite difference method. The temperature distribution is used to calculate the thermal stresses of two bodies.

2. SOLUTION PROCEDURE

2.1 Thermo-mechanical coupling

In this study we utilize the finite difference method for obtaining temperature distributions in the two contact bodies that be thermally insulated from the outside atmosphere. In the discretization scheme, we deliberately place each pair of contact nodes on the two bodies to be at the same coordinate for convenience. We construct the difference formulations by applying the energy conservation law to a volume element. And the contact coefficient on the contact region is considered to postulate as the following two cases; (1) the contact coefficient is constant and (2) the contact coefficients are proportional to the contact pressure.

The first postulate is the case that the mechanical loads and thermal flux are independent. Here the total contact load is simply the sum of all discrete contact loads. However, the second postulate means that the coefficients are determined by the mechanical loads of initial pressure distribution and the coefficients in turn influence on the initial separation and thus on the mechanical load distribution. The matrix equations by the finite difference method are solved by using Gauss-Seidel iteration.

2.2 Solution procedure

Although many methods exist for analyzing contact problems, the convenience of forming simply contact nodes compels us to use the Simplex algorithm, which is an enormous computation time saver. Fig. 1 is a schematic on which we have developed a program in this study by using MATLAB™ computer language.

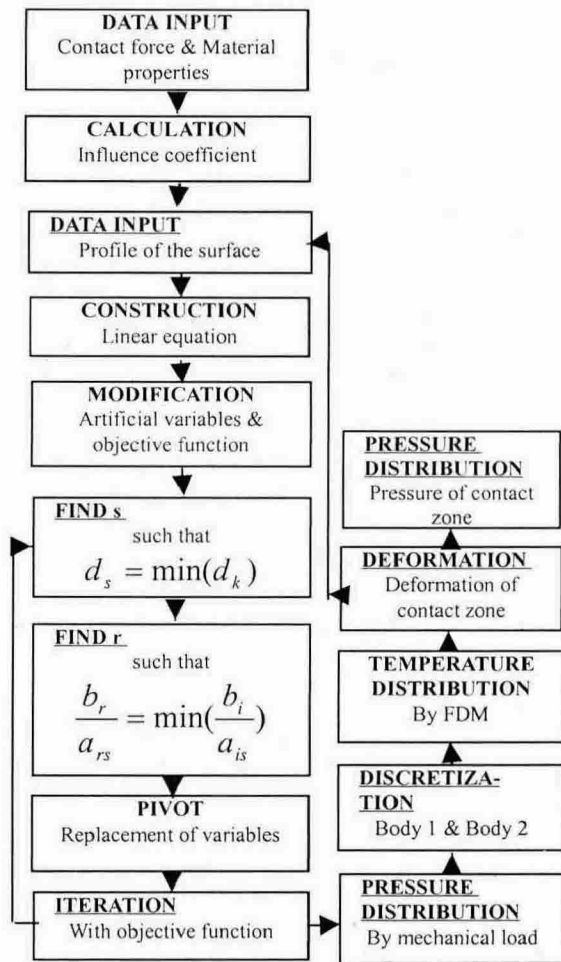


Fig. 1 Program algorithm for thermo-elastic analysis

2.3 Numerical example

As shown in the flow chart of Fig. 1, the program seeks first for a uniform pressure distribution ascribed to the applied force P and the additional pressure due to thermal load. For the each case of two, we may determine the different pressure distributions as; (1) when the contact coefficient is considered a constant, the contact pressure may be obtained by summing the respective pressures caused by the mechanical and the thermal loads.

In case of the pressure-dependent contact coefficient, since the estimated mechanical pressure influences on the temperature distributions of the punch and the half plane and in turn on the thermal distortion of contact surfaces (or initial separation), we need to repeat the analysis at each time step until a convergence is attained. In this calculation, we accept the pressure when a differential separation amount from the previous step is less than $1/1000$ mm.

Fig. 2 shows the results analyzed from the two cases above and Fig. 3 presents the initial separation distribution between contact nodes of two bodies.

3. CONCLUSIONS

An algorithm for analyzing thermo-elastic contact problems is developed and numerically validated. Only the contact nodes are established in the analysis, which is a computation

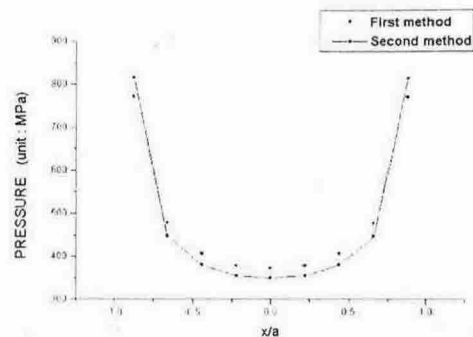


Fig. 2 Pressure distributions by two assumptions

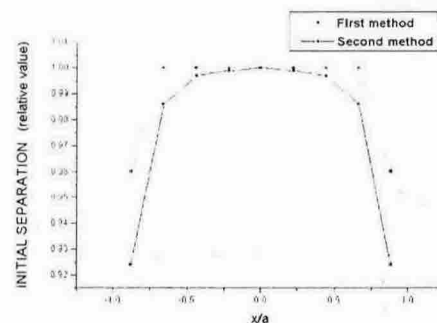


Fig. 3 Initial separations between two bodies

time saver and lead to easy convergence. Simplex algorithm can be considered as a practical estimator for thermo-elastic contact problems. This algorithm can be easily adapted to any arbitrary three-dimensional thermo-elastic contact problems.

4. ACKNOWLEDGEMENTS

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