

A Modified Method for the Boundary Fitted Coordinate Systems to Analysis of Gas Bearings Considering Upstream in Extremely High Compressibility Number Region

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An expanded scheme of direct numerical solution method for solving the Reynolds' equation in the boundary fitted coordinate systems for the gas lubrication with ultra low clearance is presented. Skewed slider is calculated by this scheme and results are compared to the original direct numerical solution. The modified scheme has advantages in stability in high compressibility number region. At the lower Λ region the difference in results of original and modified method is several percents.

Keywords: Direct Numerical Method, Boundary Fitted Coordinate System, Gas Lubrication, High Compressibility.

1. INTRODUCTION

One of the main directions of the development of computer technology is increasing their hard disk capacity. The reading/writing elements, mounted at the hard disk head, should be positioned close to disk surface to work with high density data. In dimensionless Reynolds' equation, the space between head and disk surfaces can be expressed by the number of Λ , which is called the *compressibility number*. The space is very low in the high Λ region if other design value is same.

The boundary fitted coordinate system method (BFCS method) [1] provides the numerical solutions of the Reynolds' equation for different kinds of gas bearings with complicated boundary conditions. In the BFCS method, the upstream scheme [2] is applied in high Λ . However, the original BFCS methods may provide discontinuous pressure distribution in high Λ region. In the present work, the BFCS method described in [1] is modified to provide numerical stability. The modified method is applied to skewed slider, and the continuity of the resulting pressure distribution is observed.

2. ORIGINAL METHOD

2.1 Fundamental equations

The direct numerical method solves the Reynolds' equation

$$\frac{\partial}{\partial x} \left(b^3 p \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(b^3 p \frac{\partial p}{\partial y} \right) = 6\eta \left(U \frac{\partial}{\partial x} (bp) + V \frac{\partial}{\partial y} (bp) \right) \quad (1)$$

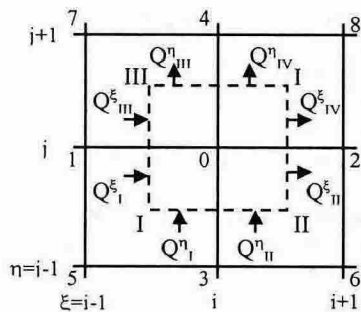


Fig. 1 Total flow in one node.

by direct application of the zero divergence conditions to the dimensionless total flow (2) in the each mesh node (Fig.1).

$$Q^\xi = Q_P^\xi + Q_C^\xi \quad (2)$$

$$Q^\eta = Q_P^\eta + Q_C^\eta$$

where Q_P is the Poiseuille flow, and Q_C is the Couette flow.

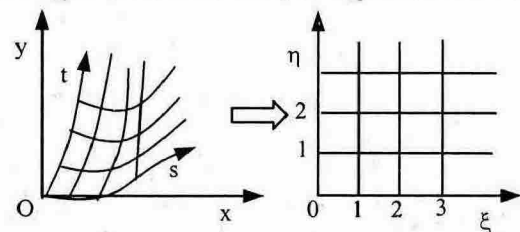


Fig.2 The physical and computational coordinate

In BFCS the flow is written in computational domain $0 \leq \xi \leq 1$ which is mapped to physical domain $0st$ by the coordinate transformation [1].

The total Poiseuille flow in the node 0 is written as

$$Q_P = \{ a_{00} p_0^2 + p_0 (a_{01} p_1 + a_{02} p_2 + a_{03} p_3 + a_{04} p_4 + a_{05} p_5 + a_{06} p_6 + a_{07} p_7 + a_{08} p_8) + p_1 (a_{11} p_1 + a_{13} p_3 + a_{14} p_4 + a_{15} p_5 + a_{17} p_7) + p_2 (a_{22} p_2 + a_{23} p_3 + a_{24} p_4 + a_{26} p_6 + a_{28} p_8) + p_3 (a_{33} p_3 + a_{35} p_5 + a_{36} p_6) + p_4 (a_{44} p_4 + a_{47} p_7 + a_{48} p_8) + a_{55} p_5^2 + a_{66} p_6^2 + a_{77} p_7^2 + a_{88} p_8^2 \} / 48 \quad (3)$$

And the total Couette flow is expressed by

$$Q_C = (\sum a_{ci} p_i) / 8 = 1/8 * (3p_0 (D_{1-} - D_{1+} + D_{III-} - D_{IV+} + E_{1-} + E_{II-} - E_{III+} - E_{IV+}) + p_1 (3D_{I+} + 3D_{III+} + E_{III-} - E_{I+}) + p_2 (-3D_{II-} - 3D_{IV-} + E_{IV-} - E_{II+}) + p_3 (+D_{II-} - D_{I+} + 3E_{I+} + 3E_{II+}) + p_4 (+D_{IV-} - D_{III+} - 3E_{III-} - 3E_{IV+}) + p_5 (D_{I+} + E_{I+}) + p_6 (-D_{II-} + E_{II+}) + p_7 (D_{III+} - E_{III-}) + p_8 (-D_{IV-} - E_{IV+})) \quad (4)$$

2.2 Couette Flow Coefficients

Coefficients D and E are proportional to Λ . They depends, correspondingly, of the flow velocities in ξ and η directions.

In full upstream scheme $D_+ = D > 0$, $D_- = 0$, $E_+ = E > 0$, and $E_- = 0$, so

$$\begin{aligned} a_{c0} &= 3(-D_{II} - D_{IV} - E_{III} - E_{IV}) \\ a_{c1} &= 3D_I + 3D_{III} - E_{III} \\ a_{c2} &= -E_{IV} \\ a_{c3} &= -D_{II} + 3E_I + 3E_{II} \\ a_{c4} &= -D_{IV} \end{aligned} \quad (5)$$

Write the multiplier of p_i in total flow as m_i

$$m_i = \sum_{j \geq i} a_{i,j} p_j + a_i \quad (6)$$

The convergence of pressure calculation requires m_0 to be negative. And it is better if $m_i \geq 0$ for $i > 0$. Notice that usually $\sum a_{0,j} p_j < 0$, and $\sum a_{i,j} p_j > 0$ for $i > 0$.

The equation (5) shows, that full upstream scheme provides the negative value of a_{c0} , but doesn't guarantee positive values of a_{c1} , a_{c2} , a_{c3} , a_{c4} . It means, that $m_0 < 0$ and the calculation converges, but some of m_i may become negative in high Λ region and this may cause problems.

3. MODIFIED METHOD

It is possible to redistribute D_+ , D_- , E_+ , and E_- between nodes of each zone by such a way, the total sums of this quantities are the same, but all D_+ and E_+ has the positive sign in a_{ci} , $i \neq 0$, and, as before, negative in a_{c0} . After these procedure, the total Couette flow changes as follows;

$$\begin{aligned} Q_c &= -b_0 p_0 + \sum b_i p_i = 1/16 * (\\ &\delta p_0 (D_I + D_{III} - D_{II+} - D_{IV+} + E_I + E_{II} - E_{III+} - E_{IV+}) + \\ &p_1 (6D_{I+} + 6D_{III+} - E_{III+} + E_{I+}) + p_1 (-6D_{II-} - 6D_{IV-} - E_{IV-} + E_{II+}) + \\ &p_3 (-D_{II-} + D_{I+} + 6E_{I+} + 6E_{II+}) + p_4 (-D_{IV+} + D_{III+} - 6E_{III-} - 6E_{IV-}) + \\ &p_5 (D_{I+} + E_{I+}) + p_6 (-D_{II-} + E_{II+}) + p_7 (D_{III+} - E_{III+}) + p_8 (-D_{IV-} - E_{IV-}) \end{aligned} \quad (7)$$

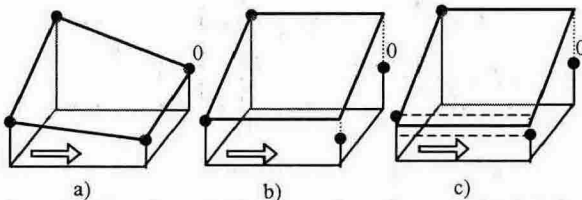


Fig. 3 Interpolation of pressure between four nodes of the I zone.
a) Linear; b) Full upstream; c) in modified method

This changes in Couette flow corresponds to changes in pressure interpolation method, that is used to determine the density distribution within each zone. The interpolation of pressure in the I zone is shown, as the example, at the Fig.3.

4. CALCULATION RESULTS

The original and modified methods were used to calculate the pressure distribution for skewed gas bearing (Fig. 4).

The result is shown at the Figure 5. It can be seen that with decreasing h_0 (that means increasing Λ) the pressure distribution, calculated by original method, becomes

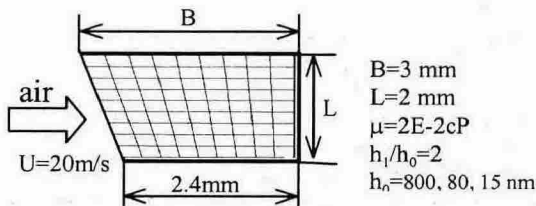


Fig 4 The skewed slider parameters

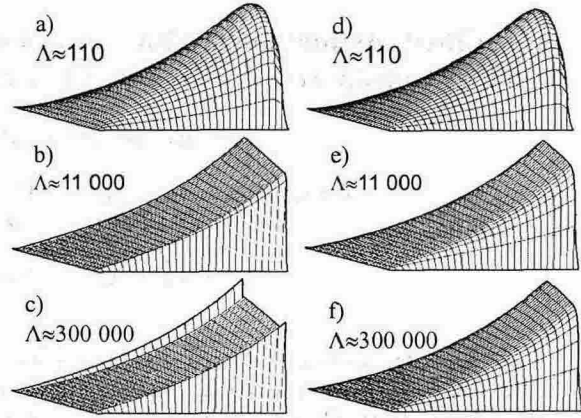


Fig.5 The pressure distribution, calculated by a, b, c) original method; d, e, f) modified method

discontinuous, but the modified methods provides smooth solution. This instability in original method appears at $h_0 < 50 \text{ nm}$, or, in other words, at $\Lambda > 30 000$.

From the equations (5), the flow in only one direction, say ξ , produces negative values in coefficients a_{c3} and a_{c4} only, because $E=0$ and $D \sim \Lambda$ in this situation. Thus the instabilities are expected in η direction only. It explains, why the discontinuities at the Fig.5c are present in only one direction, which is perpendicular to the velocity.

5. CONCLUSIONS

The considered modification of direct numerical method lets to find pressure distribution in extra high Λ region. For low Λ the result is almost the same, as for original method, but convergence is a little more slowly. The criterion of the original method stability is described.

6. NOMENCLATURE

- Λ – the compressibility number, equals $6U\mu B / (h_0^2 p_{atm})$
- h_0 – the film thickness at the bearing output
- h_1 – the film thickness at the bearing input
- U – velocity of the disk surface
- B, L – length and width of the slider or disk head
- Q_P – Poiseuille flow rate
- Q_C – Couette flow rate
- p_i – the pressure at the i -th node as shown on Fig 2.
- μ – gas dynamic viscosity
- p – pressure
- h – film thickness

7. REFERENCES

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