A Computational Analysis of Air Entrainment with a Nip Roller

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ABSTRACT

Air entrainment of a winding roll with a nip roller was studied numerically. The amount of air entrainment between two rotating rollers was obtained by solving lubrication equation, Reynolds equation, which neglect the existence of a web. However, the numerical model of this study included the web existence, therefore it considered the two lubricating air films between a winding roll and a web and also between a nip roller and the web. The pressure profiles and gap profiles of the two films were obtained by solving lubrication equation for the two air films and force balance equation of the web. Ballooning phenomenon was examined in terms of nip force, wrap angle, web stiffness, web speed, and web tension. This ballooning phenomenon caused by the back flow of the air film blocked by the nip roller. Air entrainment of the two numerical models was compared.

Keywords: ***Air entrainment, Winding, Nip roller, Reynolds equation, Hydrodynamic lubrication, Elastohydrodynamic lubrication, Foil bearing, Ballooning***

Nomenclature

$B = \dfrac{R_1 p_a}{T}$

$D = \dfrac{Et^3}{12(1-\nu^2)TR_1^2}\varepsilon^{-2/3}$

E = Modulus of elasticity of web (psi)

F = Nip force (lbf/in)

$H_1 = \dfrac{h_1}{R_1}\varepsilon^{-2/3}$

$H_2 = \dfrac{h_2}{R_1}\varepsilon^{-2/3}$

$H_o = \dfrac{h_0}{R_1}\varepsilon^{-2/3}$

$H_s = \dfrac{h_s}{R_1}\varepsilon^{-2/3}$

h_1 = Gap between web and winding roll (inch)

h_2 = Gap between web and nip roller (inch)

h_c = Amount of air entrainment, equivalent to the air film thickness after the air expands to the ambient pressure (inch)

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h_o = Gap between winding roll and nip roller at the center of nip (inch)

$h_s = \frac{x^2}{2R_e} + h_o$, Gap between winding roll and nip roller approximated to a parabolic curve (inch)

h^* = Nominal clearance: a constant air film thickness over wrapped zone for zero nip force case (inch)

L_{in} = Length of incoming web span (inch)

L_{out} = Length of outgoing web span (inch)

$P_1 = p_1 / p_a$

$P_2 = p_2 / p_a$

p_1 = Pressure between web and winding roll (psi)

p_2 = Pressure between web and nip roller (psi)

p_a = Ambient pressure, 14.7 psi

R_1 = Radius of winding roll (inch)

R_2 = Radius of nip roller (inch)

$R_e = \frac{R_1 R_2}{R_1 + R_2}$, Equivalent radius (inch)

$S = \frac{s}{R_1}\varepsilon^{-2/3}$

$s =$

T = Web tension (lbf/in)

$u =$

$W = \frac{w}{R_1}\varepsilon^{-2/3}$

$w =$

$\varepsilon = \frac{12\mu u}{T}$, Foil bearing number

$\Lambda_a = \frac{6\lambda_a}{R_1}\varepsilon^{-2/3}$

$\lambda_a = 2.65\times10^{-6}$ **(inch)**, Mean-free-path of the air molecules

$\mu = 2.6396\times10^{-9}$ (psi·s), Dynamic viscosity

$\nu =$

θ_{in} = Incoming wrap angle of winding roll (degrees)

θ_{out} = Outgoing wrap angle of winding roll (degrees)

1. INTRODUCTION

An excessive amount of air entainment in the winding roll can cause defects such as telescoping or dishing by making the web float and slide in the transverse direction. On the other hand, too little air in a winding roll can cause different types of winding defects such as buckling. Therefore, proper contact between adjacent layers of web should be achieved to prevent winding defects. Nip rollers are used for this purpose as shown in Fig 1. One annoying problem with nip rollers is the ballooning phenomenon, which can cause wrinkling problems.

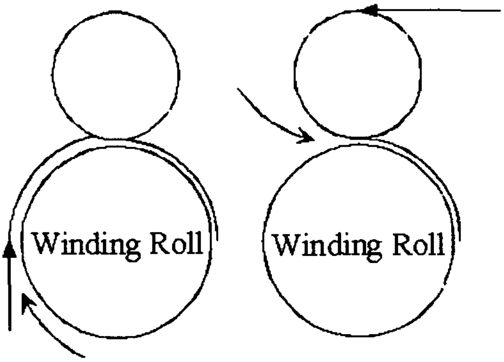

Fig. 1. Winding with a nip roller

For an unnipped winding roll, the air film thickness between the web and the winding roll can be approximated with the foil bearing equation[1].

$$\frac{h^*}{R} = 0.643\left(\frac{12\mu u}{T}\right)^{2/3} \quad (1)$$

For an winding roll wth a nip roller, the prediction equation of the air entrainment was developed by Chang neglecting the existence of the web (Hydrodynamic Model).[2]

$$\frac{h_c}{R_e} = 2.4\frac{\mu u}{F} + 2.408\left(\frac{\mu u}{p_a R_e}\right)^{2/3} - 1.8\times10^{-6} \quad (2)$$

Moreover, Chang developed the air entrainment prediction equation including the effect of the elastic deformation of roller (Elastohydrodynamic Model).[3]

$$\frac{h_c}{R_e} = 7.4\left(\frac{\mu u}{p_a R_e}\right)^{0.66}\left(\frac{F}{p_a R_e}\right)^{-0.21}\left(\frac{E}{p_a}\right)^{-0.33} \quad (3)$$

This study is an extension of the above studies including the effect of the web existence. The main objective of the present study is to develop an understanding of the effects of a nip roller on air entrainment and ballooning. This study does not include the effects of the elastic deformation of roll and the effects of asperity contact, and the surfaces of the web and the rolls are assumed perfectly smooth.

2. PROCEDURE

2.1 Governing Equations

A schematic view of the winding system with a nip roller is shown in Fig. 2.

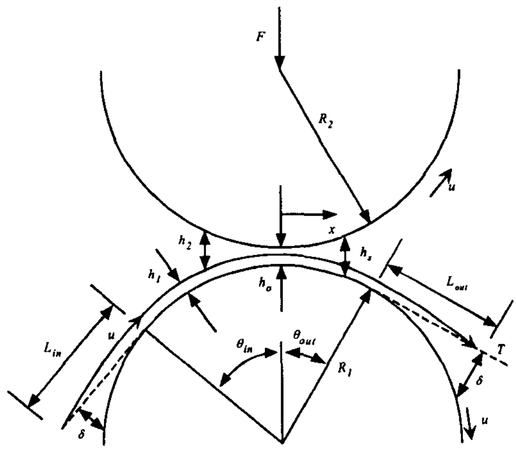

Fig. 2. Schematic of model

The model of this study consists of a rigid winding roll, a rigid nip roller, and a non-permeable web. All of the three objects have the same speed of u.

The governing equations are a web deflection equation and the modified Reynolds equation to include the effects of fluid compressibility and the effects of slip boundary. The slip boundary effect is important when the gap is not much larger than the mean-free-path of the air molecules.

The Reynolds equation with the above assumptions is

$$\frac{d}{ds}\left(ph^3\frac{dp}{ds}+6\lambda_a p_a h^2\frac{dp}{ds}\right)=12\mu u\frac{d}{ds}(ph) \quad (4)$$

where the term $6\lambda_a p_a h^2 dp/ds$ is for the slip boundary effect, λ_a is the mean-free-path of the air molecules. The boundary conditions are

$$p|_{s=0}=14.7psia \text{ and } p|_{s=L}=14.7psia \quad (5)$$

In a non-dimensional form, the above equation becomes

$$\frac{d}{dS}\left(PH^3\frac{dP}{dS}+\Lambda_a H^2\frac{dP}{dS}\right)=\frac{1}{B}\frac{d}{dS}(PH) \quad (6)$$

where

$$\Lambda_a=\frac{6\lambda_a}{R_1}\varepsilon^{-2/3},\quad B=\frac{R_1 p_a}{T},\quad S=\frac{s}{R_1}\varepsilon^{-2/3},$$

$$H=\frac{h}{R_1}\varepsilon^{-2/3},\ P=\frac{p}{p_a},\text{ and } \varepsilon=\frac{12\mu u}{T} \quad (7)$$

A force balance equation of the web can be written in a one-dimensional form including the bending stiffness of the web (Shell Equation).[4)]

$$\bar{d}\frac{d^4 w}{ds^4}-T\frac{d^2 w}{ds^2}=p_1-p_2-\bar{p}_w \quad (8)$$

where $\bar{d}$ is the bending stiffness of the web defined as

$$\bar{d}=\frac{Et^3}{12(1-\nu^2)} \quad (9)$$

and $\bar{p}_w$ is the wrap pressure defined as

$$\bar{p}_w=\begin{cases}0 & \text{inlet zone}\\ T/R_1 & \text{wrapped zone}\\ 0 & \text{outlet zone}\end{cases} \quad (10)$$

The air gaps in the inlet and outlet zone are defined as the web displacement plus inlet and outlet geometry that can be approximated as a parabolic curve,

$$h=w+\delta \quad (11)$$

where h is the air gap and δ

$$w|_{s=0}=0,\quad \left.\frac{d^2w}{ds^2}\right|_{s=0}=0,\quad w|_{s=L}=0,\text{ and}$$

$$\left.\frac{d^2w}{ds^2}\right|_{s=L}=0 \quad (12)$$

A non-dimentional form of Eq(8) can be written as

$$D\frac{d^4W}{dS^4}-\frac{d^2W}{dS^2}=B(P_1-P_2)-P_w \quad (13)$$

where

$$W=\frac{w}{R_1}\varepsilon^{-2/3},\quad D=\frac{Et^3}{12(1-\nu^2)TR_1^2}\varepsilon^{-2/3},\text{ and}$$

$$P_w=\begin{cases}0 & \text{inlet zone}\\ 1 & \text{wrapt zone}\\ 0 & \text{outlet zone}\end{cases} \quad (14)$$

For numerical calculation, those governing equations should be written in a finite-difference form[5)].

2.2 Computational Algorithm

The structure of the computation is shown in Fig. 3. For the first calculation, guessed initial pressure and gap profiles of the two air films are needed, which should be very close to a converged solution so that they can be made of foil bearing solution.

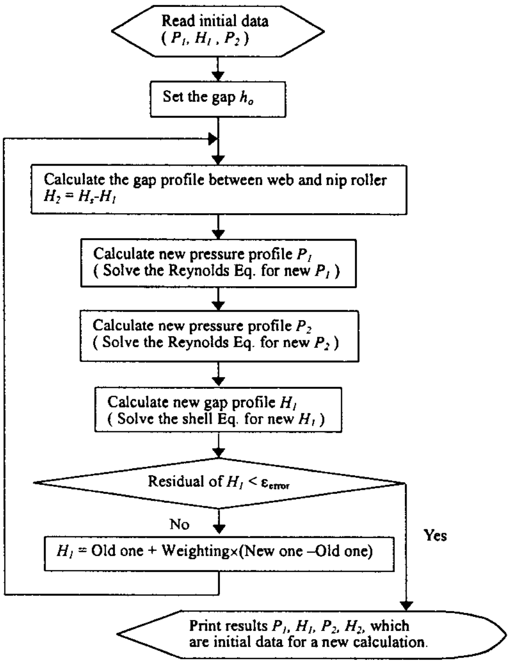

Fig. 3. Flow chart of the computation

For the first step, read initial profiles (P_1, H_1, P_2), then set the gap between winding roll and nip roll at the center of nip (h_o), calculate the gap profile between web and nip roller ($H_2=H_s-H_1$), calculate new pressure profile (P_1) by solving the Reynolds Eq. with given H_1, calculate new pressure profile (P_2) by solving the Reynolds Eq. with given H_2, and calculate new gap profile (H_1) by solving the shell Eq. Now, the new four profiles (P_1,P_2,H_1,H_2) have been calculated, then calculate residual of H_1, and a weighted profile of H_1 is used on the next iteration ($H_{1weighted}$ = H_{1old} + $Weighting\times(H_{1new}-H_{1old})$. Then iterate until the solution converges using under-relaxation weighting factor. After the solution has converged, the nip force can be calculated by integrating P_2. The amount of entrained air is determined based on the air film thickness and the pressure at the location where pressure gradient is zero. The velocity profile is uniform at that point, then the amount of air entrainment is calculated by the relationship of $h_c=ph/p_a$, which means that h_c is the equivalent air film thickness after the air expand to the ambient pressure[5)]

3. RESULTS & DISCUSSIONS

3.1 Ballooning

Gap profiles and pressure profiles of sample calculations are shown in Fig. 4 and Fig. 6 for a perfectly flexible web. It was found that there is a critical nip force above which ballooning occurs (more detailed description of critical nip force is shown in the next section). In super-critical conditions, the balloon size does not change very much with the nip force. It was also found that the maximum balloon height is nearly proportional to the incoming wrap angle as shown in Fig. 7 and Fig. 8.

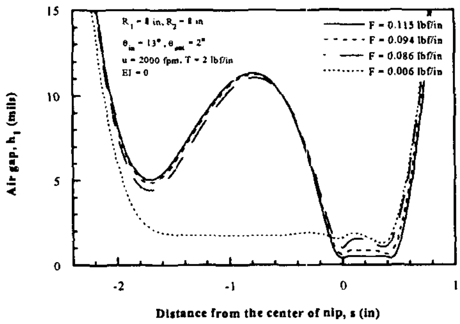

Fig. 4. gap profiles

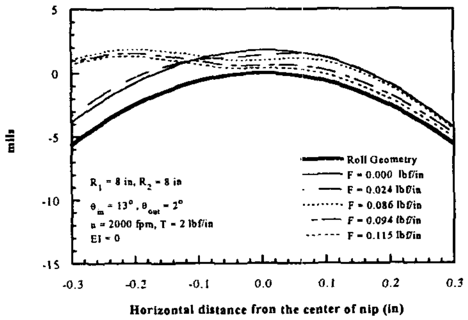

Fig. 5. web profiles

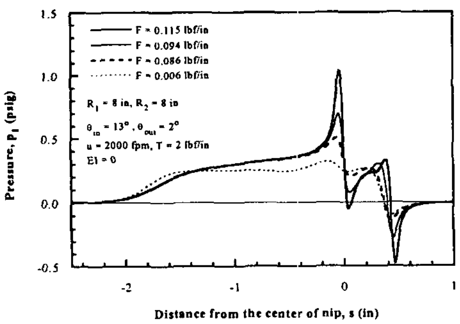

Fig. 6. pressure profiles

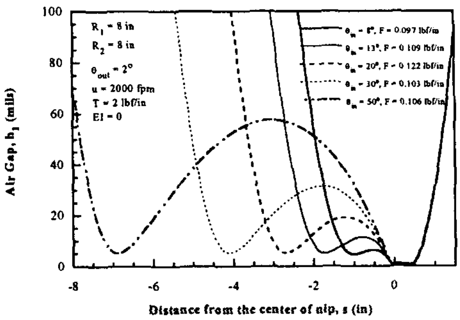

Fig. 7. Gaps for several incoming wrap angles

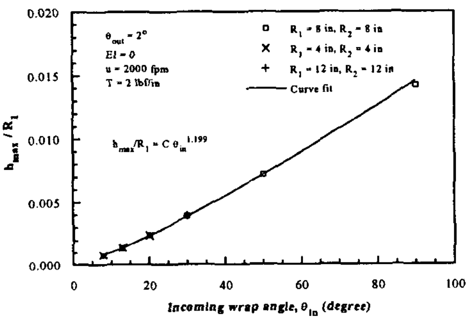

Fig. 8. effect of wrap angle on ballooning

However, the effects of web thickness or bending stiffness on ballooning are negligible as shown in Fig. 9.

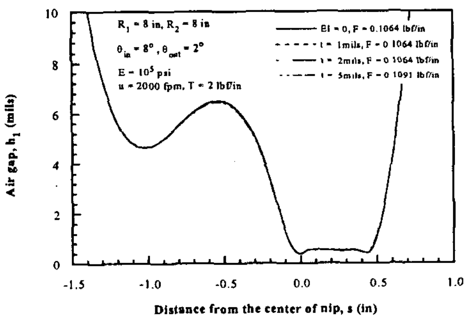

Fig. 9. effect of bending stiffness on ballooning

The maximum balloon height increases with web speed as shown in Fig. 10

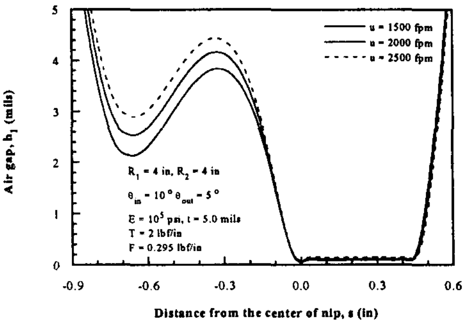

Fig. 10. effect of web speed on ballooning

The maximum balloon height decreases when web tension increases as shown in Fig. 11

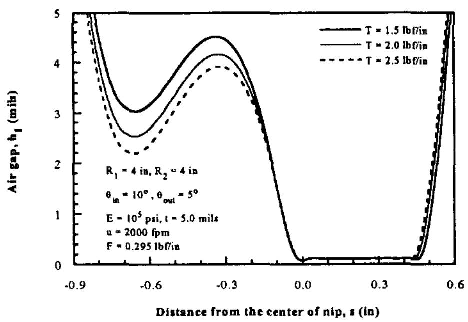

Fig. 11. Effects of web tension

3.2 Air entrainment

General trends of air entrainment versus nip force are shown for perfectly flexible web in Fig. 12, 13, and 14. Once ballooning occurs the amount of entrained air reduces dramatically as the nip force increases, which explains critical nip force. The amounts of air entrainment, h_{1c} and h_{2c}, seem to converge to each other, which cannot be verified because of insufficient data for large values of nip forces due to the convergence problem.

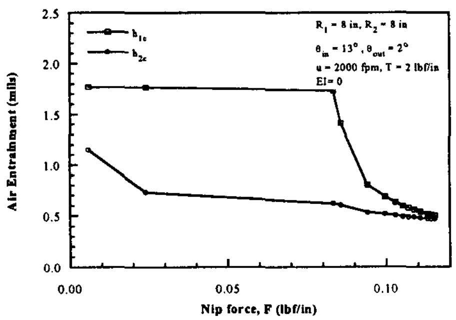

Fig. 12. General trend of air entrainment

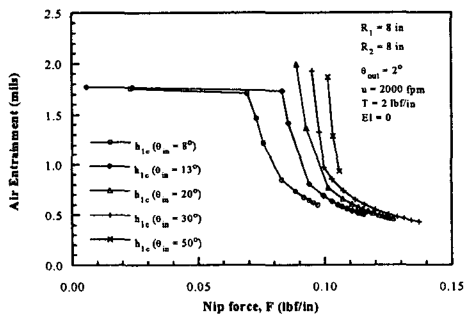

Fig. 13. Air entrainment for several wrap angles

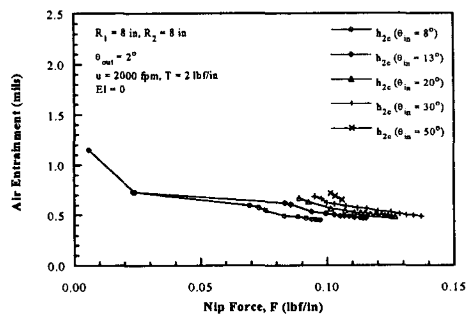

Fig. 14. Air entrainment for several wrap angles

It appears very difficult to obtain a closed form correlation equation of the air entrainment because the air entrainment varies dramatically near the critical nip force for ballooning. Therefore, the current model (Model 1) is compared with a simplified model (Model 2) shown in Fig. 15 in order to find a simple way to calculate the amount of air entrainment. The model 2 in Fig. 15 describes a rigid roller rotating near a rigid flat plate, moving at the same speed of u.

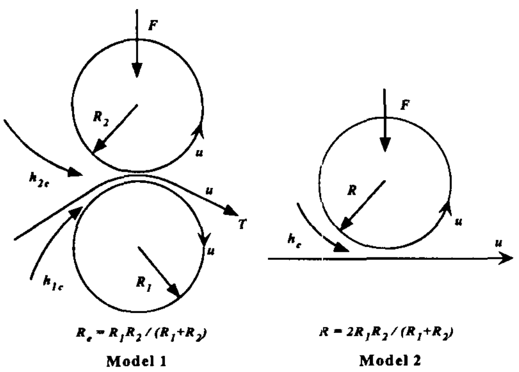

Fig. 15. Comparison of two computational models

As mentioned above, Eq (2) is the prediction equation of air entrainment for the model 2 where $R_e = R \times \infty / (R + \infty) = R$

However, the air entrainment for model 1, h_{1c} and h_{2c}, can be calculated by modifying $R = 2 \times R_e$ as

$$\frac{h_{1c}}{2R_e} = \frac{h_{2c}}{2R_e} = \frac{h_c}{R} = 2.4\frac{\mu u}{F} + 2.408\left(\frac{\mu u}{p_a R}\right)^{2/3} - 1.8 \times 10^{-6} \quad (14)$$

For example, the air entrainment for model 2 with R=4" (or R_1=R_2=8" without a web) is the same as that for model 1 with R_e=2" (R_1=R_2=4" with a web), where the equivalent radiuses of the two models have the relationship of $R = 2 \times R_e$

In Fig.. 16, 17, and 18, h_{1c} and h_{2c} for moldel 1 and h_c for model 2 approach to each other as the nip force increases regardless of the wrap angle and web stiffness.

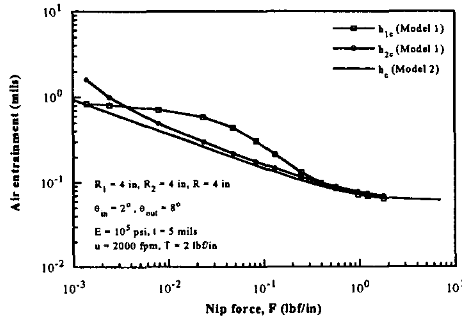

Fig. 16. Effects of nip force on air entrainment

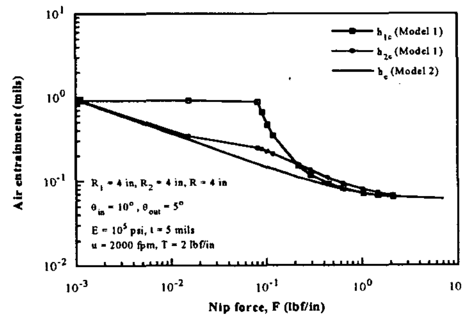

Fig. 17. Effects of nip force on air entrainment

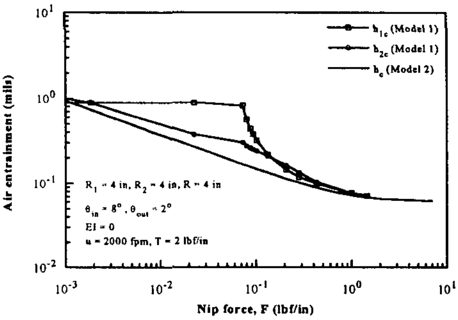

Fig. 18. Effects of nip force on air entrainment

This observation holds even when $R_1 \neq R_2$ as shown in Fig. 19, and Fig. 20

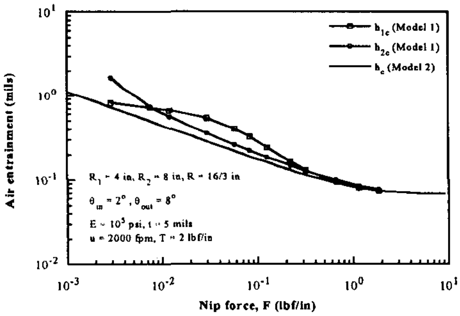

Fig. 19. Effects of nip force on air entrainment

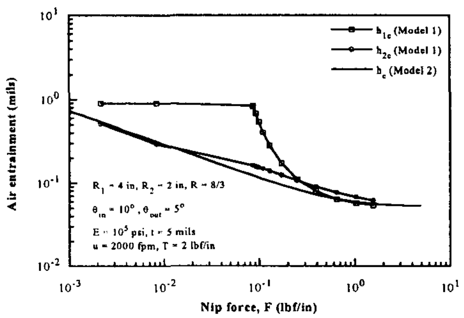

Fig. 20. Effects of nip force on air entrainment

4. CONCLUSION

The effects of a nip roller on ballooning phenomenon and the air entrainment in a winding roll have been analyzed numerically. The effects of a variety of design parameters and operating conditions have been examined. The following conclusions were obtained from this computational study:

1. There is a critical value of nip force above which ballooning occurs.
2. Once ballooning has occurred, its shape is not strongly affected by the nip force.
3. Ballooning does not occur when the incoming wrap angle is small.
4. The maximum balloon height is nearly proportional to the incoming wrap angle for a perfectly flexible web.
5. The balloon height is strongly affected by the incoming wrap angle, web tension, and web speed, but not by the bending stiffness of the web in typical applications.
6. When the nip force is smaller than the critical value for ballooning, the amount of air entrainment is nearly independent of the nip force.
7. The effect of the nip force on the air entrainment is dramatic near the critical nip force.
8. When the nip force is very large, the amounts of the air entrainment on the two sides of the web approach an asymptotic value. The asymptotic value can be predicted using Eq. (14) with

$R=2R_e=2R_1R_2/(R_1+R_2)$. This conclusion holds even when $R_1 \neq R_2$.

9. A nip roller with smaller radius is more effective than a large one.

It is recommended to extend this study by including the effects of side leakage from the ballooned incoming web, elastic deformation of the rolls, and asperity contact at the nip.

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