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Instability and Self-Sustained Oscillation of the Flow between Three-Dimensionally Cross-corrugated Plates

Lee, Seung Youp* and Choi, Young Don**

3차원 교차 주름판 내 유동의 불안정성 및 자활 진동

이승엽*·최영돈**

***Key Words** : Flow Instability(유동불안정성), Self-sustained Oscillation(자활진동), Plate Heat Exchanger(판형열교환기)*

Abstract

Energy dissipations in a general PHE flow are the compounded effects of the piled corrugate geometries and its wall pressure and temperature distributions. In addition, although the exchangers are substantial pieces of engineering equipment, they are composed of a very large number of nominally identical and small geometrical elements. In the present numerical study, the three-dimensionally complicated energy dissipation fields and those wall-shape-induced flow destabilization are investigated in the cross-corrugated passages, which result in high energy transports with comparatively low pressure drop. We revealed the critical conditions as $Re = 157.3$ for the wall-shape-induced flow destabilization in a general PHE element by initial value method, or shooting method, and compare its value to that of analytical solution of plane Poiseille flow, two-dimensional grooved flow and so on. We also observed the detailed variation of flow field and energy transportation with changes in time and flow variables such as Reynolds number. Lastly, we considered the flow natural frequency, or Strouhal number, with variation of hydrodynamic conditions for the best use of active control, such as forced mass flow rate pulsative flow, to enhance energy transportation.

1. INTRODUCTION

Plate heat exchangers have been the subject of increasing research due to the attractive possibility of improving the performance of energy transport, thus reducing volume and cost, by a relatively simple rational re-design of the basic heat transfer elements. In addition, the corrugated geometry of PHE connotes the positive possibility of the enhancement of energy transport through wall-shape induced flow destabilization.

However, the study on the unsteadiness and its application to the energy transport enhancement of PHE has not been performed yet due to the difficulty in the analysis of complicated PHE channel flow field and the previous studies on the flow instability and its affirmative applications are almost restricted within simple flow conditions, such as two-dimensional grooved or furrowed geometries,[1][2][3] axisymmetric wavy channel flows[4] or three-dimensional but geometirically symmetric condition[5]. The practical flow and heat transfer boundary conditions of high performance heat exchanger such as PHE, however, are not so simple but three-dimensionally multifarious.

In a plane Poiseille flow, the critical Reynolds number for instability was revealed theoretically as 5772 by the viscous instability analysis and the most unstable modes are two-dimensional Tollmien-Schlichting waves. It means that these traveling disturbance modes naturally decay with time for $Re < Re_c$ and so the flow through parallel flat plates has its inherent limitation for the enhancement of energy transport in general working condition. Wall boundary shapes, however, critically influence the time variance of the flow field as well as space variance. Especially, grooved geometry advances drastically the critical working point for the destabilization and grows flow fluctuation. Starting interest in destabilized flows grew from the study on resonant heat transfer enhancement in two-dimensionally grooved channels.[1][6] These studies revealed the resonant excitation of the least stable Tollmien-Schilichting modes by actively modulating the flow rate at frequencies close to the natural frequency of these modes, even at moderately low Reynolds number. In addition, Greiner et al.[2] insisted that energy transport enhancement by the passive shear destabilization using the grooved channel geometry is more reliable in practical systems. By the way, in either active or passive systems for the enhancement of the energy transportation and performance in heat exchanger, it is very important to disclose the geometry-induced destabilization and more effective operating conditions.

In the present study, we will investigate the instability of the mass-flowrate-steadily forced flow in the cross-corrugated channel

* Korea Univ., Graduate School, lsy@korea.ac.kr
** Korea Univ., Mechanical Eng., ydchoi@korea.ac.kr

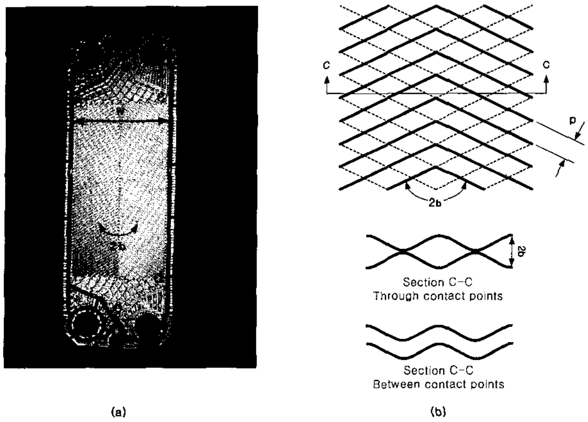

Fig. 1 The geometrical features of chevron plate: (a) typical chevron plate, (b) stack arrangements and sections through chevron troughs.

element, which represents the geometry and hydrodynamic characteristics of general PHE, and its influence on the enhancement of energy transport. For this purpose, we will reveal first the critical conditions for the wall-shape-induced flow destabilization of three dimensionally cross-corrugated channel geometry by initial value method, or shooting method, and compare its value to that of analytical solution of plane Poiseille flow. And then we will observe the detailed variation of flow field and energy transportation with changes in time and flow variables such as Reynolds number. Lastly, we consider the natural frequency, or Strouhal number of flow with variation of hydrodynamic conditions for the best use of active control, such as the mass-flowrate-pulsative flow, to enhance energy transportation.

2. NUMERICAL METHOD

2.1 Geometry and Computational Grid of the Cross-Corrugated PHE Element

Of the many different types of plate corrugations available[7], the most commonly used chevron plate is illustrated in Fig. 1. Real exchangers are composed of a large number of flow passages. However, due to the modular nature of the heat exchange matrix, it is possible to think of it as composed of a very large number ($\sim 10^5$) of nominally identical, geometrically small elements repeating themselves periodically.

For the crossed-corrugated design, the element of cross-corrugated PHE matrix shown in Fig. 2 can be identified by the following parameters; corrugation inclination angle, β, its pitch, P, external height, H, and wall thickness, s, in Fig.3. Other relevant geometrical quantities can be correlated from these as summarized in Table 1. These correlations are based on the simplifying assumption of perfectly sinusoidal corrugation of the plate center line.

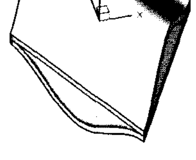

Fig. 2 Computational domain and coordinate system.

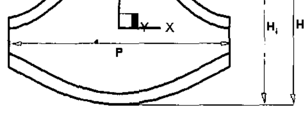

Fig. 3 Cross section normal to the upper plane corrugation.

Table 1 Geometry parameters.

Parameters	Value
Plate thickness, s	0.5mm
Corrugation angle, 2β	126deg.
Pitch, P	10.75mm
Height, H	3.5mm
Internal height, $H_i = H - s$	3.0mm
Hydraulic diameter, $D_h = 4P^2H_i/A_w sin(2\beta)$	5.125mm
P/H	3.07
P/H_i	3.58

The ratio of corrugation pitch P and corrugation depth H_i to hydraulic diameter D_h, and the corrugation inclination angle β parameterize the small-scale channel shape; a unitary cell of the PHE channel. In addition, seven representative points were selected to investigate the unsteadiness of flow field. The computational domain for flow field is fulfilled with tetrahedral control volumes to avoid singularities caused by sine-wave corrugations of a three-dimensional array of hexahedral control volumes. Fig. 4 shows the computational grids for a PHE element. The number of grid cells is more than 170,000 and that of grid faces is more than 400,000. In order to calculate a spatially periodic flow field with a specified mass flow rate derivative, we first create two pairs of inlet-outlet grids with translationally periodic boundaries that are parallel to each other and equal in size.

2.2 Boundary and Initial Condition

In PHE flow, the geometry varies in a repeating manner along the direction of the flow, leading to a periodic fully-developed flow regime in which the flow pattern repeats in successive cycles. As like examples of streamwise-periodic flows include fully-developed flow in pipes and ducts, these periodic conditions are achieved after a sufficient entrance length, which depends on the flow Reynolds number and geometric configuration. However, Stasiek et al.[8] revealed the entrance and plate end effects are limited and have died out before the fifth PHE element layer from entrance and wall boundaries and thus the most flow condition in PHE elements can be legitimately considered as representative of fully developed

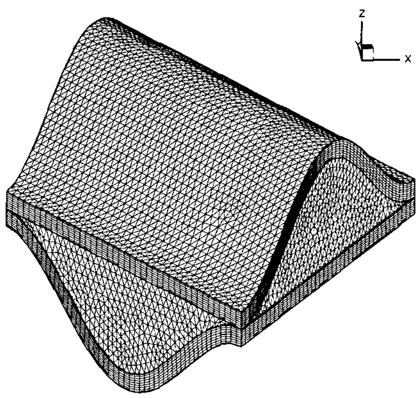

Fig. 4 Computational grid for a cross-corrugated PHE element.

conditions.

The problem is clarified for a initial value problem in which numerical solution is converged first and then the disturbance of 5% more mass flow rate is inflicted on the fully-developed steady state solution at $t=0$. Therefore, the mean perturbation velocity at $t=0$, $\overline{u}'(0)$, is approximately $0.05\overline{U}(0)$.

3. RESULTS AND DISCUSSION

In the linearly stable condition, $Re < Re_c$, although the flow may have the oscillation frequency(internal wave frequency) of the corresponding unstable mode, no matter how strong disturbance may be, it should decay with time and so it is very inefficient condition for the energy transport such as heat transfer in heat exchanger. Therefore, it is important to determine the critical Reynolds number, $Re_c = Re_L$, for the onset of instability and to quantify the time-variation of disturbances.

As the instantaneous growth rate of disturbance kinetic energy, $\frac{1}{E(t)}\frac{dE(t)}{dt}$, is independent of the disturbance amplitude, if a disturbance of finite amplitude has a certain instantaneous growth rate, so does the corresponding infinitesimal disturbance with a theory of linear growth mechanism.[9] It will be also revealed in present paragraph that the numerical solution of subcritical transition in PHE channel element follows the linear growth mechanism.

The disturbance kinetic energy, $E(t)$, can be rewritten as the function of $e^{2G_w t}$. The growing rate, G_w, can be inferred more precisely by curve fitting method of logarithmic scale of it defined linearly as shown in eq.(1).

$$y = ln[E(t)/E(0)] = f(t;\ A_0, G_w) \equiv A_0 + 2G_w t \qquad (1)$$

where, A_0 is a parameter.

Figure 5 shows the time-variation of the disturbance kinetic energy rate and the decreasing rates in low Reynolds conditions(subcritical conditions).

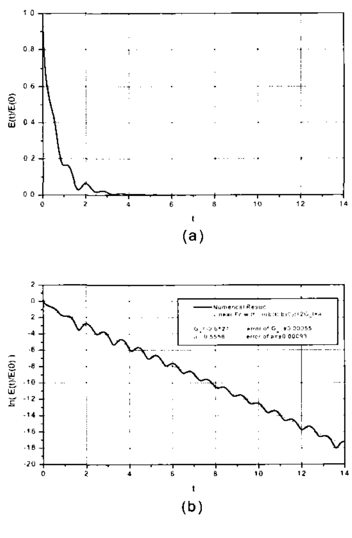

Fig. 5 Time-variation of the reference-points-sum kinetic energy rate of a disturbance in $Re = 95.24$; (a) linear scale (b) logarithmic scale.

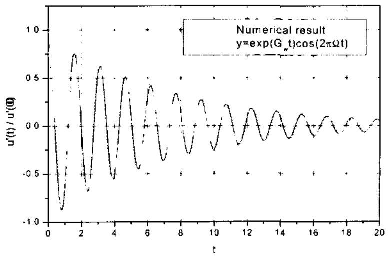

Fig. 6 Time-variation of the perturbation velocity of point 0 in $Re = 142.86$ and its analytic solution of the Orr-Sommerfeld equation.

In $Re = 47.62$, $dE(t)/dt < 0$ for all $t > 0$, and so it can be said to be monotonically stable in the applied perturbation condition. As Reynolds number increasing, the region of $dE(t)/dt > 0$ appeared in $Re = 71.43$, and so the monotonically stable Reynolds number, Re_E, in 5% perturbation condition is approximately $O(60)$. It is larger than the value of the plane Poiseille flow, 49.6, and that of the plane Couette flow, 20.7, but smaller than that of the circular pipe Poiseille flow, 81.49. In this case, there exists the indentation of disturbance kinetic energy rate and so it is revealed that the internal wave frequency can be determined over $Re \sim O(70)$.

Reynolds number increasing, the indentation of $E(t)/E(0)$ becomes growing because the disturbance kinetic energy dissipation relative to the flow inertia decreases. In addition, $ln(E(t)/E(0))$ of lower Reynolds number are closely following the linear equation and the errors of G_w are not over ±0.0005 in the cases of $Re < 166.67$. It means that the disturbance kinetic energy decays with time and the flow field becomes steady state in low Reynolds number.

Figure 6 shows the time-variation of the perturbation velocity of the center point of control volume in Re=142.86, and its analytic solution of the Orr-Sommerfeld equation. The perturbation velocity decreased with time to the steady state and so it is the linearly stable condition, $Re < Re_c$. The numerical result for the fluctuating velocity follows precisely the solution of simplified Orr-Sommerfeld equation, eq.(2) in the error of 0.4%.

$$\frac{u'(t)}{u'(0)} = e^{G_s t} cos(2\pi\Omega t) \qquad (2)$$

This result shows that the time-variation of fluctuating energy suggests precisely the disassembled vector components of the wave decrease of fluctuating velocity and so the difficulty of data aquisition and management for the wave growth and reduction can be overcome by the macroscopically synthesized method.

Figure 7 shows the variation of the wave decreasing rate with respect to Reynolds number. It shows the dependency of the decreasing rate on Reynolds number and the extrapolation of the $(Re, -G_w)$ data points intimates destabilization, $-G_w = 0$, with the critical Reynolds number for the onset of unsteadiness, Re_c, of approximately 157.3. It means that, for subcritical Reynolds number(below $Re \sim 157.3$), the perturbation energy dissipates and the flow field result in steady state as $t \to \infty$ and, for

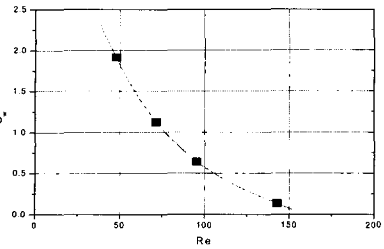

Fig. 7 The wave decreasing rate with respect to Reynolds number; $Re_c = 157.3$.

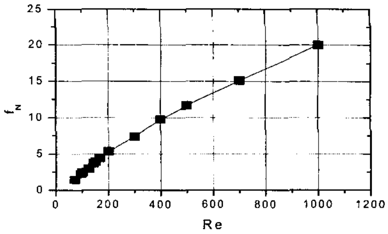

Fig. 9 The variation of flow natural frequency with respect to Reynolds number.

supercritical Reynolds number, the stable flow is unsteady and becomes time-periodic corresponding to a nonlinear self-sustained flow oscillation. The critical Reynolds number for the PHE element is 2.7% and 30% of plane Poiseille flow, 5772, and Blasius flow, 520, respectively. Especially, it is about 16% of 2-dimensionally grooved channel revealed by N. K. Ghaddar et al.[9] It infers that the 3-dimensionally cross-corrugated geometry enhances the self-sustained oscillation through wall-shape-induced flow destabilization in similar hydrodynamic condition and promotes energy transport more effectively in result.

To clarify the flow oscillation in supercritical region, the time-periodic variation of the three-dimensional streamlines of the self-sustained flow oscillation during one cycle in $Re = 500$ was shown in Fig. 8.

Figure 9 shows the variation of flow natural frequency with respect to Reynolds number by FFT method and it reveals that flow in a PHE element has an oscillatory pattern which is almost linearly-dependent on the Reynolds number and the vortex shedding occurs in the range $10^2 < Re < 10^3$, with an average Strouhal number, $\Omega \approx 0.6$.

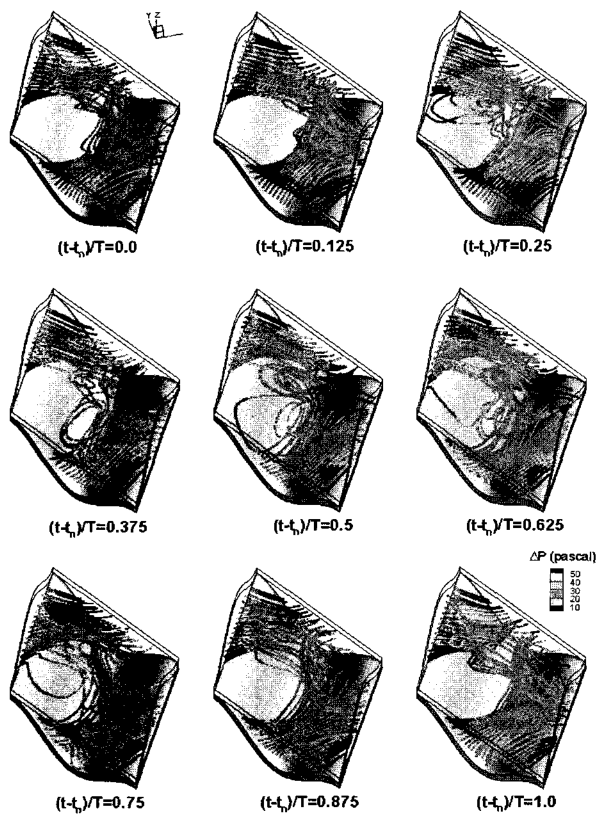

Fig. 8 The time-periodic variation of streamlines of the self-sustained flow oscillation during one cycle in $Re = 500$.

CONCLUSION

The wall-shape-induced flow destabilization and its time-periodic oscillation in the three-dimensionally complicated energy dissipation fields of the cross-corrugated passages occur in much lower Reynolds number, $Re \sim 157.3$, corresponding to other general conditions; 2.7% of plane Poiseille flow, 5772, 30% of Blasius flow, 520, 16% of 2-dimensionally grooved channel. We also clarified the time-periodic variation of flow field and energy transportation in super-critical Reynolds number in PHE. Lastly, we revealed that the flow natural frequency varies linearly with the Reynolds number in the range $10^2 < Re < 10^3$ and the flow oscillation occurs with an average Strouhal number, $\Omega \approx 0.6$.

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