Proceedings of
The Second National Congress on Fluids Engineering
August 22-24, 2002, Muju, Korea

A time-domain analysis for a nonlinear free-surface problem

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시간영역에서의 비선형 자유표면파문제에 대한 수치해석

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***Key words :** Nonlinear Free surface, Potential flow, Time-domain, water waves*

Abstract

The free surface flow problem has been one of the most interesting and challenging topic in the area of the naval ship hydrodynamics and ocean engineering field. The problem has been treated mainly in the scope of the potential theory and its governing equation is well known Laplace equation. But in general, the exact solution to the problem is very difficult to obtain because of the nonlinearlity of the free surface boundary condition. Thus the linearized free surface problem has been treated often in the past. But as the computational power increases, there is a growing trend to solve the fully nonlinear free surface problem numerically. In the present study, a time-dependent finite element method is developed to solve the problem. The initial-boundary problem is formulated and replaced by an equivalent variational formulation. Specifically, the computations are made for a highly nonlinear flow phenomena behind a transom stern ship and a vertical strut piercing the free surface.

1. Introduction

This paper describes a finite element method applied to a nonlinear free surface flow problem for a body moving in three dimensions. The exact nonlinear free-surface flow problem formulated by an initial/boundary value problem is replaced by an equivalent weak formulation. The numerical model is treated to simulate the towing tank experimental conditions. The model is assumed to be vertical wall-sided and stretched from the free surface to the bottom for simplicity. The similar problem was considered earlier by Bai, et.al.[1,2] where some irregularities were observed in the downstream waves and a transom stern ship geometry could not be treated. In the present paper, specifically, three improvements are made from the earlier work. The first improvement is the introduction of the 5-point Chebyshev filtering scheme(Loguet-Higgins & Cokelet[3]) which eliminates the irregular and saw-toothed waves in the downstream. The second improvement is that now we can treat a transom stern ship geometry. The third is the introduction of a new boundary condition to simulate a dry bottom behind a transom stern ship which is stretched from the free surface to the bottom at a high Froude number.

The first model has a wedge-shaped bow and a parallel body cut off at the stern, thus a transom stern ship model. The second model is a vertical strut piercing free surface with a normal angle of attack. Computations for the model are made to investigate the generation of a dry bottom behind the model which is highly nonlinear phenomena. Computations at higher Froude numbers show the flow behind the model detaches and results in a dry bottom. The computed results agree well with the preliminary experimental observation for a dry-bottom, which is reported in Bai,et al[4].

The present method can treat arbitrary water-depth and practical ship geometries. Thus the present method does not restrict to shallow water problem or special ship model geometry stretched from the free surface to the bottom, even though the computation is made for a simple model in shallow water in the present paper. The numerical simulation of the model can be applied to the local flow behind a sail of a submarine in cruise, a sloshing problem in LNG tankers, and a dam breaking problem.

2. Mathematical Formulation

We used the Cartesian coordinate in this paper. *Oxyz* is the coordinate system with *Oz* opposing the direction of gravity and $z = 0$ coincides the undisturbed free surface. The body moves to the negative x-direction with velocity U. The formulation is given in an inertial coordinate system. However, in the numerical procedures, the computing box is moving with the moving body. We assume that the fluid is inviscid, incompressible and its motion is irrotational. So the velocity potential exists and is defined as

$$u(\vec{x}, t) = \nabla \phi(\vec{x}, t) \qquad (1)$$

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where $\vec{x} = (x, y, z)$ and ϕ is the velocity potential. From the continuity condition we obtain the Laplace equation

$$\nabla^2\phi(\vec{x}, t) = 0 \quad \textit{in fluid domain D.} \tag{2}$$

The boundary condition on the body boundary surface, S_o, is

$$\phi_n = -Un_x . \tag{3}$$

Where the vector, $\vec{n} = (n_x, n_y, n_z)$, denotes the outward unit normal vector on the boundaries. The conditions on the free surface, $z = \zeta(x, y, t)$ can be given by the kinematic and dynamic boundary conditions as follows.

$$\zeta_t = -U\zeta_x + \frac{1}{n_z}\phi_n, \tag{4}$$

$$\phi_t = -U\phi_x - \frac{1}{2}|\nabla\phi|^2 - g\zeta - \frac{p}{\rho} . \tag{5}$$

Where g and ρ denote the gravitational acceleration and the density of fluid and the pressure, $p = p(x, y, t)$ is taken zero unless a non-zero pressure distribution is specified. The fluid motion is assumed to be at rest initially, therefore the initial condition may be given as

$$\phi = \phi_t = 0 \quad at\ t = 0 \tag{6}$$

and the radiation condition is given as

$$\phi \to 0 \quad as \quad x^2 + y^2 \to \infty \tag{7}$$

The depth of water is h, and the tank width, $y = \pm\mathrm{B}h$. The wall boundary conditions are given as follows.

$$\phi_n = 0 \quad on \quad z = -h \tag{8}$$

$$\phi_n = 0 \quad on \quad y = \pm Bh \tag{9}$$

3. Variational Formulation

We introduce a variational formulation which is equivalent to the above problem. First we define the variational functional, J and the Lagrangian L (Miles[5],Luke[6])as

$$J = \int_0^{t^*} L dt \tag{10}$$

$$L = \iint_{\bar{S}_F} \phi\zeta_t dS - \frac{1}{2}\iint_{\bar{S}_F} \zeta^2 dS - \frac{1}{2}\iiint_D |\nabla\phi|^2 dV \tag{11}$$

where $\bar{S}_F$ is the projection of S_F on Oxy plane and t^* is the final time. Taking the variations on J first with respect to ζ, we can obtain δJ_ζ as

$$\begin{aligned} \delta J_\zeta &= \int_0^{t^*} dt\left[\iint_{\bar{S}_F}\left(\phi\delta\zeta_t - \zeta\delta\zeta - \frac{1}{2}|\nabla\phi|^2\delta\zeta\right)dS\right] \\ &= \iint_{\bar{S}_F} [\phi\delta\zeta]_{t=t^*} - [\phi\delta\zeta]_{t=0} dS \\ &- \int_0^{t^*} dt\left[\iint_{\bar{S}_F}\left(\phi_t + \frac{1}{2}|\nabla\phi|^2 + \zeta\right)\delta\zeta dS\right]. \end{aligned} \tag{12}$$

Next the variations on J with respect to ϕ, δJ_ϕ can be obtained as

$$\begin{aligned} \delta J_\phi &= \int_0^{t^*} dt\left[\iint_{\bar{S}_F} \zeta_t\delta\phi dS - \iiint_D \nabla\phi\cdot\nabla\delta\phi dV\right] \\ &= \int_0^{t^*} dt\left[\iint_{\bar{S}_F}\left(\zeta_t - \frac{1}{n_z}\phi_n\right)\delta\phi\ dS + \iiint_D \nabla^2\phi\ \delta\phi dV\right]. \end{aligned} \tag{13}$$

Here $\delta J = \delta J_\zeta + \delta J_\phi$ and $n_z = 1/\sqrt{1 + \zeta_x{}^2 + \zeta_y{}^2}$.

Equation (12) means that the stationary condition on J for the variation with respect to ζ recovers the dynamic free surface condition in each time and that the wave elevation at $t=0$, t^* should be specified as the constraints. Eq(13) shows that the stationary condition on J for the variation of ϕ recovers the kinematic condition on S_F and the governing equation.

4. Finite-Element Discretization

As the first step in the numerical procedure, the fluid domain is to discretize into a number of finite elements. In this study, the finite elements are generated such that projections of x and y coordinates on the horizontal plane are fixed but the other coordinate, i.e. the z-axis, is allowed to move vertically in time. This restriction makes the regridding and computation considerably simple. But, it is not always necessary to impose this restriction in general. The trial basis is denoted by $\{N_i\}_{i=1,\dots,N}$ and ζ is approximated by the span of the restrictions of $\{N_i\}_{i=1,\dots,N}$ on S_F which is also continuous and piecewise differentiable on $\bar{S}_F$.

$$\phi(x, y, z, t) = \phi_i(t)N_i(x, y, z; \zeta) \tag{14}$$

$$\zeta(x, y, t) = \zeta_k(t)M_k(x, y) \tag{15}$$

where

$$M_k(x, y) = N_{i_k}(x, y, z; \zeta)\big|_{z=\zeta}, \quad k = 1, \dots, N_F \tag{16}$$

and N_F is the number of nodal points on S_F and i_k is the nodal number of the basis function N_i of which the node coincides with that of the free surface node k. Summation conventions for the repeated indices are used here. It should be noted that the basis function, $\{N_i\}_{i=1,\dots,N}$, is dependent on the free surface shape, $z = \zeta(x, y, t)$, but its restriction on S_F is the function of *(x,y)* and independent of ζ. The special property of $\{M_k\}_{k=1,\dots,N_F}$ is resulted from the finite-element subdivision employed here. Once the trial function is approximated by using the above basis function, the Lagrangian, L, for these trial solutions are obtained as

$$L = \phi_{i_k} T_{kj} \dot{\zeta}_j - \frac{1}{2}\phi_i K_{ij} \phi_j - \frac{1}{2}\zeta_k P_{kj} \zeta_j \tag{17}$$

$$T_{kj} = \iint_{\bar{S}_F} M_k M_j dS \tag{18a}$$

$$P_{kj} = \iint_{\bar{S}_F} M_k M_j dS \tag{18b}$$

$$K_{ij} = \iiint_D \nabla N_i \cdot \nabla N_j dV \tag{18c}$$

The tensors, K_{kj}, P_{kj} are the kinetic and potential energy tensor and T_{kj} is a tensors obtained from the free surface integral, which can be interpreted as a tensor related to the transfer rate between these two energies. It is of interest to note that in Eq(18), $T_{kj} = P_{kj}$. The stationary condition on $J = \int L dt$ gives the following Euler-Lagrange equation.

$$T_{kj}\dot{\zeta}_j = K_{i_k j}\phi_j, \tag{19}$$

$$T_{kj}\dot{\phi}_{i_j} = -\frac{1}{2}\phi_i \frac{\partial K_{ij}}{\partial \zeta_k}\phi_j - P_{kj}\zeta_j, \text{for } k=1,\dots,N_F, \tag{20}$$

$$K_{ij}\phi_j = 0, i \neq i_k. \tag{21}$$

5. Dry Bottom Condition

Here we will describe how to treat the dry-bottom condition in the numerical computations. The presence of a dry bottom causes a difficulty in the numerical scheme, especially because a strong velocity gradient occurs across the water-front line appearing in front of the dry bottom. In the dry bottom we assumed the velocity to be zero. In order to overcome the difficulty due to the change of computational domain, we adopted a 'thresh-hold thickness" method where a thin water layer is kept in the dry zone. It is a simple scheme often used to describe the bore or run-up on beaches. The dry bottom condition is given as

$$\frac{D\phi}{Dt} = 0 \text{ on the dry bottom} \tag{22}$$

$$\zeta = -h + \varepsilon \tag{23}$$

This means that the pressure on the dry bottom does not change. The value of the thresh-hold was chosen such that the Jacobian is not zero.

6. Results and Discussion

The computational model with a transom stern is shown in Fig 1. This model has also a vertical wall-sided wedge-shaped bow and a parallel middle body extending from the free surface to the bottom of the tank. The length of the wedge, the length of the parallel middle body, and the beam are nondimensionalized by the water depth, *h* and denoted by L, L_m, and *2b*, respectively. The tank geometry is the same as the first model.

Transom stern model	: *L = 30, Lm = 25, B=30*
Computation Domain	: *x = (-60,60), y=(-30,30)*
Mesh Sizes	: $\Delta x = 0.5, \Delta y = 0.5, \Delta t = 0.01$

For the transom stern ship model many cases are computed for various values of Froude numbers. Table 1 shows the computed cases for the transom stern models. In the following we present only two computed results, Fig 2 and Fig 3 show the wave contour and wave profiles at $F_n = 1.8$ for Case 1 and 2 shown in Table 1.

Table 1. Summary of the computed cases for the transom stern ship models. (All dimensions are non-dimensionalized with a water depth .)

Case	*2b/h*	*2B/h*	*L/h*	*Lm/h*
(1)	*10*	*60*	*30*	*25*
(2)	*15*	*60*	*30*	*25*

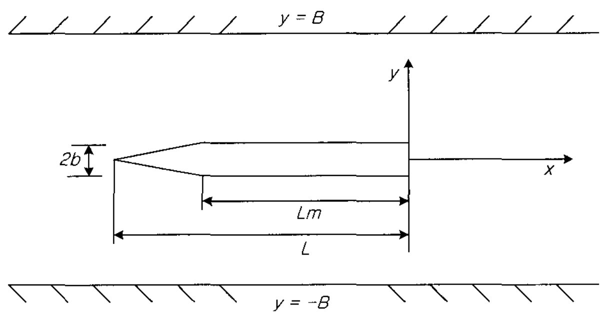

Figure 1. Sketch of transom stern ship model and tank in horizontal plane view. The z-axis is against the gravity.

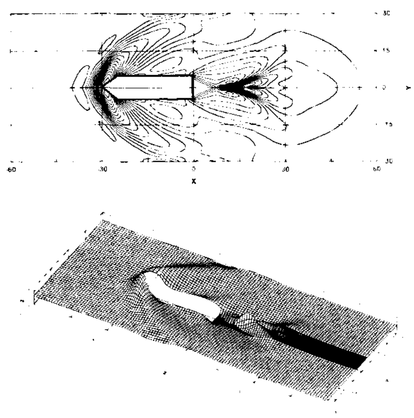

Figure 2. Contour plot and wave profile at F_h=1.8 in the case of (1) in table 2

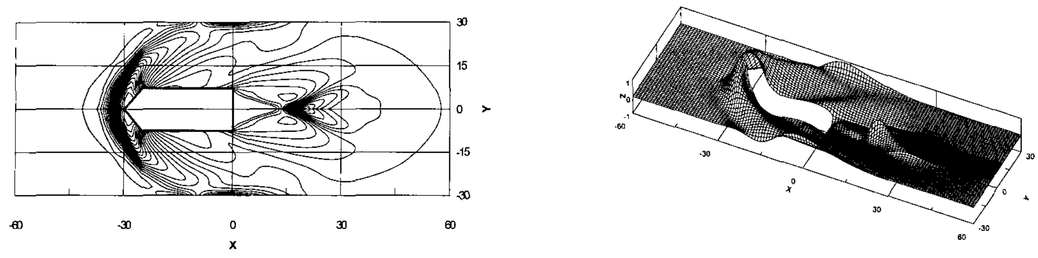

Figure 3. Contour plot and wave profile at Fn =1.8 in the case of (2) in table 2

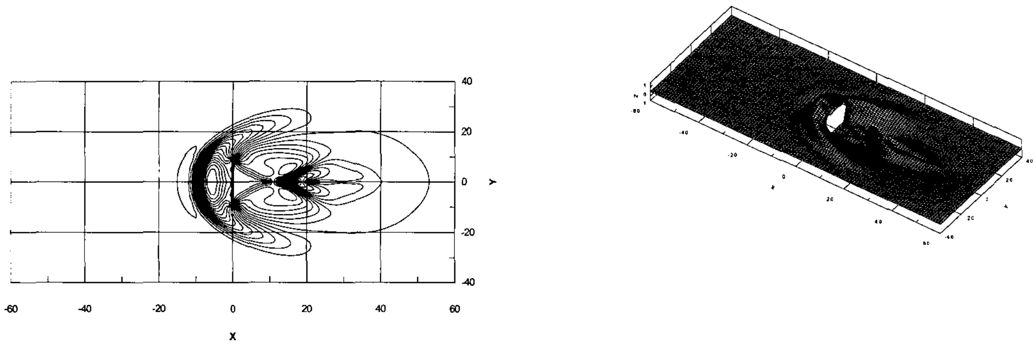

Figure 4. Wave profile and contour plot for vertical strut at $t^* = 20, F_h = 1.6$.Strut Length = 20, width of tank = 80

Fig 4 show the wave profile and contour plot for vertical strut at $F_h = 1.6$ with the normal angle of attack. The figure shows approximately triangular shapes of dry bottom generated behind the body (transom stern and vertical strut models).

As a concluding remark, based on the numerical tests, the present numerical method can be used as an efficient method to treat nonlinear free surface flow problems. Even though the sample computations are made for rather special and simple cases, i.e., a ship in shallow water tank, the present method can accommodate arbitrary water depth (i.e., tank depth) , tank width, and ship geometry even for a transom stern model. This method can also be applied to the sloshing problem, the local flow around the sail of a submarine, and the dam breaking problem.

Acknowledgments

This work was supported by a grant No.(R01-2000-00321) from Korea Science & Engineering Foundation, Research Institute of Marine Systems Engineering, College of Engineering, Seoul National University and Brain Korea 21. Bai owed partly to Korea Research Foundation in the preliminary stage of the present work.

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