Proceedings of
The Second National Congress on Fluids Engineering
August 22-24, 2002, Muju, Korea

Application of Directional Wavelet to Ocean Wave Image Analysis

S. H. Kwon*, H. S. Lee**, J. S. Park*** and M. K. Ha****

방향 웨이브렛을 적용한 해양파 이미지 분석

권순홍*, 이형석**, 박준수***, 하문근****

Key words : *Directional Wavelet, Wave Directionality, Continuous Wavelet Transform, Wave Image, Morlet Wavelet*

Abstract

This paper presents the results of a study investigating methods of interpretation of wave directionality based on wavelet transforms. Two-dimensional discrete wavelet was used for the analysis. The proposed scheme utilizes a single frame of ocean waves to detect their directionality. This fact is striking considering the fact that traditional methods require long time histories of ocean wave elevation measured at various locations. The developed schemes were applied to the data generated from numerical simulations and video images to test the efficiency of the proposed scheme in detecting the directionality of ocean waves.

1. Introduction

This study proposes a new algorithm that utilizes only a single picture of wave fields (Kwon 2000a, 2000b). The algorithm saves much time and effort. The wavelet transform is a powerful tool for solving variety of physical problems. This study tested two-dimensional directional wavelet transforms on numerically simulated data and video images. A directional wavelet is one of the continuous wavelet transform (CWT). A two-dimensional directional wavelet can be generated by introducing rotation to the CWT. A two-dimensional directional wavelet transforms can detect the direction of small amplitude waves even when the small-amplitude waves are traveling with large amplitude waves. The two-dimensional directional wavelet transforms were found to be a very efficient tool in selecting wave directionality. However, there are limitations in the proposed algorithm. The proposed scheme can provide the main direction of waves but not the exact energy spreading functions. Since the proposed scheme utilizes only one shot of wave fields, 180° ambiguity exists in the direction of the propagation. Other than these limitations, the proposed scheme can be a powerful tool for detecting directionality of ocean waves.

* 부산대학교 조선해양공학과, shkwon@pusan.ac.kr
** 현대중공업, hssy@hhi.co.kr
*** 부산대학교 대학원 조선해양공학과, jsocean@pusan.ac.kr
**** 삼성중공업, mkha@samsung.co.kr

2. The continuous wavelet transform

First, the one-dimensional case of the continuous wavelet transforms is considered.

A CWT of a signal $s(t)$ is given by

$$W_s(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} \psi^* \left(\frac{t-b}{a} \right) s(t) dt \qquad (1)$$

where $\psi(\cdot)$ is the wavelet function, a is the scaling parameter, and b is the translation parameter. * represents a complex conjugate. It can be stated that the CWT is the sum over all time of real signal $s(t)$ multiplied by the scaled, shifted wavelet function. The parameters a and b vary continuously.

The wavelet function used in the present analysis is the Morlet wavelet. The formulation of this wavelet function is shown below

$$\psi(t) = \pi^{-\frac{1}{4}} (e^{-i\omega_0 t} - e^{-\omega_0^2/2}) e^{-\frac{t^2}{2}} \qquad (2)$$

The second term is added to satisfy the so-called admissibility condition. However, for large ω_0 ($\omega_0 \geq 5.5$), the correction term is numerically negligible. Thus, the complex valued Morlet wavelet can be approximated by the following

$$\psi(t) \approx \pi^{-\frac{1}{4}} e^{-i\omega_0 t} e^{-\frac{t^2}{2}} \quad (3)$$

3. The directional wavelet

Rotation is introduced to detect directionality, as shown below

$$W_s(a,\theta,\mathrm{b}) = a^{-1} \int \psi^*(a^{-1} r_{-\theta}(\mathrm{x}-\mathrm{b})) s(\mathrm{x}) d\mathrm{x} \quad (4)$$

where $r_{-\theta}(\mathrm{x}) = (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$ at $0 \le \theta < 2\pi$.

By introducing the rotation parameter, the wavelet function now has translation, rotation, and dilation. It is convenient to perform the two-dimensional directional wavelet transforms by using Parseval's theorem because the Fourier transform of $\psi(\mathrm{x})$ has compact support in the k domain.

$$W_s(a,\theta,\mathrm{b}) = a \int e^{i\mathrm{b}\cdot\mathrm{k}} \hat{\psi}^*(a r_{-\theta}(\mathrm{k})) \hat{s}(\mathrm{k}) d\mathrm{k} \quad (5)$$

where $\hat{\psi}$ and $\hat{s}$ are the Fourier transform of $\psi(\mathrm{x})$ and $s(\mathrm{x})$, respectively.

The Directional wavelet is favored for detecting wave directionality. Examples of directional wavelets are the Morlet wavelet, multi-directional wavelet, and the Cauchy wavelet (Antoine 1996a, 1996b).

The formulation of a two-dimensional Morlet wavelet with rotation is given without a correction term, as follows

$$\psi(\mathbf{x}) = e^{i\mathbf{k}_0\cdot\mathbf{x}} e^{-\frac{1}{2}(\varepsilon^{-1}x^2 + y^2)} \quad (6)$$

where k_0 is a wave vector and $\varepsilon \ge 1$ is an anisotropy parameter.

The expression for $\psi(\mathrm{x})$ adopted in the present study is given by

$$\psi(\mathbf{x}) = e^{ik_0 y} e^{-\frac{1}{2}(\varepsilon^{-1}x^2 + y^2)} \quad (7)$$

Its Fourier transform is

$$\hat{\psi}(\mathrm{k}) = \sqrt{\varepsilon} \exp(-\tfrac{1}{2}(\varepsilon k_x^2 + (k_y - k_0)^2)) \quad (8)$$

4. Dat analysis

4.1 Numerically simulated data

A numerically simulated ocean waves is at first generated from a directional frequency spectrum that is obtained by known wave spectrum multiplied by a spreading function. It is used for an application example of the wavelet transform in this study.

The adopted spreading function was dominant for one direction, but it was spread properly, which was chosen for presenting more dramatically. Contour plot is presented in figure 1(a).

The result of the directional wavelet transforms, after appropriate thresholding, is shown in figure 1(b). The absolute value of the transformed results is represented in the coordinates $(k_0/a, \theta)$ so that the angles can be readily obtained. The analysis was performed with $k_0 = 10$ and $\varepsilon = 10$. The highest density contour appeared in the direction of 45°, which corresponded to the main direction of the simulated waves.

4.2 Video image

For a more substantial applications, data taken from video images were used. The video captured image presented in figure 2(a) has the direction of propagation 90°. The picture was taken in a small water basin with water depth of about 45mm. Figure 2(b) presents the distribution of 2-D wavelet transformed gray level. The directional analysis with $k_0 = 10$ and $\varepsilon = 10$ shows the direction of propagation of the waves is 90°. The next image which also taken in the same water basin features different direction of propagation which can be seen clearly in figure 3(a). The direction of propagation was estimated as about 77°. The result of the analysis is shown in figure 3(b). The results of the analysis seem quite accurate.

The image taken from the river is presented in figure 4(a). The waves shown in the figure have direction of propagation at 90°. The main direction of propagation at 90°could be clearly detected from figure 4(b). Figure 4(b) also reveals the fact that there exist many components of the waves. Oblique waves in the river are shown in figure 5. The directional wavelet transform shows clearly the main direction of the waves. The last example is a case when the video image was taken in the sea. The video image is shown in figure 6(a). The main direction of the waves was estimated from the image as about 34°. This estimation is in good agreement with that of the analysis that is shown in figure 6(b). Taking a close look at the figure 3(b) and figure 6(b), we can immediately notice that there is always dominant contour level at the 90° regardless of the direction

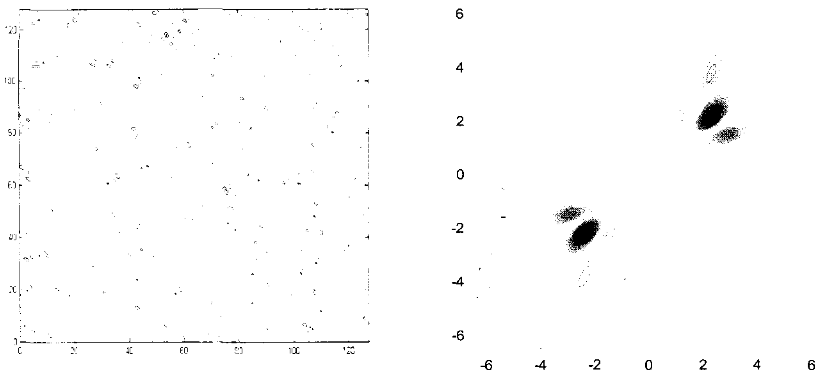

Figure 1. (a) Numerically simulated wave elevation, (b) 2-D directional wavelet transforms result

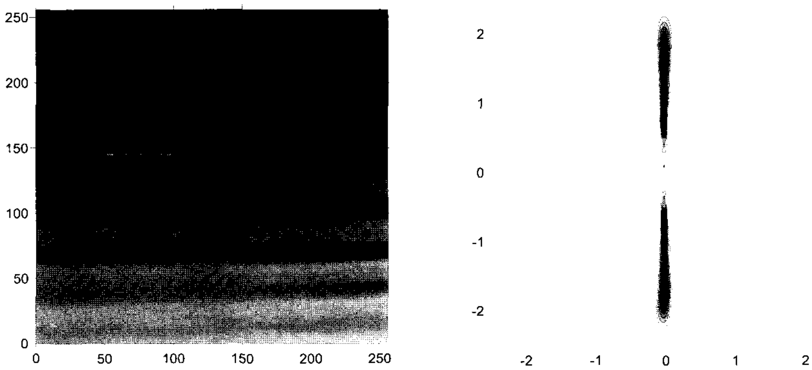

Figure 2. (a) Video image taken at small water basin, (b) 2-D directional wavelet transforms result

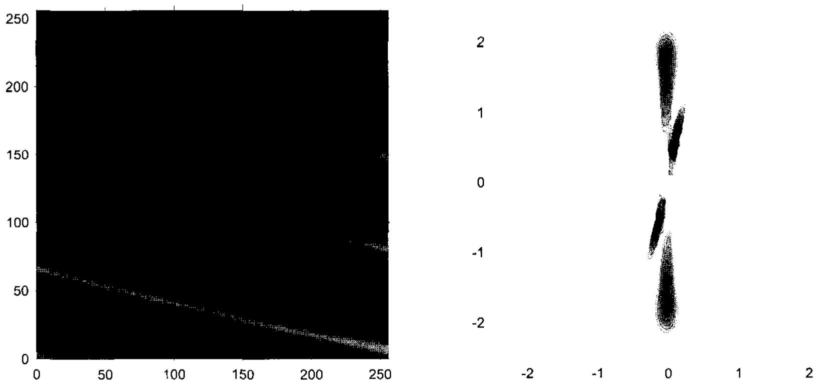

Figure 3. (a) Video image taken at small water basin, (b) 2-D directional wavelet transforms result

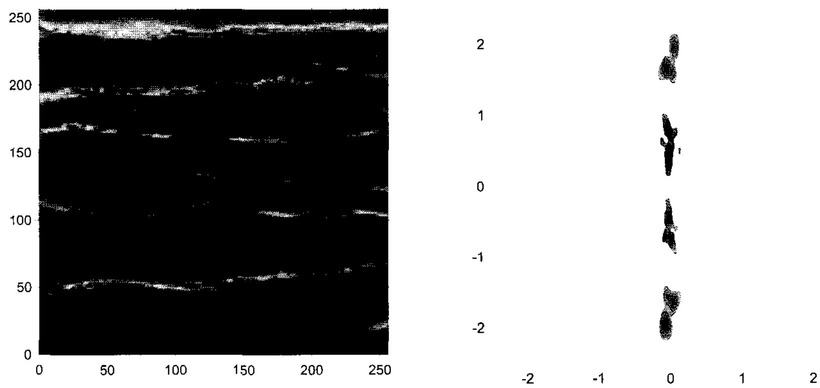

Figure 4. (a) Video image taken at river, (b) 2-D directional wavelet transforms result

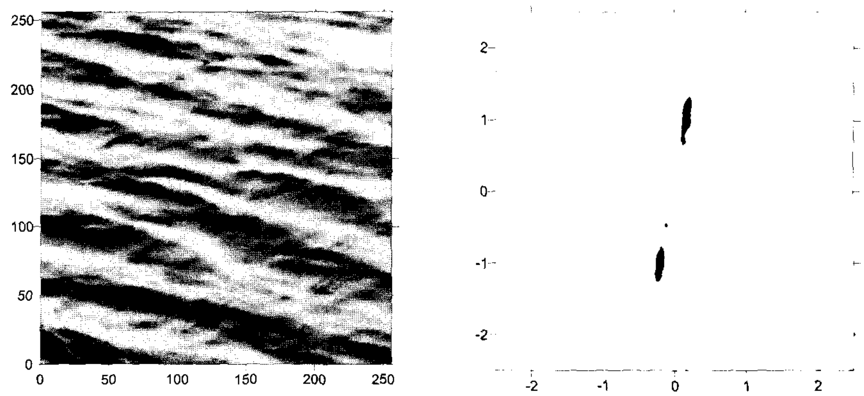

Figure 5. (a) Video image taken at river, (b) 2-D directional wavelet transforms result

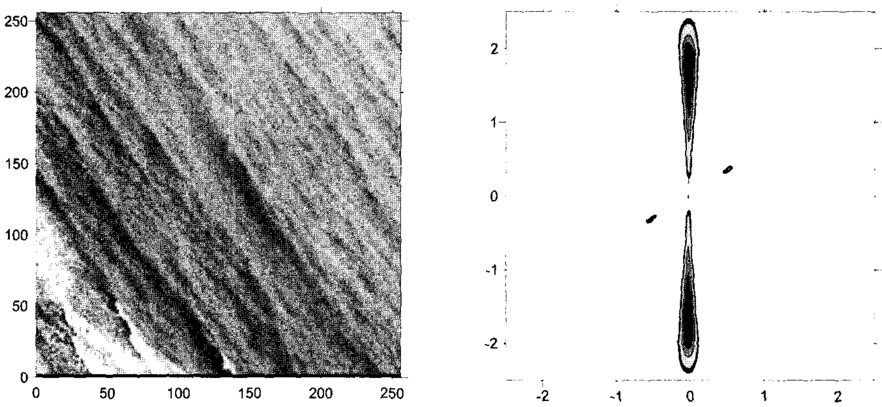

Figure 6. (a) Video image taken at sea, (b) 2-D directional wavelet transforms result

of the viewfinder. The analysis using video image always experiences this problem. It is really hard to avoid.

5. Conclusions

The directionality of waves was investigated by wavelet transforms. The directional wavelet transforms were tested with numerically simulated wave data and video images. The application of the proposed scheme to simulated wave data showed that the directional wavelet transforms can detect the direction of waves. The proposed scheme also worked very well on video images that featured the directions of the propagation of waves. The proposed scheme only utilizes a single frame of wave images, which is a distinct advantage of the scheme. This study demonstrated that two-dimensional directional wavelet transforms can be an efficient tool in the analysis of the directionality of waves when the related parameters are selected appropriately.

References

[1] ANTOINE, J. P. and MURENZI, R., Two-dimensional directional wavelets and the scale-angle representation, Signal Processing, 52, 1996a, pp. 259-281.

[2] ANTOINE, J. P. and MURENZI, R., The continuous wavelet transform, from 1 to 3 dimensions, in : AKANSU, A. N. and SMITH, M. J. T. , eds., Subband and wavelet transforms. Design and applications, 1996b, pp.149-187.

[3] KWON, S. H., LEE, H. S., PARK, J. S., HA, M. K., On wavelet-based algorithm for interpreting ocean wave directionality, Proceedings of fourth Osaka colloquium on seakeeping performance of ships, 2000a, pp. 237-241.

[4] 이형석, 박준수, 권순홍, 하문근, 방향 웨이브렛을 이용한 파동의 방향성 해석, 대한조선학회 추계학술대회 논문집, 2000b, pp. 221-224.