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Relativistic View in Hydrodynamic Waves

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流體波動에의 相對性理論 適用

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Abstract

The **relativistic** theory has not been properly taken up by the marine hydrodynamicists. To take on a relativistic view, we confine ourselves to a simple vector case of **a wave train in spacetime**, to be shown to represent **a sound wave** or **a surface wave**, and bring in an observer who is travelling on another platform. We are interested in relative position of each **event on these two worldlines**. It, then, will be shown that the velocity, the acceleration, the encounter frequency, the group velocity, and the time and the space distance between the wave and the observer on the worldlines should all be derivable in principle. This is interpreted to mean that we really have the relativistic events taking place with different values of **time dilation** in the sense of '**spacetime**', and that the well-known 「**Special Theory of Relativity**」 applies just as well in hydrodynamic waves.

1. Introductory Remarks

The **relativistic theory**, in particular the special theory of relativity as originally thought of by Albert Einstein and very much home to physicists, has not been properly taken up by us, the hydrodynamicists. To think of it, the relativistic velocity and positioning of **events** in fluids whether be it acoustic pressure propagation or gravitational surface wave, being within our biological realm, are much more easily observable and would give us some practical results if developed further. This short note is to suggest for younger students if they might delve into this. Much of this note is referred from Ref. 1.

2. Hydrodynamic Waves

2-1. Sound Waves

Sound waves, although originating from a point and thus a spherical wave originally, may for a simplicity be considered to be a plane wave at a far distance. The pressure fluctuation function p, definable as the difference between the pressure P and the mean pressure P', has a form in Newtonian space:

$$p = P - P' = A \sin(-\omega t + \mathbf{k} \cdot \mathbf{r} + \text{const}) \qquad (1)$$

where A : amplitude/constant, ω : angular frequency/constant, t : time, **k** : wave vector(note: the bold letters indicating vectors), **r** : position vector.

Following this, we define a function in **spacetime** [Eq.(1) is a simpler case of Eq.(2)]:

$$\mathbf{P} = A\left\{ \begin{matrix} \sin \\ \cos \end{matrix} \right\} (\mathbf{K} \cdot \mathcal{R} + F) \qquad (2)$$

where A: amplitude, **K**: the wave four-vector, F: constant, $\mathcal{R}$: the position vector in spacetime as in Eq.(8).

2-2. Surface Waves with a Body

Presence of a body may be represented by a collection of singularities, which in turn presents an agitation or an observation platform. For surface waves with an agitation near the surface (source location f⟶0 on z-axis), far from the point of excitation (x⟶∞), the wave elevation function Z(x,z,t) in ordinary Cartesian coordinates with z pointing downward may take a form:

$$Z \fallingdotseq U/g(\partial \Phi/\partial x) - \partial \Phi/\partial t \quad \text{at } z \to 0 \qquad (3)$$

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where U is the forward speed and g the gravitational constant, and where Φ is the solution of the differential equation $\nabla^2\Phi = 0$, to be satisfied at the boundaries. The solutions take the following forms respectively:

For the boundary condition on the surface for ***oscillating sources (or multipoles):***

$$K'\Phi + \partial\Phi/\partial z = 0 \text{ at } z = 0 \text{ with } K' = \omega^2/g \qquad (4)$$

For 2-dimensions (cylindrical coordinates $x=r\cos\theta$, $z=r\sin\theta$):

$$Z = A\left\{\begin{matrix}\sin\\ \cos\end{matrix}\right\}\omega t\left\{\begin{matrix}U/g\\ \omega\end{matrix}\right\}\left\{\begin{matrix}\sin\\ \cos\end{matrix}\right\}K'x \qquad (5)$$

where $A = \sum_{n=1}^{\infty} \frac{2\pi K'^{n}}{(n-1)!}(-1)^{n}\left[\begin{matrix}a_n\cos\alpha_n\\ b_n\sin\alpha_n\end{matrix}\right]$

For the boundary condition on the surface for ***steady case with a forward speed of U:***

$$\partial\Phi/\partial x + K''(\partial\Phi/\partial z) = 0 \text{ at } z=0 \text{ with } K'' = g/U \qquad (6)$$

For 2-dimensions:

$$Z = A\left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\}K''x \qquad (7)$$

where

$$A = \sum_{n=1}^{\infty} \frac{2\pi}{(n-1)!}K''^{n}\left[\begin{matrix}(-1)^{n-1}a_n\cos\alpha_n\\ (-1)^{n}b_n\sin\alpha_n\end{matrix}\right]$$

From the above equations, i.e. Eqs. (5), and (7), confined approximately to the free surface ($z\rightarrow 0$), for 2-dimensions, and if A, $\mathbf{R}$ and **K** are allowed to have much broader meaning in spacetime, it is seen that the equations of surface waves may be classed into Eq.(2), albeit with some manipulations. Eqs. (5), and (7) are respectively from Eqs. 207 on p.132, and derived from 222 on p.140 from Ref. 2. Waves in 3-dimensional case should be looked into, but not expected to be any different in principle.

3. Relativistic View

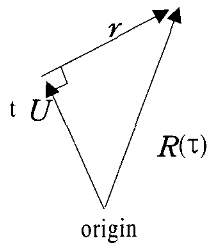
Let us now confine ourselves to a simple case in vector, **spacetime** vector, and consider a wave train. Let its **worldline** normal to the crest with respect to a fixed system be represented by a spacetime position vector $\mathbf{R}$, definable by the **proper time** parameter τ, which in turn may be given in ten (10) coordinates in Cartesian.[1] Let us bring in an observer who is travelling in another worldline given by t $\mathbf{U}$, where $\mathbf{U}$ is a **time like** unit vector. The worldline t $\mathbf{U}$ in the direction of $\mathbf{U}$ is **observer dependent**. We are interested in relative position of each events in this unique space constructed by the observer.

Then, by splitting $\mathbf{R}$ with respect to t $\mathbf{U}$ (one parallel to t $\mathbf{U}$ and the other orthogonal to, i.e. see Fig. 1),

$$\mathbf{R} = t\,\mathbf{U} + \mathbf{r} \text{ with } \mathbf{U}\cdot\mathbf{r} = 0 \qquad (8)$$

Since $\mathbf{U}$ is timelike, $\mathbf{U}^2=-1$, and $\mathbf{r}$ is orthogonal to $\mathbf{U}$ and is spacelike, i.e. $\mathbf{r}^2 > 0$.

The length of $\mathbf{r}$ or the space distance ℓ to the events $\mathbf{R}$ is: $\ell^2 = \mathbf{r}^2 = t^2 - \tau^2$



[Fig. 1]

Eq. (8) may be uniquely solved for t and $\mathbf{r}$, $t = -\mathbf{U}\cdot\mathbf{R}$ by multiplying Eq. (8) by $\mathbf{U}$, and $\mathbf{r} = \mathbf{R} + (\mathbf{U}\cdot\mathbf{R})\mathbf{U}$ by inserting the above into Eq. (8). Since the worldline $\mathbf{R} = \mathbf{R}(\tau)$ and τ is the proper time along $\mathbf{R}$,

$$d\tau^2 = -(d\mathbf{R})^2$$

From Eq. (8) differentiating, $d\mathbf{R} = dt\,\mathbf{U} + d\mathbf{r}$ with $d\mathbf{r}\cdot\mathbf{U} = 0$, and by squaring,

$$-(d\tau)^2 = -(dt)^2 + (d\mathbf{r})^2$$

$$\text{or } d\tau = dt\sqrt{1-v^2} \text{ with } v \equiv d\mathbf{r}/dt \qquad (9)$$

This is the famous equation for **time dilation**. However, it is to be noted that there is no universal time valid for this expression. This is observer dependent and is uniquely specific to our case, i.e. no relation to the constant speed of light, but related to all the different speed components of waves, etc., and therefore the time dilation takes different

1) Galileo group with three Euler angles for **x**-vectors, three components each for velocity v and the displacement of origin **d** plus time t.

values for R, r, U, etc. The concept of Minkowsky space should be applicable. For the cases of $v^2 \geq 1$, however, a proper interpretation has never been made.

Four velocity of R is defined: $V \equiv dR/d\tau$

$$V \equiv dR/d\tau = \frac{1}{\sqrt{1-v^2}} dR/dt = \frac{1}{\sqrt{1-v^2}}(U+v)$$

$$= \gamma(U+v) \quad (10)$$

with $\gamma \equiv \frac{1}{\sqrt{1-v^2}}$, and $U \cdot v = 0$.

Four acceleration is defined:

$$A \equiv dV/d\tau = d^2 R/d\tau^2 \quad (11)$$

Since the observer has the four-velocity U, split **K**:

$$K = \omega U + k \quad \text{with } k \cdot U = 0 \quad (12)$$

where **k**: wave vector, ω: angular frequency.

By multiplying U to (12), we obtain an observer dependent angular frequency $\omega°$,

$$\omega° = -K \cdot U \quad (13)$$

This expression yielding encounter angular frequency for a wave may be resolved for **K** and **k** by measuring various $\omega°$ from the observation platform U, which is certainly within our means at hand.

We may also take a superposition case where many multidirectional waves of the type given by Eq. (2) co-exist, i.e. with different A, **K**, and F in different terms, and an observer on U does in general measure different angular frequencies as in Eq. (13), and from this we may resolve for the directional wave-spectrum.

If an observer is made to follow the wave train such that the observer sees no difference in angular frequency, the four-velocity **V** satisfying the condition:

$$d(V \cdot K) = V \cdot dK = 0 \quad (14)$$

This is the condition giving the group velocity of the wave. V[dK→0] is called group four-velocity. From Eq. (12),

$$dK = d\omega U + dk \quad \text{with } dk \cdot U = 0. \quad (15)$$

Also using Eqs. (10) and (15), Eq. (14) becomes:

$$V \cdot dK = \gamma(-d\omega + v \cdot dk) = 0, \quad \text{or} \quad d\omega = v \cdot dk$$

This is an equation in a Euclidian space from which an ordinary form of group velocity may be derived, i.e., by letting $k \equiv k\kappa$, where κ being unit vector:

$$v = (d\omega/dk)\kappa^{2)} \quad (16)$$

It should then follow that from these expressions the velocity, the acceleration, the encounter frequency, the group velocity, and the time and the space distance between the wave and the observer on the worldlines should all be derivable, i.e. the relative position of events on two worldlines.

4. Concluding Remarks

Physicists in the theory of relativity (the special theory of relativity here) deal with much faster events like light and subparticles, not directly visible. Ours is very much slower. The reference speed of signal is not the speed of light but the pressure propagation for the acoustic waves, and for the surface waves the group velocity of the wave train, or the observer's speed, which may be greater than the group velocity, not at all considered possible in the case of light.[3] This is interpreted to mean that during the advent of the wave train, we also may have shifted our observation platform, and we really have the **relativistic event** taking place with different values of time dilation in the sense of '**spacetime**'.

It should, then, be obvious that the well-known 「**Special Theory of Relativity**」 applies in hydrodynamic waves just as well.

2) $k \equiv k\kappa$, then $\kappa^2 = 1$ by definition, and $\kappa \cdot d\kappa = 0$, i.e. κ and $d\kappa$ are orthogonal to each other.
Then, $dk = dk\kappa + k\, d\kappa$
$d\omega = v \cdot \kappa dk + k\, v \cdot d\kappa$
If no dispersion is assumed, i.e. v is function of k, but not of κ.
Choosing dk=0, $d\omega = 0 = v \cdot d\kappa$, then v is parallel to **k** and $d\omega/dk = v \cdot \kappa$, or by multiplying by κ,
$v = (d\omega/dk)\kappa$

3) It is certainly possible to outrun the events in hydrodynamics as we have faster means of signaling, say visual or other electronic means, but let us assume for the time being we are deprived of these means, as actually is true in the case of a submerged situation in water.

5. References

[1] Laurent, Bertel: *Introduction to Spacetime - a first Course on Relativity*, World Scientific Publishing Co., 1994.

[2] Kim, Hun Chol: *Determination of the Non-lifting Potential for an Arbitrary Ship Hull Form*, Doctoral Dissertation at The University of Michigan, 1964.