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Convergence Study of k-ω Turbulence Equations for Compressible Flows

Soo Hyung Park*, Chun-ho Sung** , Jang Hyuk Kwon*** and Seungsoo Lee****

압축성 유동을 위한 k-ω 난류방정식의 수렴성 연구

박수형*·성춘호**·권장혁***·이승수****

Key Words : *k-ω turbulence equations, multigrid method, DADI, transonic airfoil flow*

Abstract

An efficient implicit multigrid method is presented for the Navier-Stokes and k-ω turbulence equations. Freezing and limiting strategies are applied to improve the robustness and convergence of the multigrid method. The eddy viscosity and strongly nonlinear production terms of turbulence are frozen in the coarser grids by passing down the values without update of them. The turbulence equations together with the Navier-Stokes equations, however, are consecutively solved on the coarser grids in a loosely coupled fashion. A simple limit for k is also introduced to circumvent slow-down of convergence. Numerical results for the unseparated and separated transonic airfoil flows show that all computations converge well without any robustness problem and the computing time is reduced to a factor of about 3 by the present multigrid method.

1. Introduction

Turbulent flow calculations have become increasingly important in aerodynamic applications. Practically, simple algebraic turbulent models have been used to obtain the eddy viscosity because of simplicity and efficiency[1]. In view of accuracy, it is desirable to solve the transport equations for the turbulent variables[2], such as the turbulent kinetic energy(k) or the specific dissipation rate(ω).

In the present work, the compressible Navier-Stokes equations and k-ω turbulence model equations[3,4] are considered. It has advantages in that the model does not require damping functions and no distance from the wall needs to be defined. It also enables us to use freestream values as initial conditions[4] for practical applications. Despite many efforts have been devoted to accelerate the convergence of two-equation low-Reynolds number models[5,6], Navier-Stokes calculations with two-equation models consume excessive computing time.

Multigrid convergence acceleration[7,8] is the best alternative to obtain cost-effective solutions with two-equation models. However, there are several difficulties in implementation of the multigrid method. Most of all, special treatments for stiff source terms are required at coarser grids. If the terms are not properly treated, the solutions diverge in most cases. In this work, the eddy viscosity and production terms are frozen in coarser grids by passing down the values without a new calculation[6]. Next, the numerical conditions may lead to unphysically low values of ω variables. Physical limit for ω is applied to avoid this situation [5]. A simple limit for k values is also introduced to preserve the value of the eddy viscosity. Through this process, nearly linear convergence can be obtained at higher level multigrid.

To demonstrate the efficiency of the multigrid algorithm for the k-ω equations, the transonic flows[9] past RAE2822 airfoil are computed and compared with the experimental data.

2. Numerical Method

2.1 Solution algorithms

The 2-dimensional compressible Navier-Stokes equations and k-ω turbulence equations[3] are considered:

$$\frac{\partial q}{\partial t}+\frac{\partial(f_i- f_{vi})}{\partial x_i} = S \tag{1}$$

$$\begin{aligned} q &= [\rho\ ,\ \rho u_j\ ,\ \rho E\ ,\ \rho k\ ,\ \rho\omega]^T \\ f_i &= [\rho u_i,\ \rho u_i u_j+\delta_{ij}p,\ \rho u_i H,\ \rho u_i k,\ \rho u_i\omega]^T \end{aligned} \tag{2}$$

$$f_{vi}=\begin{pmatrix} 0 \\ \tau_{ij}+\tau^*_{ij} \\ u_i(\tau_{ij}+\tau^*_{ij})-q_i+(\mu_L+\frac{\mu_T}{\sigma_k})\frac{\partial k}{\partial x_i} \\ (\mu_L+\frac{\mu_T}{\sigma_k})\frac{\partial k}{\partial x_i} \\ (\mu_L+\frac{\mu_T}{\sigma_\omega})\frac{\partial \omega}{\partial x_i} \end{pmatrix} \tag{3}$$

* KAIST 기계공학과 항공우주공학전공, pish@kaist.ac.kr
** KISTI 슈퍼컴퓨터센터, page@duy.kaist.ac.kr
*** KAIST 기계공학과 항공우주공학전공, jhkwon@kaist.ac.kr.
**** 국방과학연구소, numerics@hananet.net.

where μ_L is the molecular viscosity determined by Sutherland' law, and μ_T is the turbulence eddy viscosity defined by

$$\mu_T = \frac{\rho k}{\omega} \tag{4}$$

The source vector of Eq. (1) is composed of the production rates of k and ω, denoted by P_k and P_ω, and the destruction rates of them, D_k and D_ω.

$$S = \begin{pmatrix} P_k - D_k \\ P_\omega - D_\omega \end{pmatrix} = \begin{pmatrix} \tau^*_{ij} \frac{\partial u_i}{\partial x_i} - \beta' \rho k \omega \\ (\frac{\alpha\omega}{k}) \tau^*_{ij} \frac{\partial u_i}{\partial x_i} - \beta \rho \omega^2 \end{pmatrix} \tag{5}$$

The closure constants are

$$\sigma_k = \sigma_\omega = 2.0$$
$$\alpha = \frac{5}{9}, \quad \beta' = \frac{9}{100}, \quad \beta = \frac{3}{40} \tag{6}$$

The governing equations in the physical coordinates system are transformed onto the computational coordinates and Eq. (1) is discretized with the cell-centered finite volume method. The flux residual R_{ij} of the discretized governing equations consists of the inviscid and viscous parts of numerical flux at the cell faces. In the present study, flux difference splitting(FDS) is used for upwind method and the 2nd order upwind TVD scheme with the minmod limiter is adopted to improve the solution accuracy[8]. The 9-point central differencing is applied to obtain the variable gradients of the viscous flux.

In this work, the DADI(Diagonalized-ADI) method is applied to find steady-state solutions[8]. For k-ω turbulence equations, the source vector of Eq. (5) must be implicitly treated. The contributions of the turbulent dissipation terms, D_k and D_ω, are added in the implicit parts to increase the diagonal dominance [5]. Therefore, the approximate Jacobian of the source terms can be expressed as:

$$W = \frac{-2}{J} \begin{pmatrix} Max(0, \frac{1}{3} DV) + \beta' \omega & 0 \\ 0 & Max(0, \frac{1}{3} DV) + \beta\omega \end{pmatrix} \tag{7}$$

where DV denotes the divergence $\nabla \cdot u$. The resulting scheme for turbulence equations is only a scalar ADI method if the DADI procedure is applied.

A loosely coupled algorithm is used for integrating the Navier-Stokes and k-ω equations separately. When a time stepping method is applied in loosely coupled fashion, the convergence of k-ω equations lags behind the Navier-Stokes equations. Several algorithms have been devised up to now, such as multiple iterations, to reduce the lagging[5]. In the present loosely coupled implicit algorithm, the k-ω equations are marched only one time stepping for each Navier-Stokes iteration because more iterations do not reduce the total computing time for the present test cases.

Albeit the implicit treatment of the source term greatly improves the robustness involved with the positivity of the turbulence variables, the ω equations cannot preserve the positivity constantly. A lower limit for ω is imposed for every iteration after the turbulence equations are solved[5]:

$$(\rho\omega)_{\min} = \alpha\rho\sqrt{P_d} \tag{8}$$

2.2 Multigrid method

The acceleration mechanisms of multigrid method are known as the high frequency damping and the fast wave propagation in coarser grids[7,8]. To this end, different number of time-stepping as well as CFL numbers are applied according to the grid level in the V cycle algorithm. See reference[8] for more details.

Multigrid method for the turbulence equations requires careful approach because of their high stiffness which results from the nonlinear source terms, Eq. (5). It has been noted that the strongly nonlinear terms cause divergence of the computation since they largely change the gradients in the flow and turbulence variables. As such it is important to freeze the nonlinear terms to preserve the turbulence variables in the coarse levels[5,6]. Therefore P_d and DV terms are calculated only on the finest grid and restricted as the frozen values to the coarser grids:

$$(P_d)_{2h} = \Sigma V_h (P_d)_h / V_{2h}$$
$$(DV)_{2h} = \Sigma V_h (DV)_h / V_{2h} \tag{9}$$

In the present study, the eddy viscosity is also restricted in the same way. Freezing the eddy viscosity helps the robust convergence especially when the freestream values are used for the initial conditions.

When no limit for k is applied on the coarse grids, slow-down of the convergence was found. This slow-down is related with the large level of the dissipation rate ω at the farfield shear region. Without suitable choice of the freestream values, very low values of k can be predicted consequently. Therefore, a simple limit for k is suggested as follows:

$$(\rho k)_{\min} = 0.01 \times (\rho k)_\infty \tag{10}$$

Because this limit is applied only in the coarse grids, little change is shown for solution accuracy in the finest grid.

2.3 Boundary conditions

At the solid walls, no-slip conditions for velocities are applied and the density and energy are extrapolated from the interior cells. The value of k is set to zero at the wall. Since the specific dissipation rate ω is theoretically infinite at the wall, it is required that its asymptotic value is imposed to the several interior points[3,4]. In this study, the boundary value of ω is specified as:

$$\omega_{wall} = \frac{19}{9} \frac{80(\mu_L)_w}{\rho d_1^2} - \omega_i \tag{11}$$

where d_1 is the distance of the first cell center from the wall and ω_i is the value of the first interior cell[5]

The freestream k and ω are determined by using the estimation proposed by Menter[4].

3. Numerical Results

The present method is applied to the turbulent transonic flows past the RAE2822 airfoil. A C-type computational grid is used. The farfield boundary is located at 20 chord length away. Computations are performed on the PC-cluster with 4-Pentium II 400MHz CPU. The first cell centers from the wall are located so that y+ is set to about 1[3].

Table 1 shows the used freestream conditions[9]. Computations are performed with the widely used Baldwin-Lomax(B-L) algebraic model[1] and the k-ω turbulence model[3-5]. For the B-L model used, the turbulent eddy viscosity is frozen in the coarse grid to exclude the convergence stagnation when the normalized density residual is dropped below 10^{-4}. The measured and computed lift and drag coefficients are compared in Table 2. The coefficients agree well with the experimental data for the unseparated flows(Case 1 and 6). The present k-ω turbulence model gives the excellent agreement for all freestream conditions.

Figure 1 displays the pressure coefficient distributions for each case. Cases 1 and 6 represent the subcritical and unseparated supercritical flows. Case 10 is one of the separated flows. As shown, the results with B-L and k-ω models agree well with experimental data and only a difference is found in Case 10.

Figure 2 represents comparisons of the convergence histories for Case 1. When the normalized L2 norm is reduced to 9 orders of magnitude, the 4-level multigrid DADI method takes 693 cycles while single grid takes 4219 cycles, as shown in Fig. 2-(a). As the multigrid level increases, the number of iteration decreases. Fig. 2-(b) shows that the convergence rate of the B-L model is nearly same to that of the k-ω models, while the computing time is less than that of k-ω, of course. It is noted that the convergence rates are reduced at about six order of magnitude for 3 and 4 level multigrid methods. If Eq. (10) is not used, the slow-down is so severe that the converged solution of the k equations cannot be obtained.

Figure 3 displays the convergence histories for Case 6 with k-ω turbulence model. The convergence trends are similar to Case 1. The L2 norms of density, k, and ω are shown in Fig. 3-(b). For the single grid calculation, all variables converge with the nearly same convergence rate after the error is reduced to 10^{-5}. The 4-level multigrid convergence of k equations slows down foremost and impedes others. As the convergence of Case 1, the limit for k is important to obtain the converged solutions.

The convergence for the separated transonic flow is displayed in Fig. 4. It has been found that the steady-state solution exists for this freestream condition[9]. Although the efficiency of the multigrid for Case 10 is lower than those for Cases 1 and 6, no oscillation nor stagnation of the convergence are found for all cases. On the contrary, the convergence with B-L model severely oscillates and slows down for all levels of multigrid.

4. Conclusions

An efficient multigrid method with DADI time integration method is presented for the Navier-Stokes and k-ω turbulence equations. The equations are solved separately on the coarser grids in a loosely coupled fashion. To this end, strong nonlinear source terms are restricted as the frozen variables to the coarser grids. Limits for turbulence variables can be simply imposed in the implicit scheme. Numerical results show that the present freezing and limiting strategies help fast convergence and robust computation of the k-ω turbulence equations. Especially for the separated airfoil flows, the present multigrid DADI method gives good convergence without any robustness problem.

Acknowledgement

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Case	M	α_{exp}	α_{cor}	$Re_c \times 10^{-6}$
1	0.676	2.40	1.93	5.7
6	0.725	2.92	2.54	6.5
10	0.750	3.19	2.81	6.2

Table 1. Freestream conditions for the test cases.

Case		C_L	C_D
Case 1	exp	0.566	0.0085
	B-L	0.574	0.0082
	k-ω	0.563	0.0084
Cse 6	exp	0.743	0.0127
	B-L	0.762	0.0126
	k-ω	0.745	0.0125
Case 10	exp	0.743	0.0242
	B-L	0.828	0.0283
	k-ω	0.768	0.0252

Table 2. Lift and drag coefficients for each case.

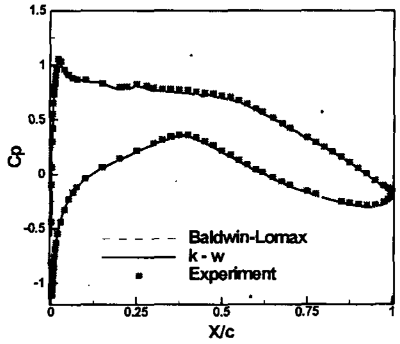

Fig. 1-(a). Wall pressure coefficient distributions for Case 1.

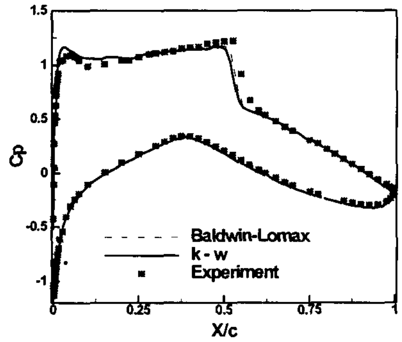

Fig. 1-(b). Wall prèssure coefficient distributions for Case 6.

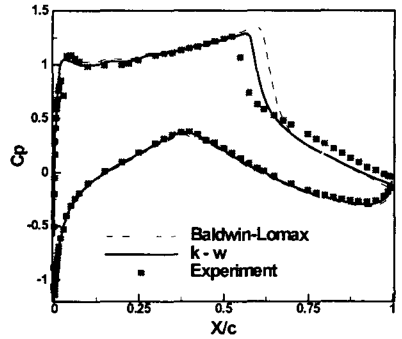

Fig. 1-(c). Wall pressure coefficient distributions for Case 10.

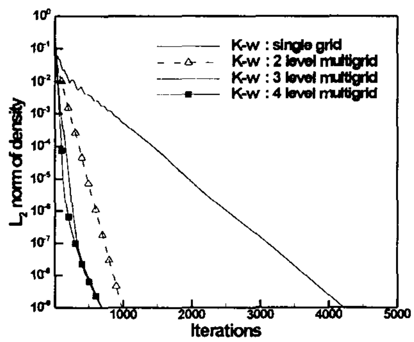

Fig. 2-(a). Case 1 : Comparisons of the convergence histories with multigrid levels.

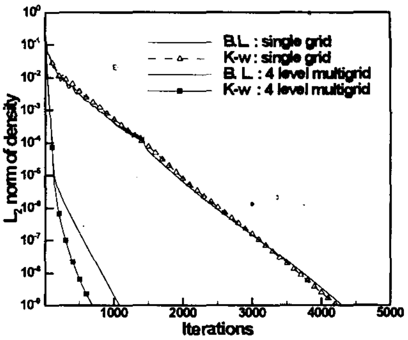

Fig. 2-(b). Case 1 : Comparisons of the convergence histories with turbulent models.

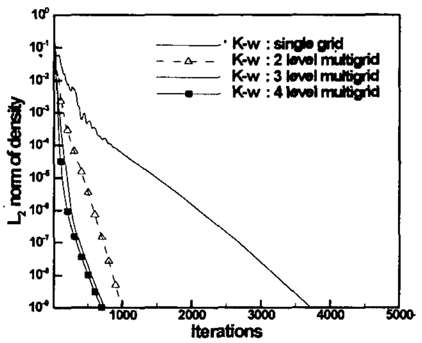

Fig. 3-(a). Case 6 : Comparisons of the convergence histories with multigrid levels.

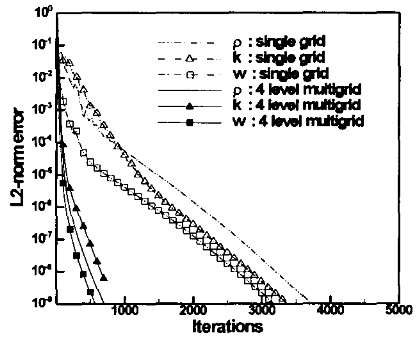

Fig. 3-(b). Case 6 : Comparisons of the convergence histories with variables.

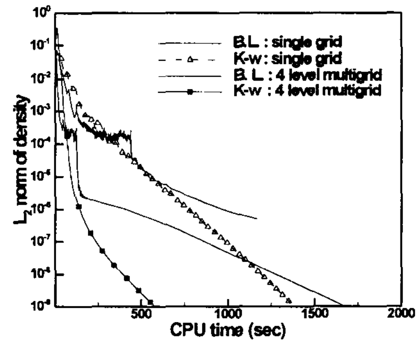

Fig. 4. Case 10 : Comparisons of the convergence histories with turbulent models.