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반복하중을 받는 균열손상 구조물의 수치해석 알고리즘

Numerical Algorithm for Cracked Structures Subjected to Cyclic Loading

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ABSTRACT

In this paper numerical algorithm for the continuum large crack model is proposed based on the return-mapping formulation. The numerical test results show that the present algorithm works appropriately under cyclic loading. It is also shown that in continuum damage models a large crack model to prevent excessive tensile plastic strain should be used to have realistic cyclic loading simulation results.

1. INTRODUCTION

The modeling of crack initiation and propagation is one of the most important aspects in the failure analysis of concrete structures. Continuum damage models have been used to simulate softening behavior of cracked concrete bodies.[1,2,3] Under severe cyclic and dynamic loading micro and mid-size cracks are developed to large cracks. In the classical continuum damage model a large crack is represented by excessive plastic strain which shows unrealistic results under cyclic loading as described in Fig. 1.

In this paper a model for large cracks in concrete and other strain-softening materials subjected to cyclic loading is presented. The suggested large crack model is based on the plastic-damage model.[1] Then numerical algorithm for the present large crack model is proposed based on the return-mapping formulation. A numerical example is presented to show the performance of the proposed algorithm.

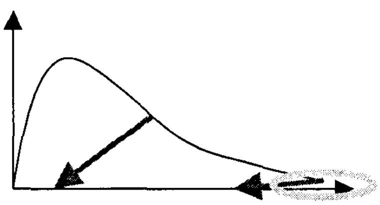

Fig. 1. Excessive Plastic Strain in Classical Damage Model

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2. PLASTIC-DAMAGE MODEL

To model the different damage states for cyclic multi-axial loading, both the tensile damage variable κ_t and the compressive damage variable κ_c are used as independent damage indexes.[1] Each damage variable is defined based on the ratio of dissipated plastic energy to the energy capacity per unit volume of materials (the specific fracture energy for tensile damage). To maintain objective results at the structural level, the characteristic length(l_t for tension, l_c for compression), which is the crack bandwidth along which the energy is dissipated, is specified as a material property. The factorization of the strength function into two functional forms, one for the effective stress and the other for the degradation damage variable, leads to the damage evolution equation described with the effective stress and damage variable vector $\boldsymbol{\kappa} = [\,\kappa_t \quad \kappa_c\,]^{\mathrm{T}}$:

$$\dot{\boldsymbol{\kappa}} = \dot{\lambda}\mathbf{H}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \tag{1}$$

The plastic strain rate is evaluated by the flow rule. In contrast with metals, a non-associative flow rule is necessary to obtain the proper dilatancy exhibited by frictional materials. If we use a Drucker-Prager type function as the plastic potential function for the present plastic-damage model, the plastic strain rate is derived from:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}(\frac{\mathbf{s}}{\|\mathbf{s}\|} + \alpha_p \mathbf{I}) \tag{2}$$

where $\dot{\lambda}$ is a non-negative function referred to as the plastic consistency parameter, $\|\mathbf{s}\| = \sqrt{\mathbf{s} : \mathbf{s}}$ denotes the norm of the deviatoric effective stress $\mathbf{s}$, and the parameter α_p is chosen to give the proper dilatancy for concrete.

For modeling the cyclic behavior of concrete, which has very different tensile and compressive yield strengths, it is necessary to use two cohesion variables in the yield function: c_t, a tensile cohesion variable, and c_c, a compressive cohesion variable. The yield function in Lubliner *et al.*[2], which only models isotropic hardening behavior in the classical plasticity sense, is modified to include two cohesion variables as follows:

$$F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) = \frac{1}{1-\alpha}[\alpha I_1 + \sqrt{3J_2} + \beta(\boldsymbol{\kappa})\langle \hat{\sigma}_{\max} \rangle - c(\boldsymbol{\kappa})] \tag{3}$$

where $\hat{\sigma}_{\max}$ denotes the algebraically maximum principal stress, and α is a parameter which is evaluated by the initial shape of the yield function. The evolution of the yield function is determined by defining β, and the cohesion parameter, c, such that:

$$\begin{aligned} \beta &= \frac{c_c(\boldsymbol{\kappa})}{c_t(\boldsymbol{\kappa})}(1-\alpha) - (1+\alpha) \\ c &= c_c(\boldsymbol{\kappa}) \end{aligned} \tag{4}$$

The mechanism of microcrack opening and closing behavior can be modeled as elastic stiffness recovery during elastic unloading from a tensile state to a compressive state. Using a multiplicative parameter $0 \le s \le 1$ on the tensile degradation variable D_t, we have the degradation damage variable $D = 1-(1-D_c(\boldsymbol{\kappa}))(1-sD_t(\boldsymbol{\kappa}))$, where D_c is the compressive degradation variable. The total stress $\boldsymbol{\sigma}$ is determined in the form of:

$$\begin{aligned}\boldsymbol{\sigma} &= (1-D)\bar{\boldsymbol{\sigma}} \\ &= (1-D_c(\boldsymbol{\kappa}))(1-sD_t(\boldsymbol{\kappa}))\mathbf{E}_0 : (\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^p)\end{aligned} \tag{5}$$

where $\bar{\boldsymbol{\sigma}} = \mathbf{E}_0 : (\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^p)$ is the effective stress and $\mathbf{E}_0$ is the initial elastic stiffness tensor. The parameter s is chosen to represent the stiffness recovery as follows:

$$s(\bar{\boldsymbol{\sigma}}) = \frac{\sum_{i=1}^{3}\langle \hat{\bar{\sigma}}_i \rangle}{\sum_{i=1}^{3}|\hat{\bar{\sigma}}_i|} \tag{6}$$

3. FORMULATION FOR LARGE CRACK OPENING/CLOSING

After a large amount of microcracking, the crack opening and closing mechanism becomes similar to discrete cracking, which cannot be appropriately represented only by the formulation described in the previous section.

In this study it is assumed that the microcracks are joined to construct a discrete large crack if $\kappa_t \ge \kappa_{cr}$, where κ_{cr} is an empirical value near unity. To model the large cracking, the evolution of the plastic strain caused by the tensile damage is stopped and the plastic strain increment is defined:

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = (1-s)\dot{\bar{\boldsymbol{\varepsilon}}}^{\mathrm{p}} \tag{7}$$

To make the effective stress based on Eq. 7 is admissible in the stress space it is necessary to introduce a new degradation variable D^{cr} and modify the definition of the effective stress in Eq. 5:

$$\tilde{\boldsymbol{\sigma}} = (1-D^{cr})\mathbf{E}_0 : (\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathrm{p}}) \tag{8}$$

The new degradation variable should be determined by the following Kuhn-Tucker type loading/unloading conditions such that:

$$\dot{D}^{cr} \ge 0; \quad \dot{D}^{cr}F(\tilde{\boldsymbol{\sigma}},\boldsymbol{\kappa}) = 0; \quad F(\tilde{\boldsymbol{\sigma}},\boldsymbol{\kappa}) \le 0 \tag{9}$$

Since during loading $F(\tilde{\boldsymbol{\sigma}},\boldsymbol{\kappa}) = 0$ which is a first-degree homogeneous function with respect to $\tilde{\boldsymbol{\sigma}}$, it is obtained:

$$D^{cr} = 1 - \frac{c_c(\boldsymbol{\kappa})}{f(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa})} \tag{10}$$

4. NUMERICAL ALGORITHM

To implement the large crack formulation described in the previous section numerically, a three-step return-mapping algorithm[4,5] based on the backward-Euler method is used in the present study. First, the following trial stress predictor is computed:

$$\tilde{\boldsymbol{\sigma}}_{n+1}^{tr} = (1 - D_n^{cr})\mathbf{E}_0 : (\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n^{\mathrm{P}}) \tag{11}$$

The trial stress evaluated by Eq. 11 is admissible as the effective stress at the current time step n+1 if:

$$F(\tilde{\boldsymbol{\sigma}}_{n+1}^{tr}, \boldsymbol{\kappa}_n) \equiv f(\tilde{\boldsymbol{\sigma}}_{n+1}^{tr}, \boldsymbol{\kappa}_n) - c(\boldsymbol{\kappa}_n) < 0 \tag{12}$$

Otherwise, the current step is inelastic and two correctors, the plastic corrector and the crack damage corrector, are required to make the effective stress admissible. In the plastic corrector the plastic strain increment in Eq. 2 is discretized using the backward-Euler method:

$$\Delta\bar{\boldsymbol{\varepsilon}}^{\mathrm{P}} = \gamma_{n+1} \frac{\partial G_{n+1}}{\partial \bar{\boldsymbol{\sigma}}_{n+1}} \tag{13}$$

and the plastic strain at the current time step becomes:

$$\boldsymbol{\varepsilon}_{n+1}^{\mathrm{P}} = \boldsymbol{\varepsilon}_n^{\mathrm{P}} + (1 - s_{n+1})\Delta\bar{\boldsymbol{\varepsilon}}^{\mathrm{P}} \tag{14}$$

where s_{n+1} is computed by Eq. 6 if $\kappa_t \geq \kappa_{cr}$, and equal to be zero otherwise. At the next step in the present algorithm the crack damage corrector, which makes the evaluated effective stress return back onto the yield surface, is obtained from Eq. 10:

$$D_{n+1}^{cr} = 1 - \frac{c_c(\boldsymbol{\kappa}_{n+1})}{f(\bar{\boldsymbol{\sigma}}_{n+1}, \boldsymbol{\kappa}_{n+1})} \tag{15}$$

Accordingly, the modified effective stress becomes:

$$\tilde{\boldsymbol{\sigma}} = (1 - D_{n+1}^{cr})\mathbf{E}_0 : (\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_{n+1}^{\mathrm{P}}) \tag{16}$$

which is used as the effective stress in Eq. 5 instead of the classical effective stress $\bar{\boldsymbol{\sigma}}$ in modeling large cracks.

5. NUMERICAL EXAMPLE

A numerical algorithm for the present large crack model has been implemented in the context of the finite element method. A single four-node plane stress quadrilateral isoparametric element is used to show the performance of the present algorithm. The loads are applied by displacement control, and the tested concrete material properties are: Poisson's ratio = 0.18, the tensile strength = 3.3MPa, the compressive strength = -22MPa, the fracture energy = 0.06N/m.

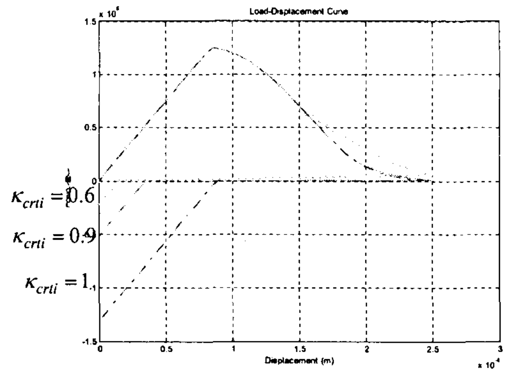

Fig. 2. Half-Cyclic Test

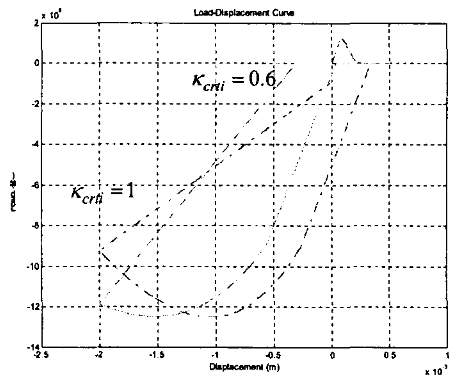

Fig. 3. Full-Cyclic Test

Two tests are performed. In the first test the concrete specimen is subjected to a half-cycle loading/unloading with three different κ_{crti} values: 0.6, 0.9, 1. It is noted that if $\kappa_{crti} = 1$ the present large crack opening/closing algorithm is not activated. In Fig. 2 the results with three κ_{crti} values are compared. It is shown that the result with $\kappa_{crti} = 1$ gives excessive palstic strain.

In the second test the concrete specimen is subjected to full-cyclic loading with two different κ_{crti} values: 0.6, 1 to evaluate the effect of the present algorithm on the overall cyclic behavior. Fig. 3 shows that the excessive plastic strain in the response curve with $\kappa_{crti} = 1$ produces unrealistic behavior when the loading direction is reversed from the tensile one to the compressive one, while the overall response curve is appropriate in the test result with $\kappa_{crti} = 0.6$.

6. CONCLUSIONS

In this paper numerical algorithm for large cracking in concrete and other strain-softening materials is presented. The algorithm is derived from the continuum large crack formulation in which the effective stress definition is modified to prevent the excessive tensile plastic strain in the case of large cracking. The numerical test results show that the present algorithm works appropriately under cyclic loading. It is also concluded that in continuum damage models a large crack formulation to prevent excessive tensile plastic strain should be used to have realistic cyclic loading simulation results.

Acknowledgement

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