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Closed Form Formulas for Equivalent Damping Ratios of a Linear Structure Equipped with Damping Devices

제진장치가 설치된 구조물의 등가감쇠비

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ABSTRACT

Hwang et al (2001) proposed a new method for an evaluation of equivalent damping ratios of a linear structure with linear or nonlinear damping devices. This procedure has a disadvantage that it requires time history analysis for the whole system including damping devices, which may be troublesome for practical application. To tackle this problem, closed form formulas for equivalent damping ratios are proposed in this study. It is assumed that the responses of MDOF system can be reproduced by an equivalent SDOF system which vibrates in a fundamental mode. The numerical analyses of a ten-story building equipped with linear viscous damper or active mass damper or friction damper show the effectiveness of equivalent SDOF model and closed form formulas.

1. INTRODUCTION

In spite of the effect of damping on dynamic response of structure, much attention has not been given to damping. This is due to the fact that damping is difficult to describe in mathematical form unlikely mass and stiffness, and it is very small in typical civil engineering structures. Thus, approximation is made to explain the effect of damping. Widely accepted method for this is concept of equivalent damping ratio.

There are many methods for estimating damping ratio from vibration test results. When damping is similar to linear viscous damping and small, logarithmic decrement method and half power method are generally used. In case damping has nonlinear hysteretic curve, damping ratio is determined to equal the dissipated energy to viscous damping in one vibration period [1-2]. Also, eigenvalue analysis of system matrix obtained from system identification technique [3] and ARMA model can present damping ratio[4].

Damping devices are additionally installed in real structure to reduce its excessive vibration under earthquakes or winds by enhancing the damping capacity of the structure. Passive types include viscoelastic damper, fluid viscous damper, friction damper, and fluid viscous walls. Active types include active mass damper (AMD), hybrid mass damper(HMD), active tendon(AT), and active variable stiffness. [5].

For the purpose of investigating the control effects of damping devices, many researches adopt the concept of equivalent damping ratio(EDR). Particularly, if needed amount of damping ratio, which reduces the response of a structure to desirable level, is given and the control efficiency of a damping device can be interpreted with EDR, the design procedure of a damping device can be much simplified. Hartog (1956) calculated the increase of damping ratios for a primary structure with tuned mass damper (TMD) as a function of mass ratios[6]. Johnson and Kienholz (1982); Soong and Lai (1991); Chang et al. (1992) applied the modal-strain-energy method to assess the effect of viscoelastic damper (VED) and predicted the EDR of the structures with added VEDs[7]. C. Li and A.M. Reinhorn (1995) derived the damping ratios of a structure with supplemental friction dampers through identification procedure using acceleration response transfer functions [8].

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In the NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273), the method to use the ratio of dissipated energy to conserved energy is recommended to estimate the damping ratio of a structure with passive energy dissipation devices[9].

Hwang et al (2001) proposed a new method for an evaluation of equivalent damping ratios of a linear structure with linear or nonlinear damping devices[10]. This procedure has a disadvantage that it requires time history analysis for the whole system including damping devices, which may be troublesome for practical application. To tackle this problem, closed form formulas for equivalent damping ratios are proposed in this study. It is assumed that the responses of MDOF system can be reproduced by an equivalent SDOF system which vibrates in a fundamental mode. The numerical analyses of a ten-story building equipped with viscous dampers (VD), active mass damper (AMD) and friction dampers (FD), show the effectiveness of equivalent SDOF model and closed form formulas.

2. EQUIVALENT DAMPING RATIO IN MODAL SPACE

The equation of motion of mass-damping-spring system under dynamic environmental load $f(t)$ and control force $u(t)$ is given by

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = L_c u(t) + L_e f(t) \tag{1}$$

where M , C and K denote, respectively, the mass, damping and stiffness matrices and $x(t)$ is displacement. L_c and L_e represent location vector of control force by supplemental damping devices and influence vector of excitation, respectively. The state-space form of modal equation for Eq.(1) is written as

$$\dot{z}_i(t) = A_i z_i(t) + B_i u(t) + H_i f(t), \quad z_i(t) = \begin{bmatrix} \eta_i(t) & \dot{\eta}_i(t) \end{bmatrix}^T \tag{2}$$

where, η_i denote generalized displacement of ith mode and A_i , B_i , H_i are system matrices. Lyapunov function e_i is defined as follows

$$e_i(t) = z_i^T(t) P_i z_i(t) \tag{3}$$

where P_i is arbitrarily chosen positive definite matrix. By differentiating the both sides of Eq.(3) and substituting Eq.(2), we get

$$\dot{e}_i(t) = z_i^T(t)\left(A_i^T P_i + P_i A_i\right) z_i(t) + 2z_i^T(t) P_i B_i u(t) + 2z_i^T(t) P_i H_i f(t) \tag{4}$$

If positive definite matrix P_i and positive scalar α_i exist to satisfy the following condition given by Eq.(5), the derivative of Lyapunov function can be expressed in autoregressive form and modal states can satisfy stability condition.

$$A_i^T P_i + P_i A_i = -\alpha_i P_i \tag{5}$$

$$\dot{e}_i(t) = -\alpha_i e_i(t) + 2z_i^T(t) P_i B_i u(t) + 2z_i^T(t) P_i H_i f(t) \tag{6}$$

Eq.(6) indicates that the amplitude of $e_i(t)$ decays exponentially with time without control force $u(t)$ and excitation $f(t)$. α_i is property related to damping rate at which free vibration decays.
By solving Eq.(5), we get the alpha and P.

$$P = \begin{bmatrix} \omega_i^2 & \xi_i \omega_i \\ \xi_i \omega_i & 1 \end{bmatrix} \tag{7}$$

$$\alpha_i = 2\omega_i \xi_i \tag{8}$$

where, ξ_i, ω_i denote damping ratio, natural radial frequency of *i*th mode, respectively.
If it is assumed that

$$u_i(t) = -\alpha_{ia} e_i(t) \tag{9}$$

where $u_i(t) = 2 z_i^T P_i B_i u(t)$.

Parameter α_{ia} is a function of time because Lyapunov function $e_i(t)$ and $u_i(t)$ are functions of time. Damping ratio increased by damping devices can be written as

$$\xi_{ia} = \frac{\alpha_{ia}}{2\omega_i} \tag{10}$$

Proposed equation for damping ratio is a function of time while general damping ratio has constant value. If external force is stationary random process, α_{ia} can be obtained by taking expectation of both sides of Eq.(9).

$$E\left[u_i(t)\right] = -\alpha_{i_{eq}} E\left[e_i(t)\right] \tag{11}$$

3. CLOSED FORM FORMULAS FOR EDRs

3.1 EDRs Based on the Deflection Shape

Previously mentioned method based on the numerical analysis for the evaluation of EDR has a disadvantage that it requires time history analysis for the whole system including damping devices, which may be troublesome for practical application. Although this procedure can identify the effect of damping devices most exactly by presenting the EDRs for all modes of a structure, time history analysis for the whole system with damping devices should be repeated whenever the design parameters of damping devices are changed. Especially, this process may require more time-effort for nonlinear damping devices.
To tackle this problem, assuming that the responses of structure are dominated by a fundamental mode normalized to the top floor displacement, a MDOF system is transformed to an equivalent SDOF system and the effect of damping device on this fundamental mode is estimated by using the proposed method. Also, based on the probabilistic properties of structural response, closed-form formulas for the estimation of EDRs of the equivalent SDOF system are proposed for MDOF system with damping devices which add stiffness or viscosity to a structure, with FD and with active damping devices using linear feedback law or bang-bang.
Displacement vector $x(t)$ can be expressed approximately by

$$x(t) = \phi d(t) \tag{12}$$

where ϕ is a vector of deflection shape of fundamental mode and normalized to a top story displacement $d(t)$. The equation of motion for the equivalent SDOF system is obtained as

$$M^* \ddot{d}(t) + C^* \dot{d}(t) + K^* d(t) = \phi^T L_u u(t) + \phi^T L_e f(t) \tag{13}$$

where $M^* = \phi^T M \phi$, $C^* = \phi^T C \phi$, and $K^* = \phi^T K \phi$. Dividing both sides of the equation by M^*, one can obtain

$$\ddot{d}(t)+2\xi_o\omega_o\dot{d}(t)+\omega_o^2 d(t)=\frac{\phi^T L_u}{M^*}u(t)+\frac{\phi^T L_e}{M^*}f(t) \tag{14}$$

where $\omega_o^2 = K^* / M^*$ and $\xi_o = C^* / 2\omega_o M^*$

Eq.(14) has the same form as Eq.(2). Thus, the increase damping ratio ξ_{eq} by damping device for equivalent SDOF system is derived by using the same procedure presented in the previous section.

$$\xi_{eq} = -\frac{E\left[2z(t)^T PBu(t)\right]}{2\omega_o E\left[z(t)^T Pz(t)\right]} = -\frac{E\left[z(t)^T PBu(t)\right]}{\omega_o E\left[z(t)^T Pz(t)\right]} \tag{15}$$

where $z(t)^T = \left[d(t) \quad \dot{d}(t)\right]^T$, $P=\begin{bmatrix}\omega_o^2 & \xi_o\omega_o \\ \xi_o\omega_o & 1\end{bmatrix}$, $B=\begin{bmatrix}0 \\ \dfrac{\phi^T L_u}{M^*}\end{bmatrix}$

For analysis of the SDOF system equivalent to MDOF system, the determination of shape vector is essential for the evaluation of damping ratio. In this study, ϕ is assumed to be the shape vector corresponding to the deflected shape under the action of statically applied lateral loads

3.2 Closed Form Formulas

Structural responses subjected to dynamic loads such earthquake and wind loads are known to have probabilistic characteristics. It means that the responses can be estimated by mean and standard deviation values. Considering that the forces of damping devices are dependent on the structural responses, EDR increased by these damping forces has probabilistic characteristics. The method proposed in this study begins with taking expectation of responses for their mean values, based on the assumption that responses are stationary and Gaussian. Then, probabilistic properties of responses are obtained or assumed and closed form formulas for EDRs can be induced. It is assumed that top floor displacement d and velocity $\dot{d}$ have following probabilistic characteristics.

$$E\left[d^2(t)\right]=\sigma_d^2 , \quad E\left[\dot{d}^2(t)\right]=\sigma_{\dot{d}}^2=\omega_o^2\sigma_d^2 , \quad E\left[d(t)\cdot\dot{d}(t)\right]=0 \tag{16}$$

where σ_d and $\sigma_{\dot{d}}$ are mean values of displacement and velocity, respectively.

Using the above equations , the denominator of Eq.(15) becomes

$$\omega_o E\left[z(t)^T Pz(t)\right]=\omega_o\left[\omega_o^2 d^2(t)+2\xi_o\omega_o d(t)\dot{d}(t)+\dot{d}^2(t)\right]=2\omega_o^3\sigma_d^2 \tag{17}$$

In order to evaluate the value of the nominator in Eq.(15), the control force $u(t)$ should be known. As $u(t)$ is generally a function of the displacement and velocity responses, EDR is evaluated for the following cases for $u(t)$; 1) $u(t)$ is linearly proportional to the relative velocity between the ends of device j along the axis of device j, 2) $u(t)$ is constant multiplied by the sign of the relative velocity between the ends of device j along the axis of device j. Case 1 mean that damping devices adds stiffness and viscosity to a structure, respectively. Case 2 means that friction damper or bang-bang control algorithm is adopted for damping device or active control algorithm. 3) $u(t)$ is a linear function of states. Active control devices using linear feedback control correspond to case 3. The jth control forces for each case are given by

$$u_j(t)=c_{Dj}\dot{d}(t)\delta_{rj}\cos\theta_j ; \quad \text{for case 1} \tag{18}$$

$$u_j(t) = u_{\max j} \operatorname{sgn}\left[\dot{d}(t)\delta_{rj}\right]\cos\theta_j; \quad \text{for case 2} \tag{19}$$

$$u(t) = -Gz(t) = -G_1\phi d(t) - G_2\phi\dot{d}(t); \quad \text{for case 3} \tag{20}$$

in which θ_j is the angle between the axis of damping device j and the floor at which the device is installed. c_{Dj} and $u_{\max j}$ is viscosity and maximum force of device j. δ_{rj} ,relative displacement between the ends of device j along the axis of device j, is determined based on the deflection shape vector ϕ. G_1 is gain for displacement feedback and G_2 is gain for velocity feedback.
Therefore, the nominator is determined

$$E\left[z(t)^T PBu(t)\right] = \frac{1}{M^*} E\left[\left(\xi_o\omega_o d(t) + \dot{d}(t)\right)\phi^T L_u u(t)\right] \tag{21}$$

The jth element of $\phi^T L_u$ for case 1, 2 is given by

$$\left[\phi^T L_u\right]_{j_{th}\_element} = -\delta_{rj}\cos\theta_j \tag{22}$$

Thus, corresponding denominators are given as

$$\xi_{eq} = \frac{1}{2M^*\omega_o}\sum_j c_{Dj}\delta_{rj}^2\cos\theta_j^2 \quad \text{for case 1} \tag{23}$$

$$\xi_{eq} = \frac{1}{\sqrt{2\pi}M^*\omega_o^2\sigma_d}\sum_j u_{\max j}\left|\delta_{rj}\right|\cos\theta_j^2 \quad \text{for case 2} \tag{24}$$

$$\xi_{eq} = \xi_o\frac{\phi^T L_u G_1\phi}{2M^*\omega_o^2} + \frac{\phi^T L_u G_2\phi}{2M^*\omega_o} \quad \text{for case 3} \tag{25}$$

While the EDR ξ_{eq} for case 1and case 3 can be easily determined only with information about ϕ and the properties or the gain of damping devices, σ_d should be known for the evaluation of ξ_{eq} for case 2. However, since σ_d is a property which should be determined by surely considering the effects of damping devices, ξ_{eq} cannot be obtained explicitly and iterative procedure will be necessary for the evaluation of ξ_{eq} for case 2. This iterative procedure can be avoided if σ_d is estimated by using an estimation technique. Kasai(1998) suggested a simplified expression which represents the response variation due to changing damping ratio from ξ_o to ξ [11].

$$\frac{S_d(\xi)}{S_d(\xi_o)} = \frac{\sqrt{1+25\xi_o}}{\sqrt{1+25\xi}} \tag{26}$$

in which S_d denotes a spectral displacement.
Since spectral response is approximately proportional to RMS response, with the help of equation (26), σ_d can be expressed as a function of σ_{d_o}. Here, d_o is displacement of a structure without damping devices.

$$\sigma_d = \sigma_{d_o}\frac{\sqrt{1+25\xi_o}}{\sqrt{1+25\left(\xi_o+\xi_{eq}\right)}} \tag{27}$$

By using equation (27), equation (24) is expressed as

$$\xi_{eq} = \frac{\sum_j U_{\max j}\left|\delta_{rj}\right|\cos\theta_j^2\sqrt{1+25\left(\xi_o+\xi_{eq}\right)}}{\sqrt{2\pi}M^*\omega_o^2\sigma_{d_o}\sqrt{1+25\xi_o}} = C_1\frac{\sqrt{1+25\left(\xi_o+\xi_{eq}\right)}}{\sqrt{1+25\xi_o}} \tag{28}$$

in which $C_1 = \dfrac{\sum_j U_{\max j} |\delta_{rj}| \cos\theta_j^2}{\sqrt{2\pi} M^* \omega_o^2 \sigma_{d_o}}$

From equation (28), ξ_{eq} can be obtained as

$$\xi_{eq} = \frac{25C_1^2 + C_1\sqrt{625C_1^2 + 4(1+25\xi_o)^2}}{2(1+25\xi_o)} \tag{29}$$

4. NUMERICAL ANALYSIS

To verify the applicability of equivalent SDOF model to a MODF system with damping devices and closed form formulas for EDRs, analyses are performed on a ten story shear building. LVD, FD, and AMD using bang-bang control algorithm are used as damping devices. It is assumed that LVD and FD are installed at every inter story of building and AMD is installed on top floor and the angles between LVDs or FDs and associated floors are zero. Each story unit of the building has identical lumped mass 100ton; story stiffness $k_{i=1,\dots,4} = 15000\,\text{kN}$, $k_{i=5,6,7} = 10500\,\text{kN}$ and $k_{i=8,9,10} = 7350\,\text{kN}$. Damping matrix is composed for modal damping ratios of all modes to be 0.02.

Figure 1 shows the variation of maximum and RMS values of top floor displacement and the 1st modal damping ratio of the building equipped with LVDs. With increasing C_o, the maximum and RMS values decrease and damping ratios increase. Modal analysis for MDOF system modeled by EDRs proposed in this study reproduce the almost same results as those for actual MDOF system, while equivalent SDOF model describes the top floor displacement with a little error. The 1st modal damping ratio obtained by using equation (23) fits the one by eigenvalue analysis well. It is worthy to note that the difference of responses becomes small with increasing C_o while the difference between EDR by equivalent SDOF model and EDR by MDOF system becomes large with increasing C_o. This fact means that the variation of response due to the effect of damping increased by damping device become less sensitive for larger damping.

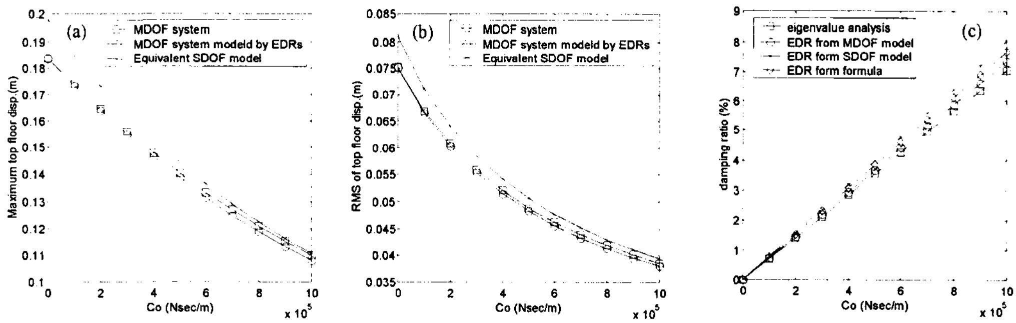

Figure 1. Model building with LVD; (a) Max.Top Floor Displ.; (b)RMS Top Floor Displ.; (c) 1st Modal Damping Ratio

Figure 2 and 3 represent the results for the building with AMD using bang-bang control algorithm and FDs, respectively. As these two damping devices generate maximum control force, regardless of response magnitude, their control effects are similar. Figure 2(a) and (b) indicate that the EDR proposed in this study represent accurately the control effects of FDs or AMD using bang-bang on rms and peak responses and, especially, rms response is closer to real value than maximum response. As control force limit u_{max} or F_c increases, the discrepancy appearing in maximum top floor displacement becomes large, that is, the EDR overestimates the control effects of FD or AMD with large force limit. Therefore, the proposed method for EDR is carefully used for Coulomb friction type damping devices such as FD and bang-bang controller with large force capacity. Compared with the 1st modal EDRs by MDOF system, EDRs given by Equivalent SDOF model in Figure 2(c) and 3(c) are overestimated, which is obvious particularly for large u_{max} or F_c. This tendency can be compensated by closed form formula given by Eq.(29). The reason why Eq.(29) can compensate the EDR by equivalent SDOF model which is a basis for Eq.(29) is that Eq.(27) for response variation due to variation of damping ratio conservatively expresses the control effect of damping on response reduction. Therefore, for a conservative design of a structure with Coulomb friction type damping devices, Eq.(29) is recommended at the stage of preliminary design.

It is noted from Figs. 2 and 3 that the control effect of single AMD at top floor with the same maximum control force to a FD corresponds to the that of ten FDs which are installed at every inter story. This is because an AMD generates control force of which sign is opposite to the relative velocity of top floor to the base floor while a FD makes control force with respect to the relative velocity between both ends of the device. AMD at top floor can be said to have larger controllability than that of a FD installed at inter story and the proposed method can reflect this fact.

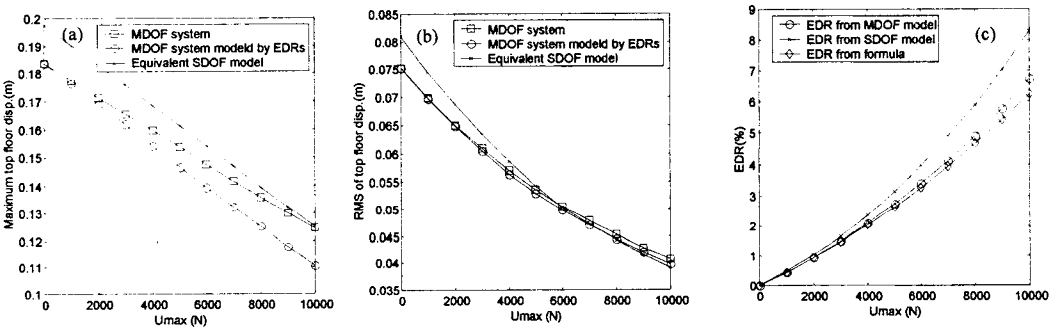

Figure 2. Model Building with AMD using bang-bang control algorithm; (a) Maximum Top Floor displacement; (b)RMS Top Floor Displacement; (c) 1st Modal Damping Ratio

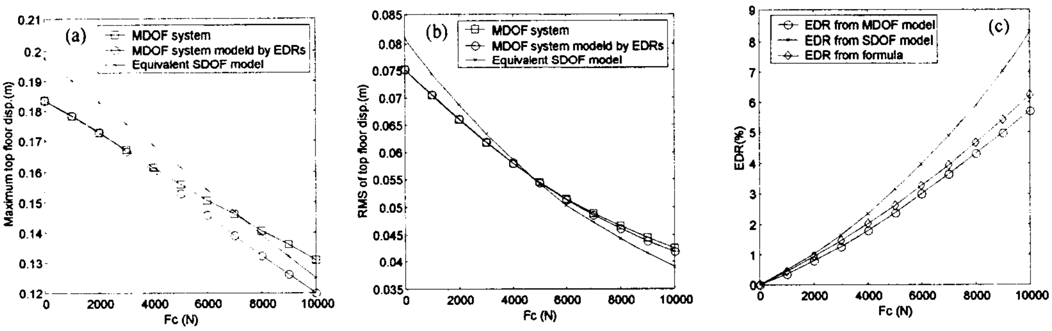

Figure 3. Model building with FD; (a) Maximum Top Floor displacement; (b)RMS Top Floor Displacement; (c) 1st Modal Damping Ratio

5. CONCLUSION

The objective of this study is to find method for evaluating equivalent damping ratios of a structure with supplemental damping devices. The findings of the study can be summarized as follows:

Lyapunov function, whose derivative is expressed in autoregressive form, is obtained and equivalent damping ratio is evaluated by using this Lyapunov function and its derivative to assess the control effect of various damping devices quantitatively. For the reduction of time efforts and preliminary design of damping devices, a MDOF system is transformed to an equivalent SDOF model and the control effect of damping device on fundamental mode is estimated by using the proposed method. Also, closed form formulas for the evaluation of EDRs are presented for linear and nonlinear damping devices. Through the numerical analysis of a ten-story building equipped with aforementioned damping devices, the effectiveness of equivalent SDOF model and closed form formulas are verified.

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