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AM-5

On the Θ-Derivations of Prime Rings

Let R be a prime ring with characteristic different two and $d: R \to R$ a nonzero derivation. I. N. Herstein*(A note on Derivations II, Canad. Math. Bull. 22(4)(1979), 509-511)* proved that if $a \in R$ and $[d(x), a] = 0$ for all $x \in R$, then $a \in Z$. Our aim in this note is to improve the above result, and obtain some analogous results as followings.

Theorem 1. Let R be a prime ring with characteristic different from two and let d be a nonzero Θ-derivation of R. If $a \in R$ and $[d(x), a]_{\Theta^2} = 0$ for all $x \in R$, then $a + \Theta(s) \in Z$.

Corollary to Theorem 1. Let R be a prime ring with characteristic different from two and let d be a nonzero Θ-derivation of R. Let S be a nonempty subset of R. If $[d(x), s]_{\Theta^2} = 0$ for all $x \in R$ and all $s \in S$, then $s + \Theta(s) \in Z$ for all $s \in S$.

Theorem 2. Let R be a prime ring with characteristic different from two and let d be a nonzero Θ-derivation of R. If $a \in R$ and $d([x, a]_{\Theta}) = 0$ for all $x \in R$, then $a + \Theta(a) \in Z$.

Corollary to Theorem 2. Let R be a prime ring with characteristic different from two and let d be a nonzero Θ-derivation of R. Let S be a nonempty subset of R. If $d([x, s]_{\Theta}) = 0$ for all $x \in R$ and all $s \in S$, then $s + \Theta(s) \in Z$ for all $s \in S$.

Theorem 3. Let R be a prime ring with characteristic different from two and let d be a nonzero Θ-derivation of R. If $a \in R$ and $Rad(R) = 0$ for all $x \in R$, then $d(a) = 0$.

Corollary to Theorem 3. Let R be a prime ring with characteristic different from two and let d be a nonzero Θ-derivation of R. Let S be a nonempty subset of R. If $Rad(S) = 0$ for all $x \in R$ and all $s \in S$, then $d = 0$ on S.

Theorem 4. Let R be a prime ring and let d be a nonzero Θ-derivation of R. If $a \in R$ and $[ad(x), x]_{\Theta} = 0$ for all $x \in R$, then either $a = 0$ or R is commutative.