Even-phase ZCD codes for MAI Cancelled DS-CDMA Systems

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Abstract: Multiple access interference (MAI) and multi path interference(MPI) degrades the system performance in the DS-CDMA(direct-sequence code-division multiple-access)systems. In this paper, a generalized construction method for $2^n(n=1,2,3)$ phase preferred pairs(PP) with zero-correlation duration (ZCD) of $(0.5N+1)$ chips is proposed. $2^n(n=1,2,3)$ phase ZCD code sets with ZCD and enlarged family sizes are generated by carrying out a chip-shift operation of the preferred pairs. The properties of the proposed codes are effective for the cancellation of MAI and MPI in DS-CDMA Systems..

1. Introduction

For any two complex spreading codes of period N, $C_N^{(x)}=(c_0^{(x)},\cdots,c_{N-1}^{(x)})$ and $C_N^{(y)}=(c_0^{(y)},\cdots,c_{N-1}^{(y)})$, the periodic correlation function with a shift τ is defined as

$$\theta_{x,y}(\tau)=\sum_{k=0}^{N-1}c_k^{(x)}c_{(k+\tau,\bmod N)}^{(y)*} \cdot \qquad (1)$$

where * denotes a complex conjugate. Eqn (1) becomes the autocorrelation function (ACF) when $x=y$ and the crosscorrelation function (CCF) when $x \neq y$.

Since the maximum magnitude of periodic ACF sidelobes (θ_{as}) and the maximum magnitude of periodic CCF (θ_c) are bounded by theoretical limits [1], spreading codes with both zero θ_{as} and zero θ_c cannot be constructed. However, it is possible to construct spreading codes with both zero θ_{as} and zero θ_c at the local duration around $\tau=0$. This local duration is defined as the zero-correlation duration (ZCD). This ZCD property can cancel MAI at the uplink of DS-CDMA systems, and it can be found in the binary codes [2-6] and a class of 4-phase codes[6]. In this paper, we propose new 8-phase ZCD codes and present a generalized construction method for $2^n(n=1,2,3)$ phase ZCD codes.

At first, we present a generalized construction method for $2^n(n=1,2,3)$ phase ZCD PP that have the ZCD of $(0.5N+1)$chips and periods of $4\times 2^i(i=0,1,2,3\cdots \textit{when } n=1,2 \;\; \textit{or} \;\; i=1,2\cdots \textit{when } n=3)$. Secondly, using the chip-shift operation to the $2^n(n=1,2,3)$ phase ZCD PP, we generate new spreading codes that have sufficient ZCD and family sizes for MAI-canceled DS-CDMA systems.

2. Construction method of $2^n(n=1,2,3)$ phase ZCD codes

The $2^n(n=1,2,3)$ phase ZCD codes are constructed by using the chip-shift operation of the $2^n(n=1,2,3)$ phase ZCD PP. $2^n(n=1,2,3)$ phase ZCD PP is started from the initial $2^n(n=1,2,3)$ phase ZCD PP generated by the initial basic matrix and its periods are extended by the period-extension matrix E as defined below.

1) Definition of the initial basic matrix

The initial basic matrices to construct preferred pairs are defined for the 2-phase, 4-phase and 8-phase cases respectively. The initial basic matrices to make 2-phase PP, *4-phase PP* and 8-phase PP are defined as Eqn (2), Eqn (3), Eqn (4), repectively

$$G_{2P}=\begin{bmatrix}1&1&1&-1\\1&-1&1&1\\1&1&-1&1\\-1&1&1&1\end{bmatrix} \qquad (2)$$

$$G_{4P}=\begin{bmatrix}1&j&1&-j\\1&-j&1&j\\1&j&-1&j\\1&-j&-1&-j\end{bmatrix} \text{ or } \frac{1}{\sqrt{2}}\begin{bmatrix}1+j&-1+j&1+j&1-j\\1+j&1-j&1+j&-1+j\\1+j&-1+j&-1-j&-1+j\\1+j&1-j&-1-j&1-j\end{bmatrix} \qquad (3)$$

$$G_{8P}=\begin{bmatrix}j&B&1&C&-j&D&1&C\\j&D&1&A&-j&B&1&A\\j&C&1&D&-j&A&1&D\\j&A&1&B&-j&C&1&B\\j&D&1&A&j&D&-1&C\\j&B&1&C&j&B&-1&A\\j&C&1&D&j&C&-1&B\\j&A&1&B&j&A&-1&D\\-j&D&1&C&j&B&1&C\\-j&B&1&A&j&D&1&A\\-j&A&1&D&j&C&1&D\\-j&C&1&B&j&A&1&B\\j&D&-1&C&j&D&1&A\\j&B&-1&A&j&B&1&C\\j&C&-1&B&j&C&1&D\\j&A&-1&D&j&A&1&B\end{bmatrix} \qquad (4)$$

In Eqn (3), G_{4P} has 4-phase elements of $\{1, j, -1, -j\}$ or $\left\{\frac{1+j}{\sqrt{2}}, \frac{-1+j}{\sqrt{2}}, \frac{-1-j}{\sqrt{2}}, \frac{1-j}{\sqrt{2}}\right\}$ with $j=\sqrt{-1}$. In Eqn (4), G_{8P} has 8-phase elements of $\{1, A, j, B, -1, C, -j, D\}$, where $A=\frac{1}{\sqrt{2}}(1+j)$ $B=\frac{1}{\sqrt{2}}(-1+j)$ $C=\frac{1}{\sqrt{2}}(-1-j)$ $D=\frac{1}{\sqrt{2}}(-1+j)$.

2) Definition of the period-extension matrix

For the convenience, we denote $2^n (n=1,2,3)$ as $2^{n=1,2,3}$. Using any given initial PP of $\{2_m^{n=1,2,3\ (a)},\ 2_m^{n=1,2,3\ (b)}\} = \{(e_0^{(a)},\cdots,e_{m-1}^{(a)}),\ (e_0^{(b)},\cdots,e_{m-1}^{(b)})\}$ that have the period m and ZCD of $(0.5\times m+1)$ chips, the period-extension matrix E is written as

$$E=\begin{bmatrix} X & Y & X & -Y \\ X & Y & -X & Y \\ X & -Y & X & Y \\ -X & Y & X & Y \end{bmatrix} \quad (5)$$

where $X = (e_0^{(a)},\cdots,e_{0.5m-1}^{(a)})$, $Y=(e_{0.5m}^{(a)},\cdots,e_{m-1}^{(a)})$ and $m=4\times 2^i$. Here i is nonnegative integer if n =1 or 2, or positive integer if n =3. Any row of $\pm E$ is $2_{2m}^{n=1,2,3\ (a)}=(e_0^{(a)},\cdots,e_{2m-1}^{(a)})$ with the period of $2m$. Furthermore, any row of $\pm jE$ can also be $2_{2m}^{n=2,3\ (a)}$. $2_{2m}^{n=1,2,3\ (b)}=(e_0^{(b)},\cdots,e_{2m-1}^{(b)})$ is generated from $2_m^{n=1,2,3\ (a)}$, where $e_\rho^{(b)} = (-1)^\rho e_\rho^{(a)}$ $(\rho=0,1,\cdots,2m-1)$. $\{2_{2m}^{n=1,2,3\ (a)},\ 2_{2m}^{n=1,2,3\ (b)}\}$ is a $2^{n=1,2,3}$ phase PP with ZCD of $(0.5\times 2m+1)$ chips.

3) Step(i): The construction of $2^{n=1,2,3}$ phase ZCD PP

The pairs of $\{2_4^{n=1\ (a)},\ 2_4^{n=1\ (b)}\}$ and $\{2_4^{n=2\ (a)},\ 2_4^{n=2\ (b)}\}$ that have ZCD of $(0.5\times 4+1)$ chips are defined as the initial 2-phase ZCD PP and the initial 4-phase ZCD PP, respectively. where $2_4^{n=1\ (a)}=(s_0^{(a)},s_1^{(a)},s_2^{(a)},s_3^{(a)})$ and $2_4^{n=2\ (a)}=(q_0^{(a)},q_1^{(a)},q_2^{(a)},q_3^{(a)})$ is any row of $\pm G_{2P}$ and $\pm G_{4P}$ or $\pm jG_{4P}$, repectively.

$2_4^{n=1\ (b)}=(s_0^{(b)},s_1^{(b)},s_2^{(b)},s_3^{(b)})$ and $2_4^{n=2\ (b)}=(q_0^{(b)},q_1^{(b)},q_2^{(b)},q_3^{(b)})$ are generated from $2_4^{n=1\ (a)}$ and $2_4^{n=2\ (a)}$ respectively, where $s_\rho^{(b)} = (-1)^\rho s_\rho^{(a)}$, $q_\rho^{(b)} = (-1)^\rho q_\rho^{(a)}$ $(\rho=0,1,2,3)$.

Moreover, a pair of $\{2_8^{n=3\ (a)},\ 2_8^{n=3\ (b)}\}$ with ZCD of $(0.5\times 8+1)$ chips is defined as the initial 8-phase ZCD PP, where $2_8^{n=3\ (a)}$ is any row of $\pm G_{8P}$ or $\pm jG_{8P}$ and $2_8^{n=3\ (b)}=(d_0^{(b)},d_1^{(b)},d_2^{(b)},\cdots,d_7^{(b)})$ is also generated from $2_8^{n=3\ (a)}=(d_0^{(a)},d_1^{(a)},d_2^{(a)},\cdots,d_7^{(a)})$, where $d_\rho^{(b)} = (-1)^\rho d_\rho^{(a)}$ $(\rho=0,1,2,\cdots,7)$.

In utilizing initial $2^{n=1,2,3}$ phase ZCD PP and Eqn.(5), $2^{n=1,2,3}$ phase ZCD PP of longer period are constructed recursively. Thus, for the period of $N=4\times 2^i$ $(i=0,1,2,3\cdots$ or $i=1,2,3\cdots)$, $\{2_N^{n=1,2,3\ (a)},\ 2_N^{n=1,2,3\ (b)}\}$ with ZCD of $(0.5N+1)$ chips can be constructed.

.The 4-phase ZCD PP $\{2_{64}^{n=2\ (a)},\ 2_{64}^{n=2\ (b)}\}$ present the ZCD of $(0.5N+1)$chips as shown in Fig. 1

Step(ii): Construction of sets of $2^{n=1,2,3}$ phase ZCD codes

Let $2^{n=1,2,3}P(N,M,Z_L)$ represent a set of $2^{n=1,2,3}$ phase *ZCD* codes having a code period of *N* and family size *M*, where any pair of $2^{n=1,2,3}P(N,M,Z_L)$ has the common ZCD-length of Z_L. By using the chip-shift operation of $\{2_N^{n=1,2,3\ (a)},\ 2_N^{n=1,2,3\ (b)}\}=2^{n=1,2,3}P(N,\ 2,\ 0.5N+1)$, a set of $2^{n=1,2,3}P(N,\ M\geq 2,\ Z_L\leq 0.5N+1)$ can be constructed. Let T^l be the chip-shift operator, which shifts a code cyclically to the left by l chips, $2^{n=1,2,3}P(N,\ M\geq 2,\ Z_L\leq 0.5N+1)$ can be generated from $\{2_N^{n=1,2,3\ (a)},\ 2_N^{n=1,2,3\ (b)}\}$ as

$$\begin{aligned} &2^{n=1,2,3}P(N, M\geq 2, Z_L\leq 0.5N+1) = 2^{n=1,2,3}P(N,\ 2k+1, |2\Delta-1|) \\ &=\{\ 2_N^{n=1,2,3\ (a)},\ 2_N^{n=1,2,3\ (b)},\ T^{\Delta}[2_N^{n=1,2,3\ (a)}],\ T^{\Delta}[2_N^{n=1,2,3\ (b)}],\cdots \\ &,T^{(k-1)\Delta}[2_N^{n=1,2,3\ (a)}],\ T^{(k-1)\Delta}[2_N^{n=1,2,3\ (b)}],\ T^{k\Delta}[2_N^{n=1,2,3\ (a)}],\ T^{k\Delta}[2_N^{n=1,2,3\ (b)}]\ \} \end{aligned} \quad (6)$$

where Δ is a chip-shift increment and k the maximum number of chip-shifts for a code. Δ and k should satisfy $|(k+1)\Delta|\leq|0.25N+1|$ where Δ is a positive and k a non-negative integer The various examples of 2-phase ZCD code sets are shown in Table 1. Fig. 2 shows the family size against ZCD of $2^{n=1}$ phase (binary) *ZCD* codes with a period of 128. When ZCD > three chips, it is clear that proposed codes have larger family sizes than that of binary code pairs with ZCD [2] or QS(OG-r) codes generated from orthogonal Gold codes [3]. Here, large family sizes produce a larger CDMA user number and large ZCDs produce longer cell radius of MAI-canceled DS-CDMA systems, respectively.

3. Conclusion

We have proposed novel $2^{n=1,2,3}$ phase codes with sufficient ZCD and family sizes. The proposed $2^{n=1,2,3}$ phase codes can be usefully employed in the DS-CDMA systems with Interference Cancellation.

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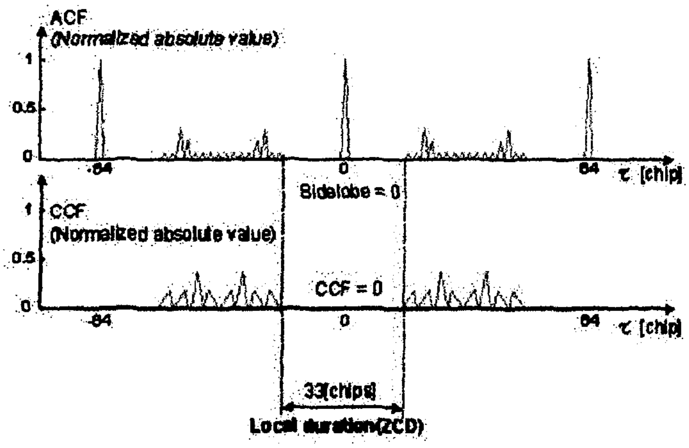

Fig. 1 ACF and CCF of a 4phase ZCD PP of period 64

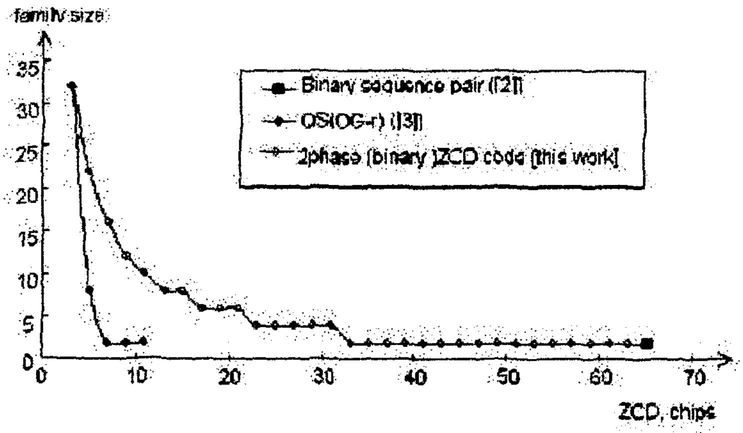

Fig. 2 Family size against ZCD of 2-phase ZCD codes of period 128

Table 1. Various examples of 2-phase ZCD code sets.

$2^{n=1}P(N=16, M=2, Z_L=9)$:

$$S_{16}^{a} = (\ +\ +\ -\ +\ +\ +\ +\ -\ -\ -\ +\ -\ +\ +\ +\ -\ )$$
$$S_{16}^{b} = (\ +\ -\ -\ -\ +\ -\ +\ +\ -\ +\ +\ +\ +\ -\ +\ +\ )$$

$2^{n=1}P(N=64, M=2, Z_L=33)$:

$$S_{64}^{(a)} = (u\ \ v\ \ u\ -v\ \ u\ \ v\ -u\ \ v\ \ u\ \ v\ \ u\ -v\ -u\ -v\ \ u\ -v)$$

$$S_{64}^{(b)} = (c\ \ d\ \ c\ -d\ \ c\ \ d\ -c\ \ d\ \ c\ \ d\ \ c\ -d\ -c\ -d\ \ c\ -d)$$

where $u=(---+)$, $v=(--+-)$, $c=(-+--)$, and $d=(-+++)$.

$2^{n=1}P(N=32, M=6, Z_L=6)$:

$$S_{32}^{1} = (--+-+++-++-++++-++-+---+++-++++-)$$
$$S_{32}^{2} = (-++++-+++---+-+++----+--+---+-++)$$
$$S_{32}^{3} = (-+++-++-++++-++-+---+++-++++---+)$$
$$S_{32}^{4} = (++-+++---+-+++----+--+---+-++-++)$$
$$S_{32}^{5} = (+-++-++++-++-+---+++-++++---+-++)$$
$$S_{32}^{6} = (+++---+-+++----+--+---+-++-++++-)$$