Impulse response shortening for DFE in single-carrier wideband transceivers

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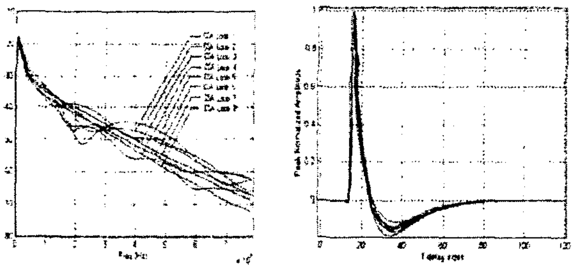
Abstract: This paper proposes an impulse response shortening algorithm applicable to decision feedback equalization of single carrier wideband signal. When the impulse response shortening methods for narrowband signaling are applied to single carrier wideband signals, they result in noise enhancement problem, significantly deterioriting the receiver performance. This problem can be alleviated by reducing the eigenvalue spread ratio of the impulse response, which can be achieved by adding additive white noise with small variance to the impulse response of the channel. The performance of the proposed scheme is verified by computer simulation.

1. Introduction

The use of decision feedback equalizer (DFE) has widely been applied to various communication systems as an asymptotically optimum one [1]. When the DFE is applied to a channel whose impulse response has a long post-cursor such as the CSA loops whose typical response is shown in Figure 1, the feedback filter (FBF) may need a long tap size [2]. The use of a large tap FBF requires a long training time as well as increased Implementation complexity. It can also suffer from so called error propagation problem. These problems can be alleviated by reducing the duration of the channel impulse response.

The concept of impulse response shortening (IRS) was originally introduced in the discrete multitone (DMT) data transmission throughput [3-5]. Since the bandwidth of each subchannel in the DMT system is very narrow, the postcursor of the channel impulse response can effectively be shortened by using a simple IRS filter (IRSF).

When this IRSF is directly applied to a wideband signal such as the high bit rate digital subscriber line (HDSL) as shown in Figure 2 [6], it can effectively reduce the post-cursor length, but it suffers form excessive noise due to large eigenvlaue spread ratio (ESR) of the channel, significantly deteriorating the receiver performance. To alleviate the noise enhancement problem with the use of the IRSF, it is necessary to make the IRSF have a reduced magnitude of frequency response in high frequency region. This can be achieved by reducing the ESR of the channel impulse response [7]. This paper proposes an efficient IRS algorithm for wideband signal transmission, that can significantly reduce the



(a) Frequency responses (b) Impulse responses

Figure 1. Response of typical CSA Loops

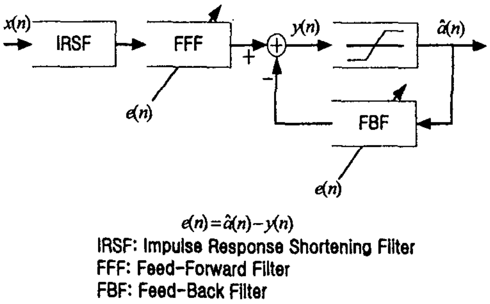


Figure 2. Structure of the DFE with the IRSF

tap size of the FBF in the DFE with minimal noise enhancement.

2. Impulse response shortening for DMT

An efficient impulse response shortening algorithm was proposed to reduce the length of the cyclic prefix in the DMT transceiver [4]. Let $h[n]$ be the impulse response of the channel. By employing an m-tap IRSF $w[n]$, the effective channel response is given by

$$\hat{h}[n] = h[n] * w[n] \tag{1}$$

where $*$ denotes the convolution process. The IRSF $w[n]$ can reduce the tap length of $\hat{h}[n]$ from L' to L, by making the first L components of $\hat{h}[n]$ have the most of the energy.

Let $\hat{h}_L[n]$ be the first L samples of $\hat{h}[n]$ and $\hat{h}_{\bar{L}}[n]$ be the remaining samples of $\hat{h}[n]$,

$$\hat{h}_L[n] = (u[n] - u[n-L])\hat{h}[n] \tag{2}$$

$$\hat{h}_{\bar{L}}[n] = \hat{h}[n] - \hat{h}_L[n] \tag{3}$$

where $u[n]$ is the unit step function. Letting $\mathbf{A}$ and $\mathbf{B}$ be the correlation matrix of $\hat{h}_L[n]$ and $\hat{h}_{\bar{L}}[n]$, respectively, we have

$$\mathbf{A} = E\left\{\hat{\mathbf{H}}_L^T \hat{\mathbf{H}}_L\right\} \tag{4}$$

$$\mathbf{B} = E\left\{\hat{\mathbf{H}}_{\bar{L}}^T \hat{\mathbf{H}}_{\bar{L}}\right\} \tag{5}$$

$$\mathbf{C} = (\sqrt{\mathbf{A}})^{-1}\mathbf{B}(\sqrt{\mathbf{A}}^T)^{-1} \tag{6}$$

where $E\{\ \}$ denotes the expectation process, $\hat{\mathbf{H}}_L$ and $\hat{\mathbf{H}}_{\bar{L}}$ are respectively $L \times m$ and $(L'-L) \times m$ matrix of $\hat{h}_L[n]$ and $\hat{h}_{\bar{L}}[n]$, and $(\sqrt{\mathbf{A}})^{-1}$ and $(\sqrt{\mathbf{A}}^T)^{-1}$ respectively denote the Cholesky decomposed upper and lower triangular matrix of $\mathbf{A}$.

Defining ξ_L by the power ratio of $\hat{h}_L[n]$ to $\hat{h}_{\bar{L}}[n]$,

$$\xi_L(L) \equiv \frac{\sum_{n=0}^{L-1} \hat{h}^2[n]}{\sum_{n=L}^{\infty} \hat{h}^2[n]} \tag{7}$$

the optimum IRSF can be designed so as to maximize the ratio ξ_L. The optimum coefficient of the IRSF is determined by [4]

$$\mathbf{w}_{opt} = (\sqrt{\mathbf{A}}^T)^{-1}\mathbf{e}(\lambda_{\min}) \tag{8}$$

where $\mathbf{e}(\lambda_{\min})$ is the eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$ of $\mathbf{C}$. The corresponding maximum ξ_L is [4].

$$\begin{aligned} \xi_L &= \frac{\mathbf{w}_{opt}^T \mathbf{A} \mathbf{w}_{opt}}{\mathbf{w}_{opt}^T \mathbf{B} \mathbf{w}_{opt}} \\ &= \frac{1}{\lambda_{\min}} \end{aligned} \tag{9}$$

3. Proposed impulse response shortening filter

When the IRSF for the DMT is applied to a wideband signal, it can cause excessive noise enhancement due to large eigenvalue spread ratio (ESR). To alleviate the noise enhancement problem with the use of the IRSF of (8), it is necessary to make the IRSF have a reduced magnitude of frequency response in high frequency region. This can be achieved by reducing the ESR of the channel response $\hat{h}[n]$.

The ESR can be reduced by adding white noise $\delta[n]$ to $\hat{h}[n]$ [7],

$$\hat{h}_\delta[n] = \hat{h}[n] + \delta[n] \tag{10}$$

where the variance σ_δ^2 of $\delta[n]$ should be small enough not to affect the impulse response of $\hat{h}[n]$. Assuming that $\delta[n]$ is independent and identically distributed, the corresponding composite matrix $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ can be represented as

$$\begin{aligned} \mathbf{A}_\delta &= E\left\{\hat{\mathbf{H}}_{\delta,L}^T \hat{\mathbf{H}}_{\delta,L}\right\} \\ &= \mathbf{A} + \sigma_\delta^2 \mathbf{I} \approx \mathbf{A} \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbf{B}_\delta &= E\left\{\hat{\mathbf{H}}_{\delta,\bar{L}}^T \hat{\mathbf{H}}_{\delta,\bar{L}}\right\} \\ &= \mathbf{B} + \sigma_\delta^2 \mathbf{I} \not\approx \mathbf{B} \end{aligned} \tag{12}$$

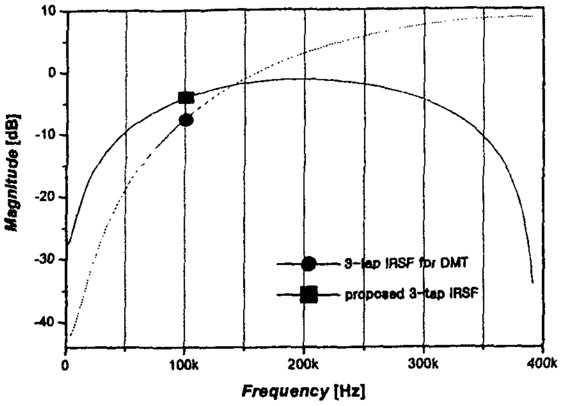


Figure 3. Frequency response of 3-tap IRSF

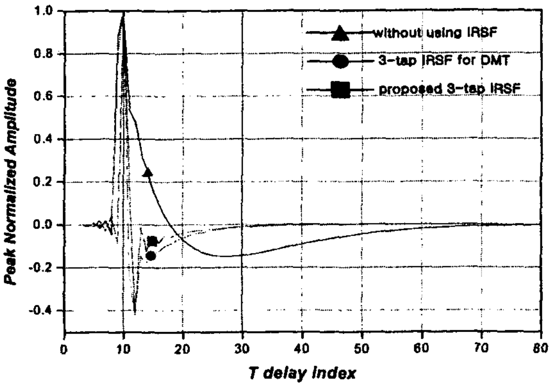


Figure 4. Impulse response of the channel with and without 3-tap IRSF

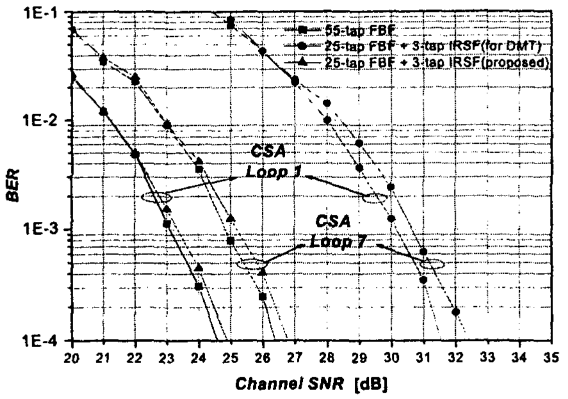


Figure 5. BER performance of HDSL with the use of 3-tap IRSFs

$$\begin{aligned}\mathbf{C}_\delta &= (\sqrt{\mathbf{A}_\delta})^{-1}\mathbf{B}_\delta(\sqrt{\mathbf{A}_\delta}^{T})^{-1}\\ &\approx (\sqrt{\mathbf{A}})^{-1}\mathbf{B}_\delta(\sqrt{\mathbf{A}}^{T})^{-1}\end{aligned} \tag{13}$$

where $\mathbf{I}$ denotes the unit diagonal matrix. The addition of $\delta[n]$ to $h[n]$ results in increase of the eigenvalues of $\mathbf{C}_\delta$ by σ_δ^2, reducing the ESR. Figure 3 depicts the frequency response of 3-tap IRSFs when applied to the CSA loop 1 in the HDSL [2], The proposed IRSF is designed using (10), where white additive noise $\delta[n]$ with $\sigma_\delta^2 = -20\text{dB}$ is added. It can be seen in Figure 4 that the proposed IRSF can shorten the impulse response as much as the IRSF for the DMT can. To evaluate the performance, both the IRSFs are applied to the HDSL receiver employing the DFE. Figure 5 depicts the the performance of the HDSL in terms of the bit error rate when applied to the CSA loop 1 and 7. It can be seen that the tap size of the FBF in the DFE can be reduced from 55 to 25 with the use of the proposed 3-tap IRSF at the expense of an SNR loss of a fractional dB, while yielding an SNR loss of about 7dB with the use of the IRSF for the DMT.

4. Conclusion

In this paper, we have designed an IRS filter applicable to wideband signal transmission. To alleviate the noise enhancement problem with the use of an IRSF for narrowband signals, the impulse response of the channel is perturbed by zero mean additive white noise with small variance. Numerical results show that the proposed IRSF with a few tap can reduce the tap size of the FBF of the DFE by more than half at the expense of an SNR loss of a fractional dB.

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