A Companding Scheme for PAPR Reduction in OFDM Systems*

Ju-Tak Han[1] and Yoan Shin[2]

[1] School of Electronic Engineering, Soongsil University,
5 Sangdo-dong, Dongjak-gu, Seoul, 156-743, Korea
Tel. : +82-2-817-1539, Fax.: +82-2-821-7653, E-mail : dynohan@amcs.ssu.ac.kr
[2] School of Electronic Engineering, Soongsil University,
5 Sangdo-dong, Dongjak-gu, Seoul, 156-743, Korea
Tel. : +82-2-820-0632, Fax.: +82-2-821-7653, E-mail : yashin@e.ssu.ac.kr

Abstract : We propose in this paper a companding scheme for peak-to-average power ratio (PAPR) reduction in orthogonal frequency division multiplexing (OFDM) systems. By exploiting statistical distribution of OFDM transmit signals, the proposed scheme effectively reduces the PAPR by compressing the peak signals, while maintaining the average power unchanged. Simulation results are provided to show good performance of the proposed scheme.

1. Introduction

Recently, there has been much interest in OFDM for digital broadcasting systems and high speed wireless data networks [1]. In the OFDM systems, a given set of signals is transmitted on several orthogonal subcarriers, where phase shift keying (PSK) or quadrature amplitude modulation (QAM) scheme is usually employed. As compared to single carrier systems, the OFDM system is more robust to severe frequency selective fading. Moreover, digital transmitter and receiver can be implemented efficiently with utilization of the fast Fourier transform (FFT) algorithm.

Most radio systems employ high power amplifiers (HPAs) in the transmitter to obtain sufficient transmit power. For the purpose of achieving the maximum output power efficiency, the HPA is usually operated at or near the saturation region and this introduces nonlinear distortion [2]. The nonlinear characteristic of the HPA is very sensitive to variation in transmit signal amplitudes. Unfortunately, the variation of OFDM transmit signal amplitudes is very wide with large PAPR [2], which requires expensive HPAs with very good linearity. In addition to this, large PAPR also demands analog-to-digital converters (ADCs) with large dynamic range. Moreover, OFDM transmit signals exhibit Gaussian distribution for large number of subcarriers [2], which means that peak signals quite rarely occur and uniform quantization by the ADCs is not desirable.

One of several schemes for PAPR reduction in OFDM systems is clipping [3] which is simple and effective, however it causes additional clipping noise that degrades the system performance. As an alternative approach, a companding (*comp*ression in transmitter and ex*panding* in receiver) technique, originally rooted from speech signal processing, has been proposed recently by Wang *et al.* in [4]. The companding scheme in [4] shows better performance than the clipping. However, the average signal power increases after the compression [5], and the compressed signals still exhibit non-uniform distributions.

In this paper, we propose a companding scheme that can remedy the drawbacks of the one in [4]. Key idea of the proposed scheme is to exploit statistical distribution of OFDM transmit signals, which is well modeled by Gaussian distribution for large number of subcarriers. The proposed compression algorithm transforms the distribution of the transmit signals, which is assumed to be Gaussian, into uniform distribution with the same average power.

The remainder of the paper is organized as follows. Section 2 describes baseband OFDM system models considered in this paper. In Section 3, the companding algorithm in [4] is discussed, and then, a new companding scheme is proposed. In Section 4, performance of the proposed scheme is demonstrated by computer simulation followed by concluding remarks in Section 5.

2. OFDM Systems

Fig. 1 shows block diagram of a QAM-modulated discrete-time baseband OFDM system under additive white Gaussian noise (AWGN) channel, where companding scheme is incorporated. In the transmitter, a group of $N\log_2 M$ input bits is encoded into a block of N QAM symbols $X[l]$ $(l=0,\cdots,N-1)$, where symbol duration is T_s (sec) and M -ary QAM is considered. These N symbols are serial-to-parallel converted and modulated using N orthogonal subcarriers $\{e^{j2\pi f_0 t},\cdots,e^{j2\pi f_{N-1}t}\}$ with the l-th subcarrier frequency $f_l \equiv l/NT_s$ (Hz). Thus, the OFDM signal $x(t)$ for a block of duration NT_s (sec) can be expressed as

$$x(t)=\frac{1}{N}\sum_{l=0}^{N-1}X[l]e^{j2\pi lt/NT_s}=\frac{1}{N}\sum_{l=0}^{N-1}X[l]e^{j2\pi lt/NT_S} \quad (1)$$

By discretizing $x(t)$ at $t=nT_s$ $(n=0,\cdots,N-1)$, we have the discrete-time equivalence of the OFDM signal as

$$x[n]\equiv x(nT_s)=\frac{1}{N}\sum_{l=0}^{N-1}X[l]e^{j2\pi nl/N} \quad (2)$$

Eq. (2) is equivalent to N-point inverse discrete Fourier transform of the N QAM symbols $X[l]$, followed by parallel-to-serial conversion. Thus, a fast implementation using inverse fast Fourier transform (IFFT) can be utilized. In the receiver of the system, converse functions of the

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* This work was financially supported by ITRC program of Korean Ministry of Information & Communication.

transmitter are performed as shown in Fig. 1. In particular, subcarrier demodulation can be efficiently implemented by N-point FFT.

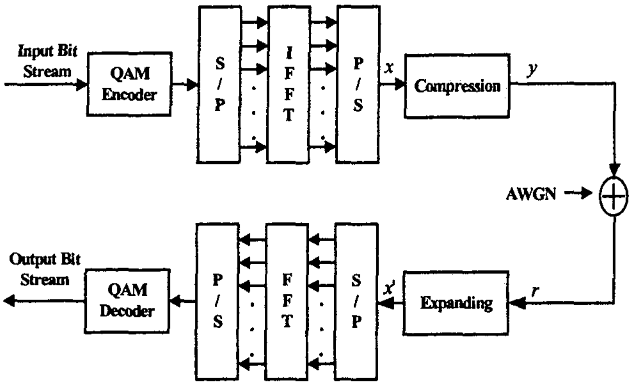

Figure 1. Discrete-time baseband OFDM system with companding scheme.

Since an OFDM transmit signal is obtained after subcarrier modulation block (i.e., IFFT block) as a sum of many subcarrier signals, variation of its amplitudes becomes wide, that is, the PAPR of the signal becomes large. The large PAPR requires expensive HPAs with highly linear characteristic and ADCs with large dynamic range. In addition, OFDM transmit signals follow Gaussian distribution for large number of subcarriers [2]. This implies that, although the PAPR is large, peak signals rarely occur and uniform quantization by the ADCs is not effective. Hence, it is important to reduce the PAPR in the OFDM systems, and we consider companding schemes for this purpose in the next section.

3. Proposed Companding Scheme

In this paper, we consider discrete multi-tone (DMT)-type OFDM systems with real transmit signals [6]. In this type of OFDM systems, the input symbols to the IFFT block in the transmitter take the following relationship to have the real IFFT outputs.

$$X[N-k]=X^*[k],\quad k=1,\cdots,N/2-1 \tag{3}$$

$$X[0]=X[N/2]=0 \tag{4}$$

As for the companding scheme, we first consider the conventional μ-law companding scheme by Wang *et al.* in [4]. Since companding is applied on a symbol-by-symbol basis, we ignore discrete-time index n and denote the OFDM symbol as $x\equiv x[n]$ for the purpose of notational simplicity. Then, the output $y\equiv y[n]$ of the compression algorithm of the μ-law companding in the OFDM transmitter can be described by [4]

$$y=\frac{A\,\mathrm{sgn}(x)\ln\left[1+\mu\left|\frac{x}{A}\right|\right]}{\ln(1+\mu)} \tag{5}$$

where $\mathrm{sgn}(\cdot)$ is signum function, A is a normalization constant such that $0\le\left|\frac{x}{A}\right|<1$, and $\mu>0$. Similarly, the output $x'\equiv x'[n]$ of the expanding algorithm in the receiver is given by [4]

$$x'=\frac{A\exp\left\{\frac{r}{A\,\mathrm{sgn}(y)}\ln(1+\mu)\right\}-A}{\mathrm{sgn}(y)\mu} \tag{6}$$

where $r\equiv r(n)$ is the input to the receiver. Note that for sufficiently large N, Eq. (5) can be approximated as [5]

$$y\approx x\frac{\mu}{\ln(1+\mu)} \tag{7}$$

This means that average signal power after the compression increases. Fig. 2 shows probability density functions (pdfs) of the signals before and after the compression algorithm in [4]. In the figure, we consider OFDM system with $N=512$ subcarriers and 16-QAM modulation. Gaussian nature of the OFDM signals can be easily observed. Moreover, the compressed signals still exhibit non-uniform distribution.

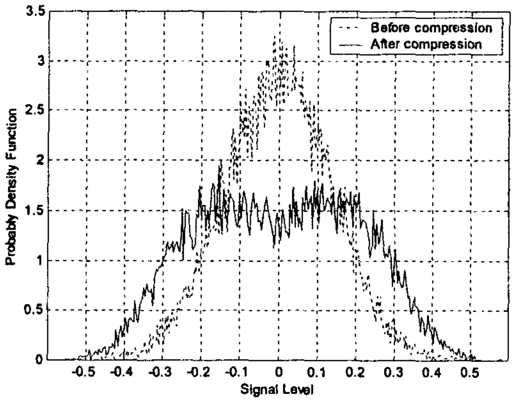

Figure 2. The pdfs of the signals before and after the compression function in [4] for OFDM system with $N=512$ subcarriers and 16-QAM modulation.

In this paper, we propose a companding scheme that can remedy the drawbacks of the one in [4]. Key idea of the proposed scheme is to exploit statistical distribution of OFDM transmit signals, which is well modeled by Gaussian distribution for large number of subcarriers. For DMT-type OFDM systems with real transmit signals, the proposed compression algorithm transforms the distribution of the transmit signals, which is assumed to be Gaussian, into uniform distribution with the same average power.

Let us denote X and Y as random variables (r.v.s) representing the inputs and the outputs of the compression function $C(\cdot)$ with the cumulative distribution functions (cdfs) $F_X(x)$ and $F_Y(y)$, respectively. Since Y is to be uniformly distributed in the interval $[-K,K]$, the cdf of Y is given by

$$F_Y(y)=\begin{cases}0 & x<-K\\ \frac{1}{2K}x+\frac{1}{2} & -K\le x<K\\ 1 & x\ge K\end{cases} \tag{8}$$

By transformation relationship of r.v.s [7], we can obtain the following identity

$$y = C(x) = F_Y^{-1}\{F_X(x)\} \qquad (9)$$

Since the r.v. X for the OFDM signals is supposed to be Gaussian with mean 0 and variance (or average power) σ^2, its cdf becomes

$$F_X(x) = 1 - \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}\sigma}\right) \qquad (10)$$

with $erfc(z) \equiv \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2}\, dt$. Hence, from Eqs. (8) – (10) we have

$$y = C(x) = K\left\{1 - erfc\left(\frac{x}{\sqrt{2}\sigma}\right)\right\} \qquad (11)$$

Note that the variance of Y is $K^2/3$ and the peak power is K^2, yielding the PAPR of the proposed scheme as

$$10\log_{10}\left(\frac{K^2}{K^2/3}\right) \approx 4.8\ \text{dB} \qquad (12)$$

regardless of K, the number of subcarriers or modulation schemes used in the OFDM transmitter. Moreover, to maintain the same average power after the compression, K may take the value as $K = \sqrt{3}\sigma$. Fig. 3 depicts the proposed compression function $C(\cdot)$ for the OFDM signals modeled as zero-mean Gaussian with the variance of 25. For the purpose of comparison, the compression function by the scheme in [4] is also depicted in the figure. Note from the figure that, by transformation of signal distribution, the proposed scheme effectively combines the conventional clipping with the companding.

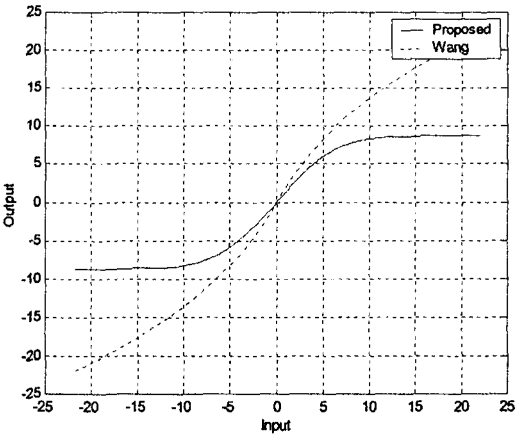

Figure 3. Compression functions $C(\cdot)$ of the proposed companding scheme and the one in [4] for the OFDM signals modeled as zero-mean Gaussian with the variance of 25.

The expanding function $E(\cdot)$ in the receiver can be obtained as an inverse of $C(\cdot)$. In addition, to prevent some numerical problem due to excessive channel noises, the magnitudes of the inputs to the expanding function are intentionally limited to $S \cdot K$ for some real value $S \approx 1$. Thus, the output x' of the expanding function with the input r is given by

$$x' = \begin{cases} \sqrt{2}\sigma erfc^{-1}\left(1 - \frac{r}{K}\right), & r < S \cdot K \\ \sqrt{2}\sigma erfc^{-1}(S \cdot \text{sgn}(r)), & \text{otherwise} \end{cases} \qquad (13)$$

4. Simulation Results

To verify the performance of the proposed companding scheme, computer simulations have been performed. In the simulation, we have considered a baseband DMT-type OFDM system as in Eqs. (3) and (4) with $N=512$ subcarriers for which subcarrier modulator and demodulator were implemented by using 512-point IFFT and FFT, respectively. For the modulation scheme, 16-QAM was employed. Moreover, we assume AWGN channels without any multipath impairment.

Figs. 4 and 5 depict actual signals and their pdfs before and after the proposed compression function. We observe that the distribution of the signals is effectively changed after the compression. Fig. 6 shows bit error rate (BER) performances according to transmit signal-to-noise ratio (Tx SNR) for different values of the saturation parameter S in the proposed expanding function. In the figure, $S = 0.975$ achieves the best performance. Fig. 7 depicts BER performances for different numbers k of quantization bits of the ADC in the receiver. We observe that the performance of the system is not significantly improved for k larger than 9. Fig. 8 compares BER performances of the proposed companding scheme with $S = 0.975$ and the one by Wang *et al.* in [4]. In both systems, we assume $k = 9$-bit uniform quantization in the ADC. The PAPR obtained by the scheme of [4] is 7.8 dB, while the proposed scheme achieves the PAPR of 4.8 dB as mentioned earlier. Moreover, the figure shows that the BER of the proposed scheme is much better than the scheme of [4], which highlights the effectiveness of the proposed companding scheme.

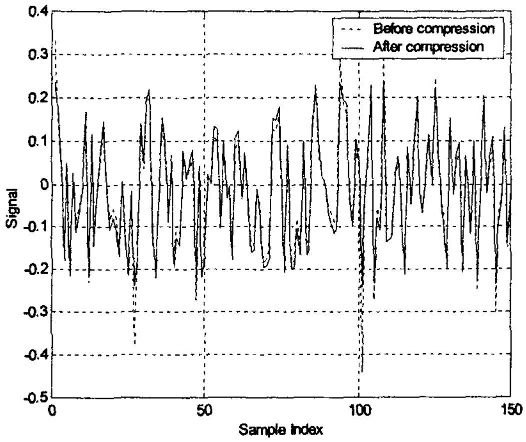

Figure 4. Actual signals before and after the proposed compression function.

5. Conclusion

In this paper, we have proposed a companding scheme for PAPR reduction in OFDM systems. The proposed scheme exploits statistical distribution of OFDM transmit signals, which is well modeled by Gaussian distribution, and transforms the distribution into uniform distribution with the same average power. Simulation results show that the proposed scheme achieves much less PAPR with better BER performance as compared to the conventional scheme in [4].

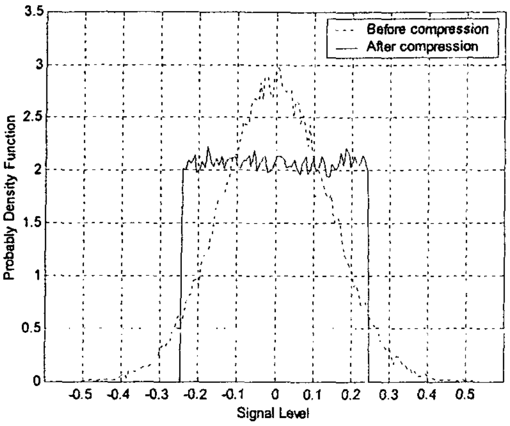

Figure 5. The pdfs of the signals before and after the proposed compression function.

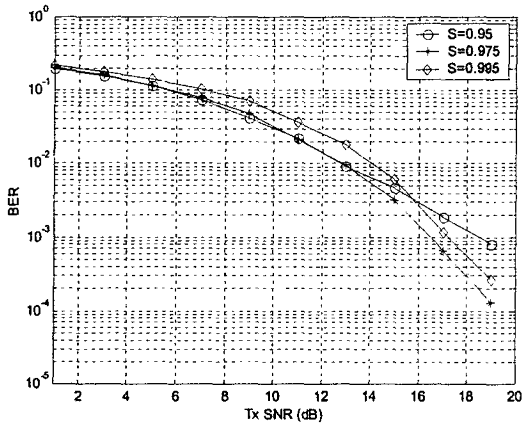

Figure 6. BER performances of OFDM systems employing the proposed companding scheme for different values of the saturation parameter S in the expanding function.

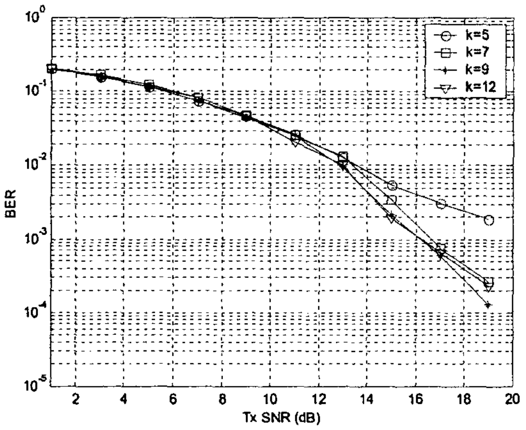

Figure 7. BER performances of OFDM systems employing the proposed companding scheme for different numbers k of quantization bits of the ADC in the receiver.

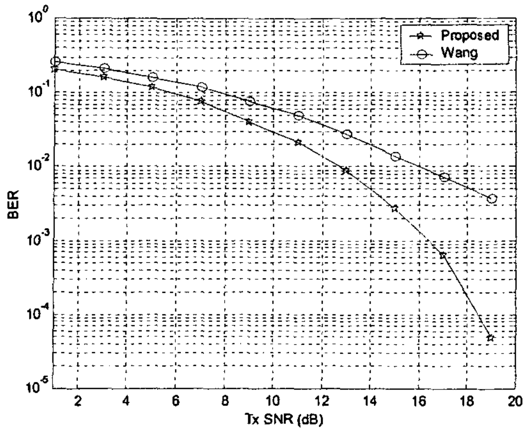

Figure 8. BER performances of the proposed companding scheme with $S = 0.975$ and the one in [4] ($k = 9$-bit uniform quantization in the ADC is assumed).

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