Synthesis of a Complex R^{i}CR filter with finite transmission zeros

Hidehiro Kikuchi*, Yukio Ishibashi**

*Doctoral Program in System & Information Engineering, University of Tsukuba
**Institute of Electronics and Information Sciences, University of Tsukuba
1-1-1 Tennoudai, Tsukuba, Ibaraki 305-8573, Japan
Phone & Fax: +81-298-53-5565 Email: ishibash@is.tsukuba.ac.jp

Abstract — This paper describes synthesis of a complex R^{i}CR filter with a finite transmission zero except zero frequency. First, a new kernel function is proposed. Secondly, how to determine the element values included in the R^{i}CR filter is described. A fifth-order R^{i}CR filter is designed. Finally, the sensitivity property of the proposed filter is evaluated through computer simulation.

Keywords: *Complex signal processing, R^{i}CR filter, finite transmission zero, complex filter and equiripple*

1. Introduction

Recently, many techniques concerned with the complex filters have been presented, not only in the field of digital circuits but also in the field of analog circuits [1],[2]. The frequency response of the complex filter is asymmetrical with respect to the D.C. axis. This property is important for the application to the orthogonal communication systems and so on.

The authors have proposed synthesis method of the complex R^{i}CR filter [1]. This filter includes imaginary resistors, capacitors and terminating resistors. In this literature, although it has the equiripple passband response, it does not have a finite transmission zero except zero frequency.

In this paper, we propose synthesis of an R^{i}CR filter with a finite transmission zero except zero frequency. First, a new kernel function is proposed. An algorithm for determining the required kernel function is presented. Secondly, how to determine the element values from the resulting kernel function is described. As an example, a fifth-order R^{i}CR filter is designed. Finally, the frequency characteristics and the sensitivity characteristics are estimated by using a computer.

2. Kernel function

2.1 Outline of $f(x)$

Figure 1 shows the proposed fifth-order kernel function $f(x)$. The required form of $f(x)$ is

$$f(x) = \frac{ax^5 + bx^4 + cx^3 + dx^2 + gx + h}{(x/z+1)^2\,(x/z-1)^2} \tag{1}$$

where a, b, c, d, g, h are real constants and z is a given constant. In order for $f(x)$ to have an outline shown in Fig.1, $f(x)$ should satisfy the following conditions.

$$f(m_n) = (-1)^{n+1} \quad n = 0..5 \tag{2}$$

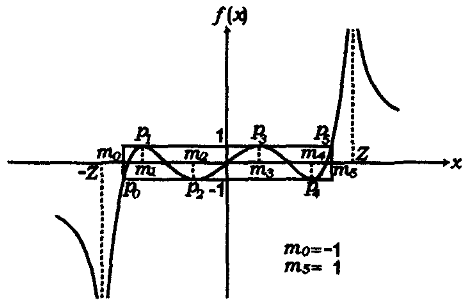

Figure 1: Kernel function $f(x)$.

$$f'(m_k) = 0 \quad k = 1..4 \tag{3}$$

where $m_0 ... m_5$ are real, $m_0 = -1$ and $m_5 = 1$. From Eqs.(1)–(3), we find that there are 10 unknown constants (a, b, c, d, g, h, m_1, m_2, m_3 and m_4) and that there are 10 conditional equations given by Eq.(2) and Eq.(3). Therefore, we can determine all the unknown constants by solving the simultaneous equations given by Eq.(2) and Eq.(3). In order to solve these simultaneous equations, we may use Newton method. When we use this method, it is necessary to try the various initial values until the correct solutions are obtained, which may come to be unacceptable in the higher order filter.

2.2 Algorithm using Lagrange interpolation

We introduce the following polynomial function $g(x)$.

$$g(x) = (x/z+1)^2\,(x/z-1)^2\,f(x) \tag{4}$$

The resulting $g(x)$ becomes a fifth-order polynomial and touches the curves C_{top} and $C_{bott.}$ as shown in Fig.2. The curves C_{top} and $C_{bott.}$ are given by the following equations, respectively.

$$\begin{aligned} C_{top}(x) &= (x/z+1)^2\,(x/z-1)^2 \\ C_{bott.}(x) &= -(x/z+1)^2\,(x/z-1)^2 \end{aligned} \tag{5}$$

Let us consider $g_1(x)$ which passes through the points p_1, p_2, p_3 and p_4 properly given as shown in Fig.3. Because p_0, p_1, p_2, p_3 p_4 and p_5 are on C_{top} or $C_{bott.}$, their coordinates are given by

$$\left.\begin{array}{ll} p_0(m_0, y_0), & y_0 = -(m_0/z+1)^2(m_0/z-1)^2 \\ p_1(m_1, y_1), & y_1 = (m_1/z+1)^2(m_1/z-1)^2 \\ p_2(m_2, y_2), & y_2 = -(m_2/z+1)^2(m_2/z-1)^2 \\ p_3(m_3, y_3), & y_3 = (m_3/z+1)^2(m_3/z-1)^2 \\ p_4(m_4, y_4), & y_4 = -(m_4/z+1)^2(m_4/z-1)^2 \\ p_5(m_5, y_5), & y_5 = (m_5/z+1)^2(m_5/z-1)^2 \end{array}\right\} \quad (6)$$

where $m_0 = -1$ and $m_5 = 1$. When we use Lagrange interpolation, $g_1(x)$ which passes through the points p_0, p_1, p_2, p_3, p_4 and p_5 is given by the following equation.

$$g_1(x) = \sum_{i=0}^{5}\left(\prod_{i\neq j}\frac{x-m_k}{m_i-m_k}\right) \quad (7)$$

The function $g_1(x)$ is shown as a solid line in Fig.3. In this stage, $g_1(x)$ differs from a desired $g(x)$. Now, some of the solutions obtained from the following equations indicate the abscissas of the points q_1, q_2, q_3 and q_4 shown in Fig.3.

$$\left.\begin{array}{lllll} dg_1(x)/dx & = & dC_{top}(x)/dx & = & 1 \\ dg_1(x)/dx & = & dC_{bott.}(x)/dx & = & -1 \end{array}\right\} \quad (8)$$

The lines lp_1, lp_2, lp_3 and lp_4 are the perpendicular lines drawn from the points q_1, q_2, q_3 and q_4 toward the x-axis, respectively. The lines lp_1 and C_{top} intersect at the point p'_1. Similarly, the lines lp_2 and $C_{bott.}$ intersect at the point p'_2. And so on. As shown in Fig.3, $g_2(x)$ is generated by using Lagrange interpolation in such a way that $g_2(x)$ passes through the point p_0, p'_1, p'_2, p'_3, p'_4 and p_5. From this figure, we find that the difference between $g(x)$ and $g_2(x)$ is much smaller than the difference between $g(x)$ and $g_1(x)$. Therefore, if we exchange $g_1(x)$ with $g_2(x)$ and continue the above procedure, $g(x)$ can be obtained.

The above procedure gives the following algorithm.

1. Properly set the initial values m_1, m_2, m_3 and m_4. (For example, $m_1 = -3/5$, $m_2 = -1/5$, $m_3 = 1/5$, $m_4 = 3/5$)

2. By using Lagrange interpolation, generate $g_1(x)$ which passes through the points p_0, p_1, p_2, p_3, p_4 and p_5 whose coordinates are given by Eq.(6).

3. Draw the perpendicular lines lp_1, lp_2, lp_3 and lp_4 whose abscissas are given by solving Eq.(8).

4. As shown in Fig.3, find the coordinates of p'_1, p'_2, p'_3 and p'_4.

5. Exchange p_1, p_2, p_3 and p_4 with p'_1, p'_2, p'_3 and p'_4, respectively.

6. Continue the above procedures **2.** - **5.** until all of m_1, m_2, m_3 and m_4 converge.

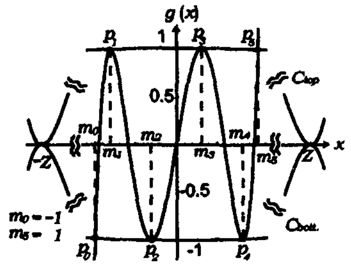

Figure 2: $g(x)$

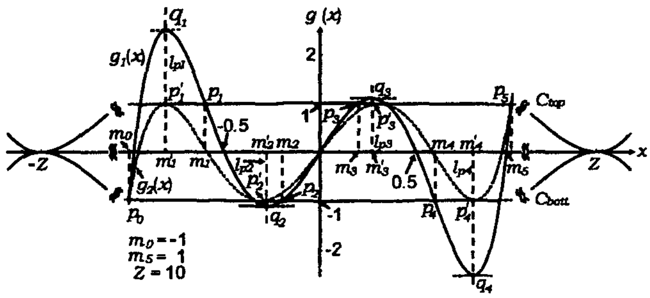

Figure 3: $g_1(x)$ and $g_2(x)$.

3. Relationship between $T(s)T^*(-s)$ and $\rho(s)\rho^*(-s)$

The kernel function $f(x)$ determined in Sec.2 is for the complex LPF. In order to obtain the kernel function which is suitable for the R[i]CR filter, we introduce $f_b(x)$ given by the following formula.

$$f_b(x) = f(x-z) \quad (9)$$

The passband edges of the resulting complex BPF are given by the following equations.

$$\begin{array}{lll} \text{Lower passband edge} & : & f_L = z-1 \\ \text{Higher passband edge} & : & f_H = z+1 \end{array} \quad (10)$$

It is should be noted that a squared magnitude function of the complex filter is given by $T(s)T^*(-s)$, where the symbol * denotes the complex conjugate. For an example, when the complex transfer function $T(s)$ is decomposed into $T_R(s) + jT_I(s)$, the complex conjugate $T^*(s)$ is given by $T_R(s) - jT_I(s)$ [2]. The squared magnitude function $T(s)T^*(-s)$ of the complex filter is given by

$$T(s)T^*(-s) = \frac{H^2}{1+\varepsilon^2 f_b^2(s/j)} \quad (11)$$

where H is real, $\varepsilon^2 = \alpha^2 - 1$. α is the passband ripple. The reflection coefficient $\rho(s)$ is given by

$$\rho(s)\rho^*(-s) = 1 - 4(R_S/R_L)T(s)T^*(-s) \quad (12)$$

where R_S and R_L are the input and the load resistances, respectively. Let us consider the relationships between $f_b(s/j)$ and $\rho(s)\rho^*(-s)$. Substituting Eq.(11) into Eq.(12) and eliminating $T(s)T^*(-s)$ give

$$\rho(s)\rho^*(-s) = \frac{\varepsilon^2 f_b^2(s/j) + 1 - 4(R_S/R_L)H^2}{1 + \varepsilon^2 f_b^2(s/j)} \tag{13}$$

We find that the transfer gain becomes maximum when $f_b(s/j) = 0$ in Eq.(11). Because the reflection coefficient $\rho(s)\rho^*(-s)$ must be zero at the maximum-gain frequency , we have

$$1 - 4(R_S/R_L)H^2 = 0 \tag{14}$$

Therefore, the relationship between $f_b(s/j)$ and $\rho(s)\rho^*(-s)$ leads to

$$\rho(s)\rho^*(-s) = \frac{\varepsilon^2 f_b^2(s/j)}{1 + \varepsilon^2 f_b^2(s/j)} \tag{15}$$

The input impedance $Z_{in}(s)$ of the R^{i}CR filter is given by

$$Z_{in}(s) = \frac{1 + \rho(s)}{1 - \rho(s)} R_S \tag{16}$$

4. Element values

4.1 Shunt arm (jG_1 and C_1)

The fifth-order R^{i}CR filter is shown in Fig.4. The input admittance $Y_{in}(s)$ seen from the port 1-1' is given by $Y_{in}(s) = 1/Z_{in}(s)$. When $s = 0$, $Y_{in}(s)$ becomes jG_1.

$$jG_1 = Y_{in}(0) \tag{17}$$

The remainder function $Y_2(s)$ is

$$Y_2(s) = Y_{in}(s) - jG_1 \tag{18}$$

Because s_∞ is a transmission zero except zero frequency, the element value of C_1 is given by

$$C_1 = \frac{Y_2(s_\infty)}{s_\infty} \tag{19}$$

The remainder function $Y_3(s)$ is

$$Y_3(s) = Y_2(s) - sC_1 \tag{20}$$

4.2 Series arm (jR_2, C_2 and C_3)

In Fig.4, the remainder impedance $Z_3(s)$ is given by $Z_3(s) = 1/Y_3(s)$. When $s \to s_\infty$, $(s - s_\infty)Z_3(s)$ becomes $C_3/C_2\,(C_2 + C_3)$.

$$\frac{C_3}{C_2\,(C_2 + C_3)} = \lim_{s \to s_\infty} (s - s_\infty)\, Z_3(s) \tag{21}$$

When $s \to 0$, $sZ_3(s)$ becomes $1/\,(C_2 + C_3)$.

$$\frac{1}{C_2 + C_3} = \lim_{s \to 0} sZ_3(s) \tag{22}$$

Solving the simultaneous equations, we have C_2 and C_3. The Resistance R_2 is obtained from the following formula.

$$R_2 = \frac{j\,(C_2 + C_3)}{C_2 C_3 s_\infty} \tag{23}$$

The remainder function is

$$Z_4(s) = Z_3(s) - \frac{1 + jsC_3R_2}{s\,(C_3 + C_2 + jsC_2C_3R_2)} \tag{24}$$

4.3 The remainder arm

Applying the same fashion as the above to the input admittance $Y_4(s)$ seen from the port 2-2', we can obtain G_3, R_4, C_4, C_5, C_6, $Y_5(s)$, $Y_6(s)$ and $Z_7(s)$. The input admittance $Y_7(s) = 1/Z_7(s)$ directly indicates the admittance of the last shunt arm.

$$Y_7(s) = 1/Z_7(s) = jG_5 + sC_7 + 1/R_L \tag{25}$$

5. Design examples

In order to show the design example, we design the R^{i}CR filter which satisfies the following specifications.

A fifth-order R^{i}CR filter

Passband ripple α	1 dB
Passband	9 rad/s – 11 rad/s

From Eq.(10), we find that $z = 10$. Using the algorithm described in Paragraph 2.2 gives the following kernel function.

$$f(x) = \frac{158402.x^5 - 198400.x^3 + 49799.5x}{(x+10)^2\,(x-10)^2} \tag{26}$$

Substituting the above equation into Eq.(9) leads to

$$f_b(x) = \frac{\left\{10^5 \left(\begin{array}{l} 1.58402x^5 - 79.201x^4 + 1582.04x^3 \\ -1580.7x^2 + 78606.3x - 156423. \end{array}\right)\right\}}{x^4 - 40x^3 + 400x^2} \tag{27}$$

From the specifications, we have $\varepsilon^2 = 0.258925$. Substituting the above equation into Eq.(15) leads to

$$\rho(s)\rho^*(-s) = \frac{\left\{\begin{array}{l} (s - 9.04858j)^2(s - 9.41067j)^2(s - 10.j)^2 \\ (s - 10.5893j)^2(s - 10.9514j)^2 \end{array}\right\}}{\left\{\begin{array}{l} s^{10} - 100.js^9 - 4497.49s^8 + 119800.0js^7 \\ +2.09299 \times 10^6 s^6 - 2.50599 \times 10^7 js^5 \\ -2.0825 \times 10^8 s^4 + 1.18602 \times 10^9 js^3 \\ +4.43019 \times 10^9 s^2 - 9.80091 \times 10^9 js \\ -9.75169 \times 10^9 + 148104s^2 \\ -589051js - 975193 \end{array}\right\}} \tag{28}$$

Factorizing $\rho(s)\rho^*(-s)$ so that all the poles of $\rho(s)$ are in the left half of the s-plane leads to

$$\rho(s) = \frac{\pm\left\{\begin{array}{l} s^5 - 50.js^4 - 998.75s^3 + 9962.4js^2 \\ +49625.s - 98751.j \end{array}\right\}}{\left\{\begin{array}{l} s^5 + (0.9366 - 50.j)\,s^4 \\ -(998.3 + 37.46j)\,s^3 - (561. - 9949.j)\,s^2 \\ +(49493. + 3727.j)\,s + (9268. - 98315.j) \end{array}\right\}} \tag{29}$$

We adopt the plus sign one as $+\rho(s)$. Substituting this equation into Eq.(16), we have

$$Z_{in}(s) = \frac{\left\{\begin{array}{l} 0.9366s^4 + (0.4386 - 37.46j)s^3 \\ -(561. + 13.16j)s^2 - (131.3 - 3727.j)s \\ +(9268. + 435.9j) \end{array}\right\}}{\left\{\begin{array}{l} 2.s^5 + (0.93656 - 100.j)s^2 \\ -(1997. + 37.46j)s^3 - (561. - 19912.j)s^2 \\ +(99118. + 3727.j)s + (9268. - 197065.j) \end{array}\right\}} \tag{30}$$

Table 1: Element values.

Element	Value	Element	Value
R_S	$1\ \Omega$	jG_1	$-21.28j$ S
C_1	2.128 F	jR_2	$10.84j\ \Omega$
C_2	0.009224 F	C_3	0.009224 F
jG_3	$-29.87j$ S	C_4	2.987 F
jR_4	$10.84j\ \Omega$	C_5	0.009224 F
C_6	0.009224 F	jG_5	$-21.28j$ S
C_4	2.128 F	R_L	$1\ \Omega$

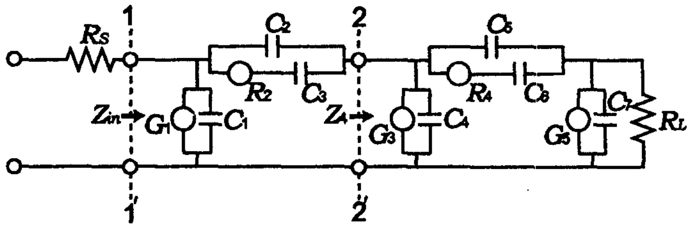

Figure 4: R^{i}CR filter.

Using the proposed method described in Sec.4 gives the element values included in the R^{i}CR filter shown in Fig.4. Table 1 summarizes the element values included in the resulting R^{i}CR filter. The frequency response of this filter is shown in Fig.5. We find that the proposed filter has the equiripple property and that the proposed filter has finite transmission zeros. Figure 6 shows the sensitivity characteristics of the proposed filter and the conventional one. We find that both of them have the similar sensitivity property.

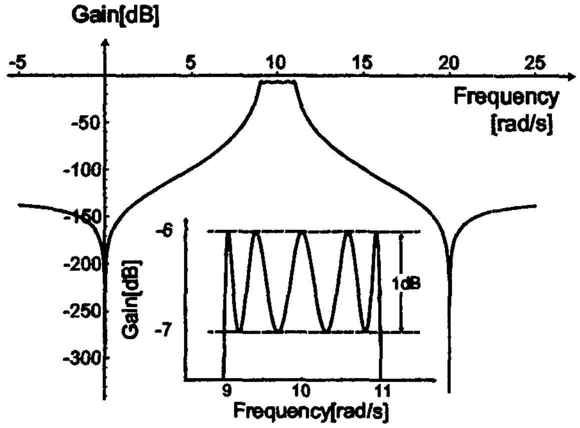

Figure 5: Frequency-gain characteristics

6. Conclusions

Synthesis of an equiripple R^{i}CR filter with finite transmission zeros have been proposed. In order to realize finite transmission zeros, we define a new kernel function and show how to determine the kernel function using Lagrange interpolation. How to determine the element values is described. It is shown that the resulting R^{i}CR filter has finite transmission zeros and that the proposed filter has almost the same sensitivity characteristics as the conventional one.

The further investigation is required to realize the proposed filter by using the active elements.

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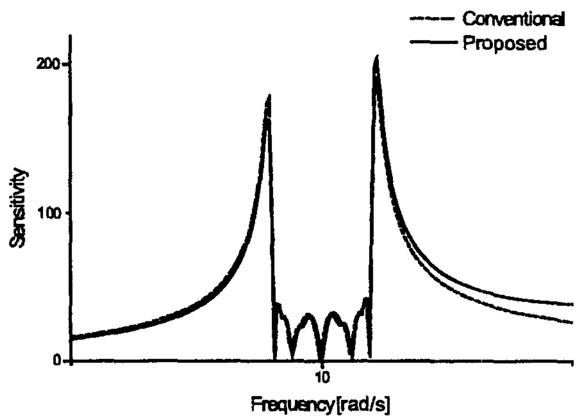

Figure 6: Sensitivity characteristics.

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