A Predistorter for OFDM Systems with a Raised Cosine Pulse Shaping Filter and a High Power Amplifier*

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Abstract: In this paper, a predistorter is presented for the orthogonal frequency division multiplxing (OFDM) systems with a transmit raised cosine (RC) pulse shaping filter and a high power amplifier (HPA). By exploiting zero-intersymbol interference (ISI) nature of the RC filter, the proposed predistorter placed before the filter only utilizes memoryless nonlinearity of the HPA, not the overall nonlinearity with memory induced by a combination of the filter and HPA. The predistirtion is realized upon fixed point iteration on OFDM symbols. Simulation results show that the proposed predisorter can be effectively employed to achieve significant performance improvement.

1. Introduction

The OFDM systems have recently drawn much attention as a radio transmission technology for digital terrestrial audio and TV broadcasting, and high speed wireless LAN[1]. In the OFDM system, a given set of signals is transmitted on several orthogonal subcarriers, where quadrature amplitude modulation (QAM) or phase shift keying (PSK) scheme is usually employed. As compared to single carrier systems, the OFDM system is more robust to severe multipath fading[1]. Moreover, digital transmitter and receiver can be implemented efficiently with utilization of the fast Fourier transform (FFT) algorithm[1].

Most radio systems employ HPAs such as a traveling wave tube amplifier (TWTA) in the transmitter to obtain sufficient transmit power. For the purpose of achieving the maximum output power efficiency, the HPA is usually operated at or near the saturation region and this introduces memoryless nonlinear distortion into the overall system[2]. The nonlinear characteristic of the HPA is quite sensitive to variation in transmit signal amplitudes. Unfortunately, the variation of OFDM signal amplitudes is known to be very wide with large peak-to-average power ratio (PAPR)[3]. For this reason, the OFDM system with the HPA suffers from significant performance degradation if some compensation techniques are not employed. In addition to this, in practical OFDM systems, the OFDM signal is passed through a transmit (TX) pulse shaping filter such as the RC filter before amplification by the HPA. The pulse shaping filter introduce memory into the system, and cascade of the TX filter and HPA forms a *nonlinear system with memory*, resulting in severe degradation of system performance.

In this paper, we consider a predistorter for compensation of nonlinear distortion with memory induced by a TX RC filter and a HPA in OFDM systems. The TX RC pulse shaping filter satisfies Nyquist's zero-ISI condition[4]. The proposed predistorter is placed before the RC filter, however *only* utilizes memoryless nonlinearity of the HPA, not the overall nonlinearity with memory induced by a combination of RC filter and HPA. The predistortion is relied upon fixed point iteration[5] proposed by the authors in other literature[6]. The organization of the paper is as follows. Section 2 describes a discrete-time baseband OFDM system model considered in this paper. In Section 3, the predistorter design scheme based on the fixed point iteration is described. Section 4 shows effectiveness of the the proposed predistorter by computer simulation. Finally Section 5 concludes the paper.

2. Discrete-Time Baseband OFDM System

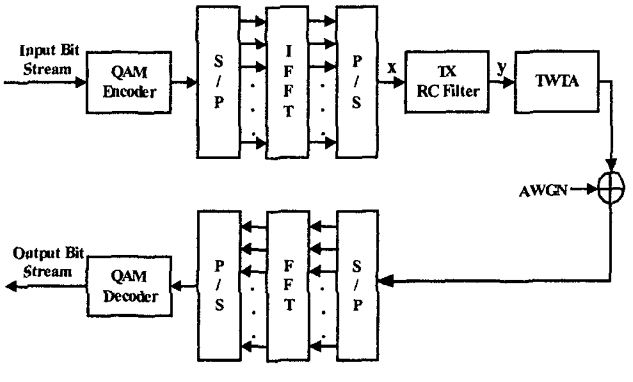

Figure 1 : Discrete-time baseband OFDM system with a TX RC pulse shaping filter and a TWTA.

Fig. 1 shows block diagram of a QAM-modulated discrete-time baseband OFDM system under additive white Gaussian noise (AWGN) channel, where a TX RC pulse shaping filter and a traveling wave tube amplifier (TWTA)

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[2] as an HPA are employed. In the transmitter of the system, a group of $N\log_2 M$ input bits is encoded into a block of N QAM symbols $X[l](l=0,\cdots,N-1)$, where symbol duration is T_s and M -ary QAM is considered. These N symbols are serial-to-parallel converted and modulated using N orthogonal subcarriers $\{e^{j2\pi f_o t},\cdots,e^{j2\pi f_{N-1}t}\}$ with the l -th frequency $f_l \equiv l/NT_s$ Hz. Thus, the OFDM signal $x(t)$ for a block of duration NT_s can be expressed as

$$x(t)=\frac{1}{N}\sum_{l=0}^{N-1}X[l]e^{j2\pi lt/NT_s}=\frac{1}{N}\sum_{l=0}^{N-1}X[l]e^{j2\pi lt/NT_s} \quad (1)$$

By discretizing $x(t)$ at $t=nT_s\,(n=0,\cdots,N-1)$, we have the discrete-time equivalence of the OFDM signal as

$$x[n]\equiv x(nT_s)=\frac{1}{N}\sum_{l=0}^{N-1}X[l]e^{j2\pi ln/N} \quad (2)$$

Eq. (2) is equivalent to N -point inverse discrete Fourier transform of the N QAM symbols $X[l]$, followed by parallel-to-serial conversion. Thus, a fast implementation using inverse fast Fourier transform (IFFT) can be utilized. In the receiver of the system, inverse functions of the transmitter. In particular, subcarrier demodulation can be efficiently implemented by N -point FFT.

To compensate for distortions, we consider a predistorter placed just before the RC filter as considered in [7,8]. In general, the predistorter employed in this type of system requires inverse transformation of *combined* filter and TWTA characteristics[7,8]. However, the proposed predistorter *only* utilizes memoryless nonlinear characteristic of the TWTA by exploiting zero-ISI nature of the RC filter. We consider the impulse response of the RC filter which is given by

$$h(t)=\operatorname{sinc}\left(\frac{\pi t}{T_s}\right)\frac{\cos(\pi\beta t/T_s)}{1-4\beta^2t^2/T_s^2} \quad (3)$$

with roll-off factor $\beta\,(0\le\beta\le 1)$. For the simulation in this study, appropriate truncation and sampling of the impulse response at every δ seconds are applied, where $\delta\equiv T_s/R$ for some positive integer R.

We consider the TWTA in this study to exploit its well-established mathematical models. Typical nonlinear characteristics of a TWTA may be described by Saleh's two-parameter formulae[2]. In this study, we adopt amplitude-phase polar model given by

$$A(r)=\frac{\alpha_a r}{1+\beta_a r^2} \quad (4)$$

$$\phi(r)=\frac{\alpha_\phi r^2}{1+\beta_\phi r^2} \quad (5)$$

where $\alpha_a=1.9638, \beta_a=0.9945, \alpha_\phi=2.5293, \beta_\phi=2.8168$, and $0\le r\le 1$ is normalized amplitude of the input signal to the TWTA. For $r>1$, $A(r)$ and $\phi(r)$ are assumed to be saturated to their maximum values at $r=1$. Eq. (4) describes nonlinear amplitude modulation to amplitude modulation (AM-AM) conversion of the TWTA with $A(r)$ representing the amplitude of the TWTA output signal. Similarly, (5) models nonlinear amplitude modulation to phase modulation (AM-PM) conversion and $\phi(r)$ is the phase shift of the TWTA output signal. Thus, for an input signal $x\equiv re^{j\theta}$, the output $\tilde{x}$ of the TWTA is expressed as

$$\tilde{x}=A(r)e^{j(\theta+\phi(r))} \quad (6)$$

3. Proposed Predistorter

Assume R -times oversampling of the baseband OFDM signals $x[n]$ at every δ interval and let us denote the oversampled signals as $x[n,m]$. Note in this case that $x[n,m]\equiv x[n]$ for $m=0,\cdots,R-1$. In Figure 1, the output $y[n,m]$ at $t=nT_s+m\delta$ of the RC filter with coefficients $h[i]\,(i=0,\cdots,L-1)$ and tap spacing of δ, is obtained as

$$y[n,m]=\sum_{i=0}^{L-1}h[i]x[n,m-i] \quad (7)$$

Then, the output $\tilde{x}[n,m]$ of the TWTA is given by

$$\tilde{x}[n,m]=N(y[n,m])\equiv H(x[n,m],\cdots,x[n,m-(L-1)]) \quad (8)$$

where $N(\cdot)$ represents the overall memoryless nonlinear characteristic of the TWTA, and $H(\cdot)$ denotes the nonlinear mapping with memory induced by cascade of the RC filter and the TWTA. The proposed predistorter is placed just before the RC filter. The predistortion of the OFDM signal $x[n,m]$ can be considered as a problem of finding predistorted signals $x_f[n,m]$ satisfying the following condition

$$\begin{aligned}\tilde{x}[n,m]&=H(x_f[n,m],\cdots,x_f[n,m-(L-1)])\\&\equiv gx[n,m]\end{aligned} \quad (9)$$

where $g>0$ is the linear gain of the TWTA determined by amplifier output back-off level. The approach taken in this paper regards finding the predistorted signal as obtaining a fixed point of a nonlinear mapping.

For a given arbitrary mapping $T(\cdot)$, the fixed point problem is to find x_f which satisfies $x_f\equiv T(x_f)$. As x_f is invariant for the mapping $T(\cdot)$, x_f is called a fixed point of $T(\cdot)$. In general, the fixed point may not exist nor be unique for the given mapping. However, if the mapping $T(\cdot)$ is a contraction mapping, the contraction mapping theorem[5] states that a unique fixed point for $T(\cdot)$ exists and can be found iteratively by the following *fixed point iteration (FPI)*

$$x_f\equiv\lim_{k\to\infty}x_{(k)}=\lim_{k\to\infty}T^k(x_{(0)}) \quad (10)$$

where $x_{(k+1)}\equiv T(x_{(k)})\,(k=0,1,\cdots)$ and $x_{(0)}$ is an arbitrary element.

Figure 2 shows the block diagram of the proposed predistorter. In the figure, the mapping $P(\cdot)$ which is applied iteratively to an OFDM signal $x[n,m]=x_{(0)}[n,m]$, is defined as

$$\begin{aligned} x_{(k+1)}[n,m] &= P(x_{(k)}[n,m]) \\ &= x_{(k)}[n,m]+\alpha\{g x_{(0)}[n,m]-N(x_{(k)}[n,m]))\} \end{aligned} \tag{11}$$

with $k=0,\cdots,K-1$, where K is a pre-determined maximum number of iterations and $\alpha>0$ is a constant. Eq. (11) does *only account for memoryless nonlinearity of the TWTA*, and is identical to the one used in the authors' previous literature[6] where a predistorter for an OFDM system with only TWTA is considered. However, we show in this paper that the same efficient predistorter without considering memory nature of the RC filter can be utilized in the OFDM systems with TWTA and RC filter, although it is placed *before* the RC filter.

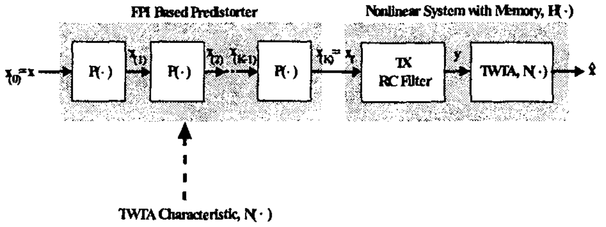


Figure 2 : Block diagram of the FPI-based predistorter considered in study.

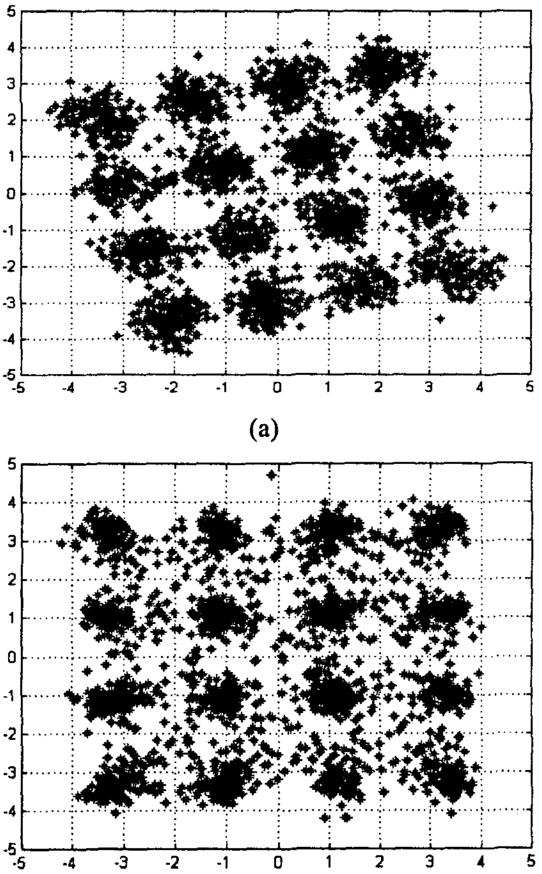
4. Simulation Results

In the simulation, we have considered a discrete-time baseband OFDM system with $N=256$ subcarriers. The subcarrier modulator and demodulator were implemented by using 256-point IFFT and FFT, respectively. For modulation scheme 16-ary QAM was employed with the largest signal amplitude before normalization of $3\sqrt{2}$. The AWGN channels without any multipath channel impairments were assumed to clearly observe the effect of nonlinearity and performance improvement by the proposed predistorter. Saleh's two-parameter formulae were employed for modeling of the TWTA[2]. The maximum number K of FPIs for the proposed predistorter was set to 20, and we have used $\alpha=0.85$[6]. For the pulse shaping filter, we have used a 35-tap RC filter with $\beta=0.5$ and $\delta=T_s/10$.

Figure 3 depicts constellations of QAM decoded output symbols (a) without a predistorter and (b) with the proposed predistorter, respectively, where bit energy-to-noise power spectral density $E_b/N_0=$ 20 dB and the output back-off (OBO) level is 6.1 dB. Here, OBO in decibels is defined as

$$\mathrm{OBO}=10\log_{10}\left(\frac{P_{sat}}{P_{out}}\right)\ [\mathrm{dB}] \tag{12}$$

where P_{sat} and P_{out} represent the maximum power and average power of the TWTA outputs, respectively. Although the RC filter does not exhibit any ISI, nonlinear distortion by the TWTA is not fully eliminated by small level of OBO and the combination of this nonlinearity with memory effect of the RC filter creates amplitude/phase distortions for the case without any predistortion. However, with the proposed predistorter these distortions are effectively compensated and the original constellation is closely recovered. For the purpose of comparison, we consider the predistorter which is placed *after* the RC filter as in Figure 4. Since the predistorter in Figure 4 requires to compensate for the TWTA only, the same predistortion algorithm described by Eq. (11) as the proposed one, can be employed. Figures 5 and 6 show the results of the proposed predistorter and the one in Figure 4. Figure 5 compares constellations of QAM decoder output symbols when E_b/N_0=20 dB and OBO = 9.5 dB. It is interesting to observe that both predistorters can effectively compensate for the distortions and achieve similar constellations. In Figure 6, bit error rate (BER) performance is depicted for the OBO of 9.5 dB. We observe that the proposed predistorter ("Pred-RCF-HPA") and the one in Figure 4 ("RCF-Pred-HPA") show quite good performance which is comparable to the performance of ideal AWGN channel ("Linear Channel").



(a)

(b)

Figure 3 : Constellations of QAM decoder output symbols (E_b/N_0=20 dB and OBO = 6.1 dB) (a) without any predistorter and (b) with the proposed predistorter.

5. Conclusions

In this paper, we have presented a predistorter based on the FPI for the OFDM systems with the HPA and the TX RC pulse shaping filter. By exploiting zero-ISI nature of the RC filter, the proposed predistorter placed before the RC filter utilizes memoryless nonlinearity of the HPA only, not the overall nonlinearity with memory induced by a combination of RC filter and HPA. Simulation results on 16-QAM, 256-subcarrier OFDM system show that the proposed predistorter can effectively compensate for nonlinear distortion by the HPA, and achieve significant BER improvement over that without any predistortion. Moreover, for the purpose of comparison, we have also considered the predistorter which is placed *after* the RC filter. The simulation results show that both predistorters can effectively compensate for the distortions and achieve similar performance.

References

[1] J. A. C. Gingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Magazine*, vol. 28, no. 5, pp. 5-14, May 1990.

[2] A. M. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. Commun.*, vol. 29, no. 11, pp. 1715-1720, November 1981.

[3] M. -G. Benedetto and P. Mandarini, "An application of MMSE predistortion to OFDM systems," *IEEE Trans. Commun.*, vol. 44, pp. 1417-1420, November 1996.

[4] B. Sklar, *Digital Communications*, 2nd Ed., Prentice-Hall, 2001.

[5] D. G. Luenberger, *Optimization by Vector Space Method*, John Wiley & Sons, 1969.

[6] M. C. Kim, Y. Shin, and S. Im, "Compensation of nonlinear distortion using a predistorter based on the fixed point approach in OFDM systems," *Proc. Vehicular Technol. Conf. (VTC '98)*, vol. 3, pp. 2145-2149, Ottawa, Canada, September 1998.

[7] H. W. Kang, Y. S. Cho, and D. H. Youn, "On compensating nonlinear distortions of an OFDM system using an efficient adaptive predistorter," *IEEE Trans. Commun.*, vol. 47, no. 4, pp. 522-526, April 1999.

[8] A. Brajal and A. Chouly, "Compensation of nonlinear distortion for orthogonal multi-carrier schemes using predistortion," *Proc. GLOBECOM '94*, vol. 3, pp. 1909-1914, San Francisco, USA, November 1994.

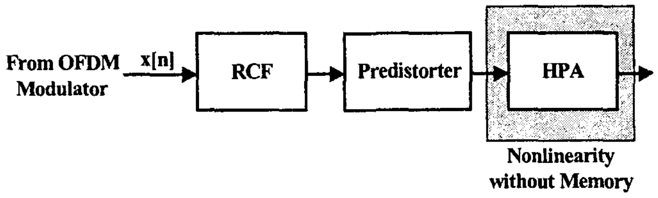
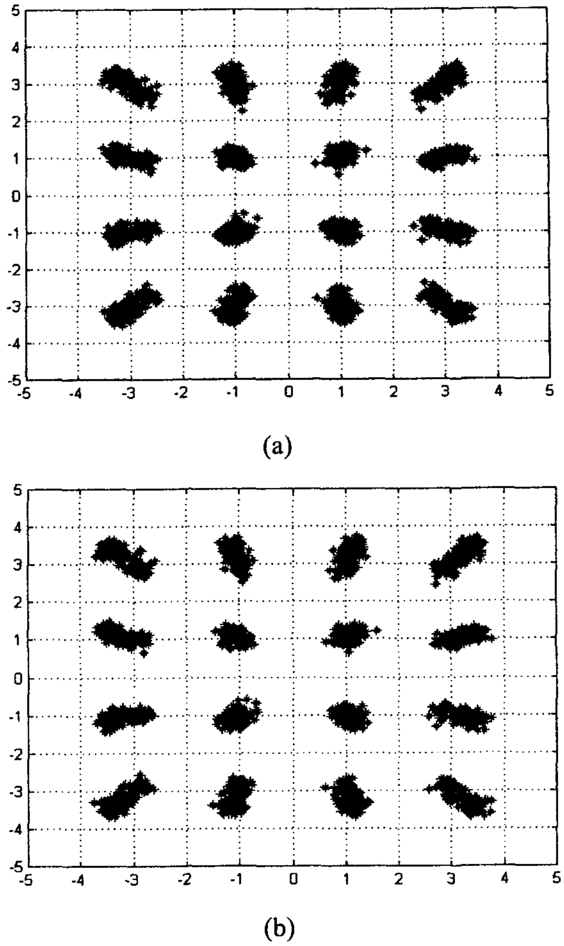


Figure 4 : The predistorter placed after the RC filter for the purpose of comparison.



(a)

(b)

Figure 5 : Constellations of QAM decoder output symbols (E_b/N_0=20 dB and OBO = 9.5 dB) (a) with the predistorter and (b) with the predistorter in Figure 4.

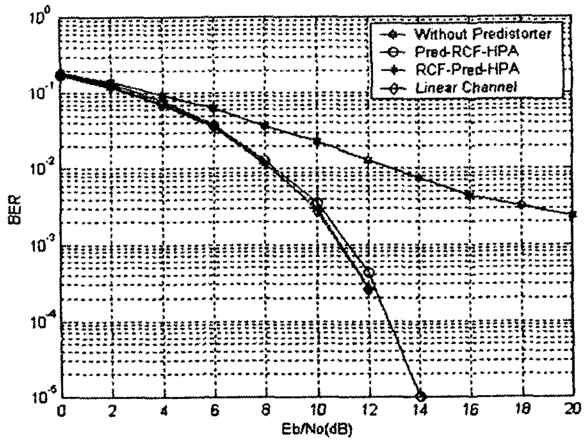


Figure 6 : BER performances of the predistorters (OBO = 9.5 dB).