Efficient Design of SVD-Based 2-D Digital Filters Using Specification Symmetry and Order-Selecting Criterion

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Abstract: Two-dimensional (2-D) digital filters are widely useful in image processing and other 2-D digital signal processing fields, but designing 2-D filters is much more difficult than designing one-dimensional (1-D) ones. This paper provides a new insight into the existing singular value decomposition (SVD)-based design approach in the sense that the SVD-based design can be performed more efficiently by exploiting the symmetries of the given 2-D magnitude specifications. By using the specification symmetries, only half of the 1-D filters (sub-filters) need to be designed, which significantly simplifies the design process and reduces the computer storage required for 1-D sub-filter coefficients. Another novel point of this paper is that an objective criterion is proposed for selecting appropriate sub-filter orders in order to reduce the hardware implementation cost. A design example is given to illustrate the effectiveness of the SVD-based design approach by exploiting specification symmetry and new order-selecting criterion.

1 Introduction

Two-dimensional (2-D) digital filtering is one of the most fundamental and most important processing techniques in digital image processing and other 2-D digital signal processing fields. Up to this point, many methods have been developed for implementing and designing 2-D digital filters, among the developed techniques, the indirect approaches that decompose the original 2-D problems into 1-D ones have received considerable attention. The reason is that the 2-D problem can be easily attacked by solving a set of easier 1-D problems through using the accumulated 1-D techniques, thus the original 2-D problems can be indirectly solved in an elegant way. The SVD-based approaches have been developed in the frequency-domain by a few researchers in an increasingly improved manner as follows:

⟨1⟩ Separable 2-D filter with only one section [1].
⟨2⟩ SVD-based 2-D filters with biased circuits [2].
⟨3⟩ ISVD-based 2-D filters without biased circuits [3].
⟨4⟩ Nonnegative decomposition-based design [4].
⟨5⟩ SVD-based non-quadrantal symmetric design [5].
⟨6⟩ SVD-based design with different sub-filter orders [6].

This paper is aimed to further advance the SVD-based design methods ⟨5⟩ and ⟨6⟩ in the following aspects:

- By exploiting the symmetries of the desired 2-D magnitude responses, we show that the design of 1-D sub-filters can be significantly simplified, which means that only one 1-D sub-filter in each parallel section needs to be designed, and the other one has identically the same filter coefficients as the designed one.
- An objective error criterion is proposed for selecting the appropriate orders for different 1-D sub-filters such that each sub-filter contributes to the final 2-D filter design accuracy at the same extent.

A design example is given to illustrate the effectiveness of the above two points.

2 Design Using Symmetries

In this section, we briefly review the mirror-image symmetry and mirror-image anti-symmetry exsiting in the SVD of the desired 2-D zero-phase frequency response [5], and then exploit a new symmetric property that can be efficiently utilized in the SVD-based design for simplifying the 2-D filter design process.

2.1 Symmetry and Anti-symmetry

As proved in [5], both quadrantally symmetric and non-quadrantally symmetric 2-D zero-phase frequency responses can be approximated by using the singular value decomposition (SVD) method, which decomposes the original zero-phase 2-D digital filter design problem into the problems of designing zero-phase or $-\pi/2$-phase 1-D sub-filters. The design approach can be briefly reviewed as follows. Assume that $H_d(\omega_1, \omega_2)$ is the desired zero-phase 2-D frequency response, and $M_d(\omega_1, \omega_2)$, $\theta_d(\omega_1, \omega_2)$ are the corresponding 2-D magnitude and phase specifications, respectively,

$$H_d(\omega_1, \omega_2) = M_d(\omega_1, \omega_2)e^{j\theta_d(\omega_1, \omega_2)} = M_d(\omega_1, \omega_2) \quad (1)$$

where $\omega_1, \omega_2 \in [-\pi, \pi]$. By using the equally-spaced samples of $H_d(\omega_1, \omega_2)$. we can form a specification matrix

$$\boldsymbol{A} = [H_d(\omega_{1l}, \omega_{2m})] \in \boldsymbol{R}^{L\times M}$$

where

$$\omega_{1l} = -\pi + \frac{2\pi(l-1)}{L-1}$$
$$\omega_{2m} = -\pi + \frac{2\pi(m-1)}{M-1}.$$

The SVD of the matrix $\boldsymbol{A}$ results in

$$\boldsymbol{A} = \sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^t = \sum_{i=1}^{r} \tilde{\boldsymbol{u}}_i \tilde{\boldsymbol{v}}_i^t \quad (2)$$

where r is the rank of the matrix $\boldsymbol{A}$, and the singular values $\sigma_1 \geq \sigma_2 \cdots \geq \sigma_r > 0$.

$$\tilde{\boldsymbol{u}}_i = \sqrt{\sigma_i}\boldsymbol{u}_i \qquad \tilde{\boldsymbol{v}}_i^t = \sqrt{\sigma_i}\boldsymbol{v}_i^t. \tag{3}$$

The column vectors $\tilde{\boldsymbol{u}}_i$ and $\tilde{\boldsymbol{v}}_i$ are *either* mirror-image symmetric *or* mirror-image anti-symmetric simultaneously [5]. In the SVD-based design of zero-phase 2-D filters, only the first K pairs of $\tilde{\boldsymbol{u}}_i, \tilde{\boldsymbol{v}}_i$ are used, and the others are neglected as

$$\boldsymbol{A} \approx \hat{\boldsymbol{A}} = \sum_{i=1}^{K} \tilde{\boldsymbol{u}}_i \tilde{\boldsymbol{v}}_i^t. \tag{4}$$

The normalized root-mean-squared (RMS) decomposition error is

$$E_K = \frac{\|\boldsymbol{A} - \hat{\boldsymbol{A}}\|}{\|\boldsymbol{A}\|} = \frac{\left\| \sum_{i=K+1}^{r} \tilde{\boldsymbol{u}}_i \tilde{\boldsymbol{v}}_i^t \right\|}{\left\| \sum_{i=1}^{r} \tilde{\boldsymbol{u}}_i \tilde{\boldsymbol{v}}_i^t \right\|} = \frac{\left(\sum_{i=K+1}^{r} \sigma_i^2 \right)^{1/2}}{\left(\sum_{i=1}^{r} \sigma_i^2 \right)^{1/2}} \tag{5}$$

where $\|\cdot\|$ denotes the Euclidean norm.

After truncating the last several $\tilde{\boldsymbol{u}}_i, \tilde{\boldsymbol{v}}_i$, the remaining $\tilde{\boldsymbol{u}}_i, \tilde{\boldsymbol{v}}_i$ can be regarded as the desired frequency responses of zero-phase or $-\pi/2$-phase 1-D filters $F_i(z_1)$ and $G_i(z_2)$ respectively, and the sub-filters $F_i(z_1)$ and $G_i(z_2)$ are separately designed by using the existing 1-D design techniques.

2.2 New Symmetry

If $M_d(\omega_1, \omega_2)$ is symmetric with respect to the straight lines $\omega_1 = \omega_2$ and $\omega_1 = -\omega_2$, then the SVD in (4) generates the vectors $\tilde{\boldsymbol{u}}_i$, $\tilde{\boldsymbol{v}}_i$ that satisfy either $\tilde{\boldsymbol{u}}_i = \tilde{\boldsymbol{v}}_i$ or $\tilde{\boldsymbol{u}}_i = -\tilde{\boldsymbol{v}}_i$.

Proof: Let

$$\tilde{\boldsymbol{H}} = [\tilde{h}_{i,j}], \quad 1 \leq i, j \leq 2N \tag{6}$$

be a real 2N-by-2N matrix whose elements satisfy

$$\tilde{h}_{i,j} = \tilde{h}_{2N+1-i, 2N+1-j}$$

and assume that the matrix

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \tilde{\boldsymbol{H}}_1 & \tilde{\boldsymbol{H}}_2 \\ \tilde{\boldsymbol{H}}_3 & \tilde{\boldsymbol{H}}_4 \end{bmatrix} \tag{7}$$

has distinct singular values. If matrices $\hat{\boldsymbol{I}}_N$, $\boldsymbol{I}_N$ are the N-by-N backward permutation matrix and N-by-N identity matrix defined by

$$\hat{\boldsymbol{I}}_N = \begin{bmatrix} \mathbf{0} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \mathbf{0} \end{bmatrix} \tag{8}$$

$$\boldsymbol{I}_N = \begin{bmatrix} 1 & & & \mathbf{0} \\ & 1 & & \\ & & \ddots & \\ \mathbf{0} & & & 1 \end{bmatrix} \tag{9}$$

and the matrix $\tilde{\boldsymbol{I}}$ is formed by using the matrices $\hat{\boldsymbol{I}}_N$ and $\boldsymbol{I}_N$ as

$$\tilde{\boldsymbol{I}} = \begin{bmatrix} \boldsymbol{I}_N & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{I}}_N \end{bmatrix} \tag{10}$$

then we can verify that the matrix

$$\boldsymbol{H} = \tilde{\boldsymbol{I}}\tilde{\boldsymbol{H}}\tilde{\boldsymbol{I}}$$

can be expressed in the form of

$$\boldsymbol{H} = \tilde{\boldsymbol{I}}\tilde{\boldsymbol{H}}\tilde{\boldsymbol{I}} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 \\ \boldsymbol{H}_2 & \boldsymbol{H}_1 \end{bmatrix} \tag{11}$$

where $\boldsymbol{H}_1$, $\boldsymbol{H}_2$ are N-by-N matrices,

$$\begin{aligned} \boldsymbol{H}_1 &= \tilde{\boldsymbol{H}}_1 = \hat{\boldsymbol{I}}_N \tilde{\boldsymbol{H}}_4 \hat{\boldsymbol{I}}_N \\ \boldsymbol{H}_2 &= \tilde{\boldsymbol{H}}_2 \hat{\boldsymbol{I}}_N = \hat{\boldsymbol{I}}_N \tilde{\boldsymbol{H}}_3. \end{aligned} \tag{12}$$

The SVD of $\tilde{\boldsymbol{H}}$ results in

$$\begin{aligned} \tilde{\boldsymbol{H}} &= \tilde{\boldsymbol{I}}\boldsymbol{H}\tilde{\boldsymbol{I}} \\ &= \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^t \\ &= [\ \boldsymbol{u}_1 \ \ \boldsymbol{u}_2 \ \ \cdots \ \ \boldsymbol{u}_{2N}\ ]\boldsymbol{\Sigma}[\ \boldsymbol{v}_1 \ \ \boldsymbol{v}_2 \ \ \cdots \ \ \boldsymbol{v}_{2N}\ ]^t \end{aligned}$$

where $\boldsymbol{u}_i$, $\boldsymbol{v}_i$ are the normalized eigenvectors of $\boldsymbol{H}\boldsymbol{H}^t$ and $\boldsymbol{H}^t\boldsymbol{H}$, respectively, and $\boldsymbol{\Sigma}$ is a diagonal matrix with the singular values σ_i as its diagonal elements, i.e.,

$$\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{2N}).$$

From (13) we obtain

$$\begin{aligned} \boldsymbol{H} &= \tilde{\boldsymbol{I}}\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^t\tilde{\boldsymbol{I}} \\ &= (\tilde{\boldsymbol{I}}\boldsymbol{U})\boldsymbol{\Sigma}(\tilde{\boldsymbol{I}}\boldsymbol{V})^t \end{aligned} \tag{13}$$

where

$$\begin{aligned} \tilde{\boldsymbol{I}}\boldsymbol{U} &= \tilde{\boldsymbol{I}}[\ \boldsymbol{u}_1 \ \ \boldsymbol{u}_2 \ \ \cdots \ \ \boldsymbol{u}_{2N}\ ] \\ \tilde{\boldsymbol{I}}\boldsymbol{V} &= \tilde{\boldsymbol{I}}[\ \boldsymbol{v}_1 \ \ \boldsymbol{v}_2 \ \ \cdots \ \ \boldsymbol{v}_{2N}\ ]. \end{aligned} \tag{14}$$

If $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are simultaneously mirror-image symmetric, then

$$\tilde{\boldsymbol{I}}\boldsymbol{u}_i = \begin{bmatrix} \boldsymbol{x}_i \\ \boldsymbol{x}_i \end{bmatrix}, \qquad \tilde{\boldsymbol{I}}\boldsymbol{v}_i = \begin{bmatrix} \boldsymbol{y}_i \\ \boldsymbol{y}_i \end{bmatrix}. \tag{15}$$

Otherwise, $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are simultaneously mirror-image antisymmetric, i.e.,

$$\tilde{\boldsymbol{I}}\boldsymbol{u}_i = \begin{bmatrix} \boldsymbol{x}_i \\ -\boldsymbol{x}_i \end{bmatrix}, \qquad \tilde{\boldsymbol{I}}\boldsymbol{v}_i = \begin{bmatrix} \boldsymbol{y}_i \\ -\boldsymbol{y}_i \end{bmatrix}. \tag{16}$$

If $M_d(\omega_1, \omega_2)$ is symmetric with respect to the straight line $\omega_1 = \omega_2$, then we can verify that $\tilde{\boldsymbol{H}}_1$ is a symmetric matrix, i.e.,

$$\tilde{\boldsymbol{H}}_1 = \tilde{\boldsymbol{H}}_1^t. \tag{17}$$

Similarly, if $M_d(\omega_1, \omega_2)$ is also symmetric with respect to the straight line $\omega_1 = -\omega_2$, then $\tilde{H}_2\hat{I}_N$ is symmetric, i.e.,

$$\tilde{H}_2\hat{I}_N = (\tilde{H}_2\hat{I}_N)^t. \quad (18)$$

The symmetries (17) and (18) together with (12) lead to

$$\begin{aligned} H_1 &= H_1^t \\ H_2 &= H_2^t. \end{aligned} \quad (19)$$

Since

$$\begin{aligned} HH^t &= \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix} \begin{bmatrix} H_1^t & H_2^t \\ H_2^t & H_1^t \end{bmatrix} \\ &= \begin{bmatrix} H_1H_1^t + H_2H_2^t & H_1H_2^t + H_2H_1^t \\ H_2H_1^t + H_1H_2^t & H_2H_2^t + H_1H_1^t \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} H^tH &= \begin{bmatrix} H_1^t & H_2^t \\ H_2^t & H_1^t \end{bmatrix} \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix} \\ &= \begin{bmatrix} H_1^tH_1 + H_2^tH_2 & H_1^tH_2 + H_2^tH_1 \\ H_2^tH_1 + H_1^tH_2 & H_2^tH_2 + H_1^tH_1 \end{bmatrix} \end{aligned}$$

it is evident that H is a normal matrix, i.e.,

$$HH^t = H^tH.$$

Substituting (13) into (20) and (20) obtains

$$\begin{aligned} HH^t &= (\tilde{I}U)\Sigma(\tilde{I}V)^t\left[(\tilde{I}U)\Sigma(\tilde{I}V)^t\right]^t \\ &= (\tilde{I}U)\Sigma(\tilde{I}V)^t(\tilde{I}V)\Sigma(\tilde{I}U)^t \\ &= (\tilde{I}U)\Sigma^2(\tilde{I}U)^t \end{aligned} \quad (20)$$

and

$$\begin{aligned} H^tH &= \left[(\tilde{I}U)\Sigma(\tilde{I}V)^t\right]^t(\tilde{I}U)\Sigma(\tilde{I}V)^t \\ &= (\tilde{I}V)\Sigma(\tilde{I}U)^t(\tilde{I}U)\Sigma(\tilde{I}V)^t \\ &= (\tilde{I}V)\Sigma^2(\tilde{I}V)^t \end{aligned} \quad (21)$$

thus

$$(\tilde{I}U)\Sigma^2(\tilde{I}U)^t = (\tilde{I}V)\Sigma^2(\tilde{I}V)^t \quad (22)$$

which implies

$$\tilde{I}U = \tilde{I}V \iff U = V \quad (23)$$

or

$$\tilde{I}U = -\tilde{I}V \iff U = -V. \quad (24)$$

Consequently, we can conclude that

$$u_i = v_i \quad \text{or} \quad u_i = -v_i \quad (25)$$

where u_i and v_i are *either* mirror-image symmetric *or* mirror-image anti-symmetric as shown in (15) and (16). The new symmetry (25) can be utilized to design 1-D sub-filters $F_i(z_1)$ and $G_i(z_2)$ efficiently. If we use $F_i(z_1)$ to approximate $\tilde{u}_i$, and set the coefficients of another sub-filter $G_i(z_2)$ identically the same as those of $F_i(z_1)$, i.e.,

$$G_i(z_2) = F_i(z_2)$$

then $T_iF_i(z_2)$ approximates $\tilde{v}_i$ just as $F_i(z_1)$ approximates $\tilde{u}_i$, where

$$T_i = \begin{cases} 1 & \text{if } \tilde{u}_i = \tilde{v}_i \\ -1 & \text{if } \tilde{u}_i = -\tilde{v}_i. \end{cases} \quad (26)$$

As a result, only sub-filters $F_1(z_1), F_2(z_1), \cdots, F_K(z_1)$ need to be designed, and $T_iF_i(z_2)$ can be readily obtained. This symmetry exploitation can

- reduce the design work by 50%,
- save the computer storage for sub-filter coefficients by 50%.

Replacing the sub-filters $G_i(z_2)$ in Fig. 1 by $T_iF_i(z_2)$ leads to the new parallel structure as shown in Fig. 1.

2.3 Order-Selecting Criterion

An important step in SVD-based 2-D filter design is how to select the orders of 1-D sub-filters for approximating different vectors $\tilde{u}_i$ and $\tilde{v}_i$. Most existing SVD-based designs use the same order for different 1-D sub-filters [5], but the only one exception proposed in [6] utilizes different orders for different $\tilde{u}_i$ and $\tilde{v}_i$. That is, low-order sub-filters are used for low-energy vectors, and high-order filters are for high-energy vectors. This paper will show that this order-selecting policy is not appropriate since lower order sub-filters cannot achieve good approximations to the last several vectors whose elements become more and more irregular (zigzag) as the number of parallel sections increases. Instead, we propose a new objective criterion for selecting appropriate sub-filter orders not only based on the vector energy but also based on the irregularity of vector elements.

First, let us define a set of approximation errors. Assume that f_i and g_i are the actual vectors for approximating $\tilde{u}_i$ and $\tilde{v}_i$, respectively, and that the approximation error vectors are

$$\begin{aligned} \Delta\tilde{u}_i &= \tilde{u}_i - f_i \\ \Delta\tilde{v}_i &= \tilde{v}_i - g_i. \end{aligned} \quad (27)$$

Clearly, the normalized RMS errors are

$$e_{\tilde{u}_i} = \frac{\|\tilde{u}_i - f_i\|}{\|\tilde{u}_i\|} = \frac{\|\Delta\tilde{u}_i\|}{\sqrt{\sigma_i}} \quad (28)$$

$$e_{\tilde{v}_i} = \frac{\|\tilde{v}_i - g_i\|}{\|\tilde{v}_i\|} = \frac{\|\Delta\tilde{v}_i\|}{\sqrt{\sigma_i}}. \quad (29)$$

It should be noted here that relatively large approximation errors $e_{\tilde{u}_i}$ and $e_{\tilde{v}_i}$ do not necessarily affect the final 2-D design accuracy significantly. On the other hand, too large errors $e_{\tilde{u}_i}$ and $e_{\tilde{v}_i}$ do not contribute to the improvement of the final 2-D design accuracy anymore, and thus those extra f_i and g_i should be completely removed. Based on this philosophy, we should select the orders of 1-D sub-filters $F_i(z_1)$ and $G_i(z_2)$ by considering how the individual errors $e_{\tilde{u}_i}$ and $e_{\tilde{v}_i}$ affect the whole design accuracy.

In this paper, we define the following normalized RMS approximation error

$$E_{f_j} = \frac{\|A - A_{f_j}\|}{\|A\|} \times 100\% \quad (30)$$

$$E_{g_j} = \frac{\|A - A_{g_j}\|}{\|A\|} \times 100\% \quad (31)$$

where

$$A_{f_j} = \sum_{i=1(i\neq j)}^{K} \tilde{u}_i\tilde{v}_i^t + f_j\tilde{v}_j^t \quad (32)$$

$$A_{g_j} = \sum_{i=1(i\neq j)}^{K} \tilde{u}_i\tilde{v}_i^t + \tilde{u}_jg_j^t. \quad (33)$$

The orders of 1-D sub-filters $F_i(z_1)$ and $G_i(z_2)$ are selected such that the approximation errors E_{f_j}, E_{g_j} are almost the same for all the vectors f_j and g_j, where $j = 1, 2, \cdots, K$.

3 Design Example

[Elliptical Filter]: The desired $M_d(\omega_1, \omega_2)$ is

$$M_d(\omega_1, \omega_2) = \begin{cases} 1 & 0 \le \omega_g \le \omega_p \\ \dfrac{(\omega_a - \omega_g)}{(\omega_a - \omega_p)} & \omega_p \le \omega_g \le \omega_a \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

where

$$\omega_g = \sqrt{\tilde{\omega}_1^2 + \frac{\tilde{\omega}_2^2}{2}} \quad (35)$$
$$\omega_p = 0.35\pi, \quad \omega_a = 0.50\pi$$

$$\begin{bmatrix} \tilde{\omega}_1 \\ \tilde{\omega}_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \quad \alpha = -\frac{\pi}{4}. \quad (36)$$

The the 2-D magnitude specification satisfies the new symmetry, and thus the 2-D filter can be efficiently designed.

To form the specification matrix $\boldsymbol{A}$, the frequencies $\omega_1, \omega_2 \in [-\pi, \pi]$ are equally sampled at the step size $\pi/40$, and then the corresponding samples $M_d(\omega_{1l}, \omega_{2m})$ are used to construct $\boldsymbol{A} \in \boldsymbol{R}^{81 \times 81}$.

In [6], the first 12 sections are approximated, i.e., K=12, the normalized RMS decomposition error is 1.0983%. When the sub-filter orders in [6] are used, the normalized magnitude response error of the designed 2-D filter is 2.0777%. In our design, we just use the first 8 channels, the decomposition error E_8 is 1.7094%, and the new order-selecting criterion is applied to the selection of 1-D sub-filter orders such that the errors E_f and E_g defined in (30) and (31) are below 1.7200%. The magnitude response of our designed 2-D filter is plotted in Fig. 2, whose normalized RMS error is 1.8373%. By comparing our design results with those in [6], we can make the following conclusions:

- Exploting the new symmetry in the SVD-based 2-D filter design enables us to accomplish the design by designing only 8 sub-filters, but 24 sub-filters need to be designed in [6].
- The number of our total multiplier coefficients is 142, which is less than 50% of the total multiplier coefficients (292) used in [6].
- Our design error (1.8373%) is smaller than that (2.0777%) by the method [6].

That is, the new SVD-based technique can achieve higher design accuracy with significantly reduced design complexity and much less hardware implementation cost than the design approach [6].

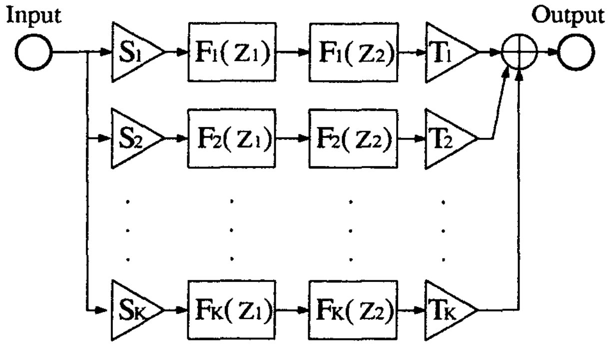

Fig. 1. SVD-based 2-D filter using new symmetry.

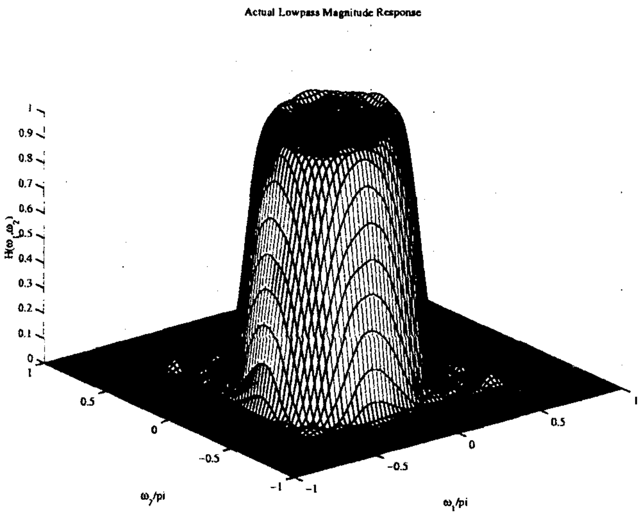

Fig. 2. Actual 2-D magnitude response.

References

[1] R. E. Twogood and S. K. Mitra, "Computer-aided design of separable two-dimensional digital filters", *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-25, no. 2, pp. 165-169, April 1977.

[2] A. Antoniou and W.-S. Lu, "Design of two-dimensional digital filters by using the singular value decomposition", *IEEE Trans. Circuits Syst.*, vol. CAS-34, no. 10, pp. 1191-1198, Oct. 1987.

[3] T.-B. Deng and M. Kawamata, "Frequency-domain design of 2-D digital filters using the iterative singular value decomposition", *IEEE Trans. Circuits Syst.*, vol. 38, no. 10, pp. 1225-1228, Oct. 1991.

[4] T.-B. Deng and T. Soma, "Successively linearized non-negative decomposition of 2-D filter magnitude design specifications", *Digital Signal Processing*, vol. 3, no. 2, pp. 125-138, April 1993.

[5] W.-S. Lu, H.-P. Wang, and A. Antoniou, "Design of two-dimensional digital filters using singular value decomposition and balanced approximation method", *IEEE Trans. Signal Processing*, vol. 39, no. 10, pp. 2253-2262, Oct. 1991.

[6] N. Yazdi, M. Ahmadi, and J. J. Soltis, "Efficient design of 2-D linear phase FIR filters using singular value decomposition (SVD)" *Electronics Letters*, vol. 28, no. 24, pp. 2256-2258, Nov. 1992.