A class of multi-level spreading codes with enhanced ZCD property

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Abstract: Spreading codes with ZCD(zero-correlation duration) property have the MAI(multiple access interference) rejection ability in the CDMA systems. A class of multi-level spreading codes, i.e., Zero padded ternary ZCD codes are introduced in this paper. By the selected zero-padding method to the binary ZCD codes, ternary ZCD codes are generated and they have enhanced ZCD property and family size than the binary ZCD codes.

1. Introduction

For any two spreading codes of period N ; $S_N^{(x)} = (s_0^{(x)}, \cdots, s_{N-1}^{(x)})$ and $S_N^{(y)} = (s_0^{(y)}, \cdots, s_{N-1}^{(y)})$, the periodic correlation function for a shift τ is defined as

$$\theta_{x,y}(\tau) = \sum_{n=0}^{N-1} s_n^{(x)} s_{(n+\tau, \bmod N)}^{(y)} \quad (1)$$

This function becomes the autocorrelation function (ACF) when $x = y$ and the crosscorrelation function (CCF) when $x \neq y$. Since the maximum magnitude of periodic ACF sidelobes (θ_{as}) and the maximum magnitude of periodic CCF (θ_c) are bounded by theoretical limits [1], spreading codes with both zero θ_{as} and zero θ_c cannot be constructed. However, it is possible to construct spreading codes with both zero θ_{as} and zero θ_c at the local duration around $\tau = 0$. This local duration is defined as the zero-correlation duration (ZCD). This ZCD property enables approximate synchronization without multiple access interference (MAI) at the uplink of CDMA systems, i.e., approximately synchronized CDMA (AS-CDMA) systems [2]. AS-CDMA was proposed to eliminate MAI at the uplink of CDMA[2,3]. One of the most important problems of spreading code theory of AS-CDMA is to construct code sets with the larger family sizes and the wider ZCD. Especially, to eliminate MAI of CDMA, wider ZCD is preferred. The ZCD property in the binary codes has been reported for some documents [4-8]. However, the maximum ZCD of binary codes is bounded in $(0.5N+1)$ chips for the period N [4,9], and the larger family sizes with ZCD is still required to increase the system capacity of CDMA. In this paper, we introduce a new class of ternary spreading codes with wide ZCD. At first, we present a generalized construction method for ternary preferred pairs (TPP) that have the ZCD of $(0.75N+1)$ chips and periods of $4 \times 2^n \ (n = 1,2,3 \cdots)$. Secondly, using the chip-shift operation to the TPP, we generate new ternary ZCD codes that have sufficient ZCD and family sizes for AS-CDMA systems.

2. Construction method of ternary ZCD codes

Step (I): Method for constructing TPP

The TPP of period $N = 4 \times 2^n \ (n = 1,2,3 \cdots)$ with ZCD of $(0.75N+1)$ is denoted as $\{C_N^{(a)}, C_N^{(b)}\}$, where $C_N^{(a)} = (c_0^{(a)}, c_1^{(a)}, \cdots, c_{N-1}^{(a)})$, and $C_N^{(b)} = (c_0^{(b)}, c_1^{(b)}, \cdots, c_{N-1}^{(b)})$

The initial basic matrix to construct $\{C_N^{(a)}, C_N^{(b)}\}$ is defined as

$$GA = \begin{bmatrix} + & z & + & z & + & z & - & z \\ + & z & + & z & - & z & + & z \\ + & z & - & z & + & z & + & z \\ - & z & + & z & + & z & + & z \end{bmatrix} \text{ or } GB = \begin{bmatrix} + & + & z & z & + & - & z & z \\ + & + & z & z & - & + & z & z \\ + & - & z & z & + & + & z & z \\ - & + & z & z & + & + & z & z \end{bmatrix} \quad (2)$$

where $+$, $-$, and z denote 1 , -1 , and zero, respectively.

Any row of $\pm GA$ or $\pm GB$ is denoted as

$C_8^{(a)} = (c_0^{(a)}, \cdots, c_7^{(a)}) = (e_0, z, e_1, z, e_2, z, e_3, z)$ or $(e_0, e_1, z, z, e_2, e_3, z, z)$.

$C_8^{(b)} = (c_0^{(b)}, \cdots, c_7^{(b)}) = (v_0, z, v_1, z, v_2, z, v_3, z)$ or $(v_0, v_1, z, z, v_2, v_3, z, z)$) is generated from $C_8^{(a)}$, where $v_q = (-1)^q e_q \quad (q = 0,1,2,3)$. A pair of $\{C_8^{(a)}, C_8^{(b)}\}$ has ZCD of $(0.75 \times 8 + 1)$ chips and is defined as the initial ternary pair. By using $\{C_8^{(a)}, C_8^{(b)}\}$, the TPP with longer period are constructed recursively as the following extension method. Using any given TPP of $\{C_m^{(a)}, C_m^{(b)}\}$ with period $m = 4 \times 2^i \ (i = 1,2,3 \cdots)$, the extension matrix DA or DB with period $2m$ is generated as Eqn (3), where DA is resulted from $\pm GA$ and DB is resulted from $\pm GB$ respectively.

$$DA = \begin{bmatrix} X & Z & Y & Z & X & Z & -Y & Z \\ X & Z & Y & Z & -X & Z & Y & Z \\ X & Z & -Y & Z & X & Z & Y & Z \\ -X & Z & Y & Z & X & Z & Y & Z \end{bmatrix} \text{ or }$$

$$DB = \begin{bmatrix} V & W & Z & Z & V & -W & Z & Z \\ V & W & Z & Z & -V & W & Z & Z \\ V & -W & Z & Z & V & W & Z & Z \\ -V & W & Z & Z & V & W & Z & Z \end{bmatrix} \quad (3)$$

where $Z = \frac{m}{4}$ zeros, $X = (c_0^{(a)}, \cdots, c_{\frac{m}{8}-1}^{(a)},\ c_{\frac{2m}{8}}^{(a)}, \cdots, c_{\frac{3m}{8}-1}^{(a)})$, $Y = (c_{\frac{4m}{8}}^{(a)}, \cdots, c_{\frac{5m}{8}-1}^{(a)},\ c_{\frac{6m}{8}}^{(a)}, \cdots, c_{\frac{7m}{8}-1}^{(a)})$, $V = (c_0^{(a)}, \cdots, c_{\frac{m}{4}-1}^{(a)})$, and $W = (c_{\frac{2m}{4}}^{(a)}, \cdots, c_{\frac{3m}{4}-1}^{(a)})$.

Any row of $\pm DA$ or $\pm DB$ is

$C_{2m}^{(a)} = (c_0^{(a)}, c_1^{(a)}, c_2^{(a)}, \cdots, c_{2m-1}^{(a)})$ with period 2m.

$C_{2m}^{(b)} = (c_0^{(b)}, c_1^{(a)}, c_2^{(a)}, \cdots, c_{2m-1}^{(b)})$ is generated from $C_{2m}^{(a)}$, where $c_q^{(b)} = (-1)^q c_q^{(a)}$ $(q = 0, 1, \cdots, 2m-1)$. $\{C_{2m}^{(a)},\ C_{2m}^{(b)}\}$ is a TPP with ZCD of $(0.75 \times 2m + 1)$ chips. Thus, for the period of $N = 4 \times 2^n$ $(n = 1, 2, 3 \cdots)$, $\{C_N^{(a)},\ C_N^{(b)}\}$ that have ZCD of $(0.75N + 1)$ chips can be constructed.

Example(i):

A TPP with a period $4 \times 2^2 = 16$ and ZCD=13 becomes

$$\begin{Bmatrix} C_{16}^{(a)} = (++zz+-zz++zz-+zz) \\ C_{16}^{(b)} = (+-zz++zz+-zz--zz) \end{Bmatrix} \text{ or}$$

$$\begin{Bmatrix} C_{16}^{(a)} = (+++-zzzz++-+zzzz) \\ C_{16}^{(b)} = (+-++zzzz+---zzzz) \end{Bmatrix}$$

where $+$, z, and $-$ denote 1, 0, and -1, respectively.

Step (II): Construction of sets of ternary ZCD code

A set of M ternary ZCD codes means a ternary code set that has ZCD $\leq (0.75N + 1)$ chips and a family size of M. This set can be constructed by the chip-shift operation using $\{C_N^{(a)}, C_N^{(b)}\}$. Let T^l be the chip-shift operator, which shifts a code cyclically to the left by l chips, a set of M ternary ZCD codes of period N can be generated from $\{C_N^{(a)}, C_N^{(b)}\}$ as

$$\{ C_N^{(a)}, C_N^{(b)}, T^{\Delta}[C_N^{(a)}], T^{\Delta}[C_N^{(b)}], T^{2\Delta}[C_N^{(a)}], T^{2\Delta}[C_N^{(b)}], \cdots, T^{(k-1)\Delta}[C_N^{(a)}], T^{(k-1)\Delta}[C_N^{(b)}], T^{k\Delta}[C_N^{(a)}], T^{k\Delta}[C_N^{(b)}] \} \quad (4)$$

where Δ *is a chip-shift increment and* k *the maximum* number of chip-shifts for a code.

Δ and k should satisfy $|(k+1)\Delta| \leq \left|\frac{3N}{8} + 1\right|$, where Δ is a positive and k a nonnegative integer. M and ZCD of the generated code become

$$\text{M} = 2(k+1) \text{ and } ZCD = |2\Delta - 1| \quad (5)$$

The generated codes have the normalized ACF peak value of 0.5. The family sizes of the proposed ternary ZCD codes are listed in Table 1.

Fig. 1 shows the family size against ZCD of the proposed codes with a period of 128. It is clear that proposed codes have larger family sizes than that of binary ZCD code pairs with ZCD [4] or QS(OG-r) codes generated from orthogonal Gold codes [5]. Here, large family sizes produce a larger CDMA user number and large ZCDs produce longer cell radius of AS-CDMA systems, respectively..

System Application Example:

The system capacity of AS-CDMA system[4] is estimated. *Required ZCD of an AS-CDMA system is determined from* cell radius R and chip rate of the spreading code Rc as

$$\text{ZCD} = (4R / c) \cdot Rc + 1 \quad (6)$$

where c is the light speed.

For example, when R=150m and Rc =3Mcps, the required ZCD is $((4 \cdot 150)/(3 \cdot 10^8)) \cdot 3 \cdot 10^6 + 1 = 17$ chips.

An example of the AS-CDMA system specification using the proposed codes is listed in Table 2. It is noted that the uplink of AS-CDMA system with 215 users is available, where bit rate is 16 kbps and $R = 155$ [m].

3. Conclusion

New ternary ZCD codes that have superior family size than that of binary ZCD codes were constructed. The proposed ternary ZCD codes can be usefully employed in AS-CDMA systems with sufficient system performance without creating.
MAI.

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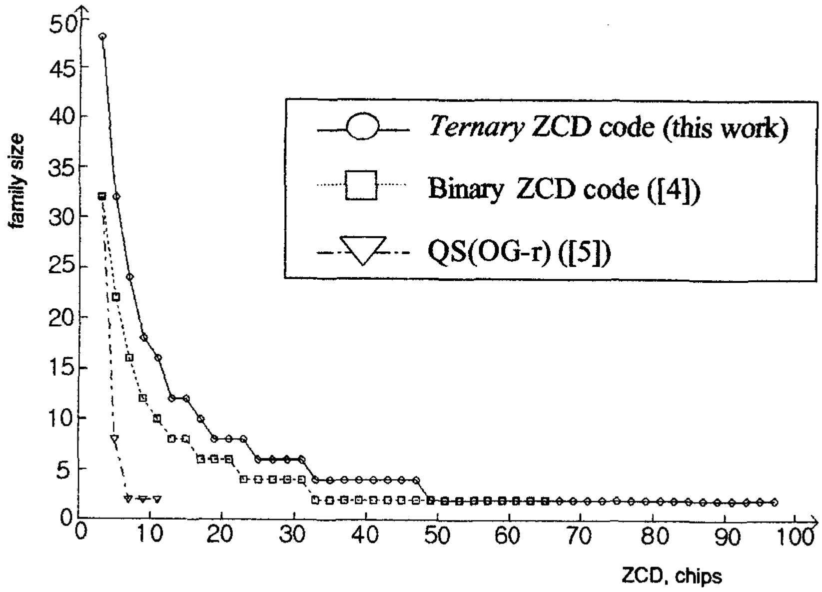

Fig. 1. Family size against ZCD of the ternary ZCD codes of period 128

Table 1: Family sizes of the *ternary* ZCD codes for N = 32, 64,128 and 256

N \ ZCD	*3*	*5*	*7*	*9*	*11*	*13*	*15*	*17*	*19*	*21*	*23*	*25*	*27*	*29*	*31*	...	*95*	...	*193*
32	12	8	6	4	4	2	2	2	2	2	2	2	-	-	-	-	-	-	-
64	24	16	12	8	8	6	6	6	4	4	4	2	2	2	2	...	-	-	-
128	48	32	24	18	16	12	12	10	8	8	8	6	6	6	6	...	2	-	-
256	96	64	48	38	32	26	24	20	18	16	16	14	12	12	12	...	4	...	2

Table 2: An example of the AS-CDMA system specification

	System A	System B
Access, Duplex	CDMA/FDMA, TDD	
Spreading code	Ternary ZCD codes (N=128)	
	M=12, ZCD=15	M=24, ZCD=7
Cell radius R	350 m	155 m
Available user number /cell	104	215
Bit rate Rb	16 k bit/s	16 k bit/s
Chip rate Rc	3.0 M chip/s	2.9 M chip/s
Information modulation	BPSK	

$$L=(ZCD-1)/2\ ,\ R_C = N/MFt\ ,\ \delta = L/R_C\ ,\ R=(c\cdot\delta)/2$$

$$User\ number = (M\cdot BW)/R_C\ ,\ Rb=(Rc\cdot rT)/(N+2L)$$

L : guard chip, M : family size, ZCD : zero correlation duration
δ : maximum propagation delay time, BW : 26 MHz (2.4 GHz ISM band)
MFt : N chip-processing time of matched filter
Period of spreading code : Tx : $N+2L$ $(=L+N+L)$, Rx :N
rT : uplink share of TDD, i.e., 0.75 [4]