Bayes Rule for MAC State Sojourn Time Supporting Packet Data Service in CDMA Wireless Celluar Networks *

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Abstract

MAC state models appeared with an effort to overcome technical demerits of CDMA in provisioning packet data service. In the scenario of sojourn and transition on MAC states, the design of state sojourn time is a critical issue for an efficient utilization of limited recource; a longer sojourn time leads to more resource being preserved for inactive stations, while more connection components should be recovered with a shorter sojourn time. Thus, the sojourn time at each MAC state must be optimized in consideration of these two conflicting arguements. In this paper, we first present a generic MAC state model. Secondly, based on the generic model, we reveal a relation of inactive period and the delay time of the last packet served in preceding active period and specify a loss function reflecting two antinomic features that result from a change of state sojourn time. Using the proposed loss function, we construct a decision problem to find an optimal rule for state sojourn times. Finally, we present a way of computing Bayes rule by use of the posterior distribution of inactivity duration for given observation on the delay time of last packet. Furthermore, Bayes rules are explicitly expressed for special arrival processes and investigated with respect to traffic load and loss parameters.

1. Introduction

For voice service, code division multiple access (CDMA) technology has been successfully employed at second generation wireless cellular networks. In the progress to third generation networks, the provision of high speed packet data service is conceived to be of necessity, while CDMA was reported to have inherently some technical demerits for transporting bursty data traffic [6] [7] [8]. With an effort to overcome such technical challenges, several medium access control (MAC) states were functionally defined and sojourn and transition scenario was presented on these MAC states in provisioning packet data service. Distinct channel assignments and connection status were also addressed at each MAC state. Specifically, active and dormant states are defined at IS-95B and control hold and suspended states are added at cdma2000 [6] [7]. In designing MAC state transition scheme, the following must be considered for effective resource utilization. Due to limited air interface capacity, limited base station equipment and constriant on mobile station power consumption, dedicated channels for packet data users must be allocated on demand and released immediately after the end of inactivity period. On the other hand, releasing the dedicated channels and reestablishing them introduces latency and signaling overhead due to the renegotiation process that has to take place between base station and mobile station before user data exchange [6]. Thus, the state sojourn time duting inactivity period must be carefully determined with consideration of the two antinomic arguements given above.

In this paper, we first propose a generic MAC state model consisting of an arbitrary integral number of states. For each state specified in the proposed MAC state model, we define a loss incurred by making a decision on the sojourn time at the state during an inactivity period of given length. Such loss itself is the sum of two losses; a constant loss reflecting the cost of reassigning channels and restoring a connection and a time-linear loss representing the invasion of channel and network resource by inactive station. Then, we construct a decision problem based on the observation of the delay time experienced by the last packet served in an active period preceding the inactivity period. Finally, we solve the decision problem and find the Bayes rule for state sojourn time with respect to a prior distribution for the length of inactivity period.

In section 2, we describe a generic MAC state model. In section 3, we formulate a decision problem by specifying loss function and present a way of computing Bayes rule with respect to a prior distribution. Furthermore, we present Bayes rules in a closed form for special processes of packet arrivals. In section 4, from numerical examples, we demonstrate posterior expected losses resulting from various traffic types and

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investigate the effect of traffic and network parameters on optimal sojourn time.

2. Generic MAC State Model

As shown in figure 1, our generic MAC state model consists of m MAC states, identified as $S_1, \cdots, S_m$. (One might think that S_1 and S_m correspond to active and dormant states defined in cdma2000, respectively.) Each state represents its own connection status.

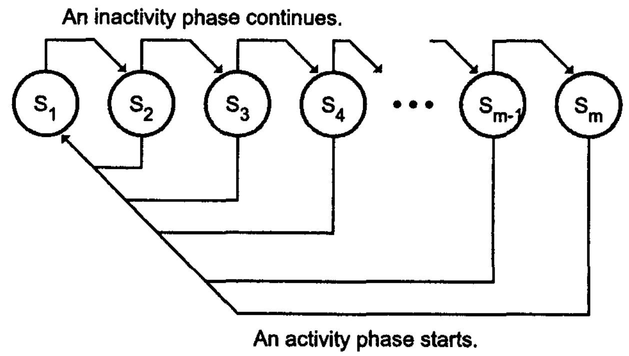

Figure 1: MAC state transition diagram

A lower index of state indicates that more connection components are maintained between transmitting and receiving stations. Consequently, at a state with higher index, more resource is preserved for the inactive transmitting station. For $k \in \{1, \cdots, m-1\}$, let τ_k denote the sojourn time at state S_k while the transmitting station is in an inactivity period. Among the m MAC states, state transitions occur as follows: Suppose that an inactivity phase durates longer than τ_k after a transition to S_k happens. Then, a transition to S_{k+1} takes palce immediately. On the other hand, once an activity phase starts, a transition to S_1 immediately occurs from any state.

3. Bayes Rule for State Sojourn Time

In this section, we first describe three components of a decision problem to find a decision rule for state sojourn times. Secondly, we present a way of computing Bayes rule from the decision problem. Finally, we illustrate Bayes rules in conjunction with two exemplary processes of packet arrivals at the transmitting node.

3.1 Packet Arrival and Service Model

Let $\{A_n, n = 0, 1, \cdots\}$ denote the sequence of packet arrival times at the transmitting station. Such arrival sequence is modeled as a non-explosive point process [1]. Packets are assumed to be transmitted according to an FCFS discipline and any packet is not instantaneously transmitted. Furthermore, no intentional vacation is allowed in the process of transmitting packets [9]. Then, the transmitting station can be viewed as a single-server queueing system with single-arrival and single-departure [3] [9]. In such single-server queueing system, busy period and idle period are alternatively repeated in accordance with the buffer occupancy. Let D denote the delay time of the last packet served in a busy period and U be the length of succeeding idle period. Let T denote the time elapsed from the moment the last packet arrives until the next packet arrives at the transmiiting station. Then, we have

$$T \overset{d}{=} D + U \,. \tag{1}$$

Since $P(D > d \mid U) = P(T > d + U \mid T > U, U)$, we have

$$\begin{aligned} & F(d \mid U) \\ & \triangleq \; P(D \le d \mid U) \\ & = \; \frac{P(T \le d + U \mid U) - P(T \le U \mid U)}{1 - P(T \le U \mid U)} \,. \end{aligned} \tag{2}$$

3.2 Decision Problem for State Sojourn Time

As described in section 2, there are m states, denoted by $S_1, \cdots, S_m$, in our MAC state model. For $k \in \{1, \cdots, m-1\}$, let τ_k denote the time to sojourn at S_k while the transmitting station is in an idle period. Let a be an action of making a decision on the state sojourn times. Then, $a = (\tau_1, \cdots, \tau_{m-1}) \in \mathcal{A}$, where the action space $\mathcal{A}$ is set to be $[0, \infty]^{m-1}$. Let $L(u, a)$ denote the loss incurred by action a when the length of idle period is equal to u. For $k \in \{1, \cdots, m-1\}$, we set

$$L(u, a) \triangleq \alpha_k + \sum_{i=1}^{k-1} \beta_i \tau_i + \beta_k \Big(u - \sum_{i=1}^{k-1} \tau_i\Big) \tag{3}$$

if $\sum_{i=1}^{k-1} \tau_i < u \le \sum_{i=1}^{k} \tau_i$, and

$$L(u, a) \triangleq \alpha_m + \sum_{i=1}^{m-1} \beta_i \tau_i$$

if $u > \sum_{i=1}^{m-1} \tau_i$. Note that the loss function given in (3) consists of two parts. One is a constant loss α_k which occurs at the transition from S_{k-1} to S_k. The constant loss represents the cost of recovering connection components at S_k. Since more components must be recovered at a state with higher index, $\alpha_k \le \alpha_{k+1}$ for $k \in \{1, \cdots, m-1\}$. The other is a time-linear loss $\beta_k t$ which is made while sojourning at S_k. The time-linear loss indicates the amount of resource which is available but preserved for the inactive transmitting station. At a state with higher index, the inactive station occupies less resource so that $\beta_k \ge \beta_{k+1}$ for $k \in \{1, \cdots, m-1\}$. Let $\mathcal{U}$ denote the parameter space which includes all possible values that the length of idle period U can realize. We set $\mathcal{U} = (0, \infty)$. Then, from the paramter space, action space and the loss function in (3), we construct a decision problem represented by triplet $(\mathcal{U}, \mathcal{A}, L)$ [5].

3.3 Bayes Rule for State Sojourn time

Suppose that we observe the delay time experienced by the last packet served in a busy period, (denoted by D). Let the support of D be $\mathcal{D}$. A decision rule δ is a mapping of an observation on the delay time to an action on state sojourn time such that $\delta : \mathcal{D} \to \mathcal{A}$. Suppose that the length of idle period U has a prior distribution G supporting parameter space $\mathcal{U}$. A Bayes rule with respect to the prior distribution G is a decision rule which minimizes the Bayes risk, and the Bayes rule is optimal in the sense that it takes the highest rank in the Bayes ordering of decision rules [5]. Note that the Bayes rule is also equivalent to an action which minimizes the posterior expected loss [2]. Let H be the posterior distribution for U given D. Then, from the prior distribution G and the conditional distribution F in (2), we have

$$H(u \mid d) = \frac{\int_0^u f(d \mid y) dG(y)}{\int_0^\infty f(d \mid x) dG(x)} \tag{4}$$

where $f(d \mid x)$ *is the conditional density of* D. From the posterior distribution H in (4), the posterior expected loss (incurred by action a) is defined as [2]

$$\rho(G,d,a) \triangleq \int_0^\infty L(u,a)\, dH(u \mid d) \tag{5}$$

for an observation d. Let δ^* denote a Bayes rule with respect to G. Then, $\delta^*(d)$ is equal to an action a^* such that

$$\rho(G,d,a^*) = \min\{\rho(G,d,a) \,:\, a \in \mathcal{A}\}\,.$$

Usually, a Bayes rule is not obtained in a closed form. In the following section, however, we will explicitly present Bayes rules for special point processes of packet arrivals.

3.4 Bayes Rule for Special Arrival Process

A. Poisson Arrival Process

Suppose that the sequence of packet arrival times is a Poisson point process with intensity λ. Then, the inter-arrival time T has an exponential distribution with paramter λ. Thus, we easily calculate

$$F(d \mid U) = \left[1 - e^{-\lambda d}\right] \cdot I_{\{d \geq 0\}} \tag{6}$$

where I_A is an indicator function such that $I_A = 1$ if A is true and $I_A = 0$ otherwise. Assume that the prior distribution for U is an exponential distribution with parameter λ, i.e., $G(u) = \left[1 - e^{-\lambda u}\right] \cdot I_{\{u \geq 0\}}$. Then, the posterior distribution is computed as

$$H(u \mid d) = \left[1 - e^{-\lambda u}\right] \cdot I_{\{u \geq 0\}} \tag{7}$$

for all $d \in \mathcal{D}$. For computational simplicity, set $\alpha_1 = 0$, $\alpha_k = (k-1)\alpha$, $\beta_m = 0$, $\beta_k = (m-k)\beta$ for constants α and β. Assume that $\tau_k = \tau$ for all $k = 1, \cdots, m-1$. (Thus, the action space $\mathcal{A} = [0, \infty]$.) Then, the posterior expected loss incurred by action τ is

$$\begin{aligned}&\rho(G,d,\tau)\\ &= \frac{(m-1)\beta}{\lambda} + [\alpha - \frac{\beta}{\lambda}]\frac{e^{-\lambda\tau}[1 - e^{-(m-1)\lambda\tau}]}{1 - e^{-\lambda\tau}}\end{aligned} \tag{8}$$

for an observation $d \in \mathcal{D}$. From the postrior expected loss in (8), we calculate the Bayes rule δ^* as follows:

$$\delta^*(d) = 0 \cdot I_{\{\alpha\lambda > \beta\}} + \infty \cdot I_{\{\alpha\lambda < \beta\}} \tag{9}$$

for all $d \in \mathcal{D}$. Note that the Bayes rule does not depend on the observation d. In fact, the Bayes rule is equivalent to an action minimizing prior expected loss for Poisson arrival processes.

B. Deterministic Arrival Process

Suppose that the inter-arrival time T is fixed to $\frac{1}{\lambda}$. Then, we obtain the conditional distribution for D given U as follows:

$$F(d \mid U) = I_{\{d \geq \frac{1}{\lambda} - U\}}\,. \tag{10}$$

Assume a prior distribution for U which is proper and strictly positive in $(0, \frac{1}{\lambda})$. Then, the posterior distribution is computed as

$$H(u \mid d) = I_{\{u \geq \frac{1}{\lambda} - d\}} \tag{11}$$

for all $d \in \mathcal{D}$. For illustrative purposes, consider an MAC state model consisting of two states, (denoted as S_1 and S_2). Set $\alpha_1 = 0$, $\alpha_2 = \alpha$, $\beta_1 = \beta$, $\beta_2 = 0$ for constants α and β. Then, the posterior expected loss incurred by action τ_1 is calculated to be

$$\begin{aligned}\rho(G,d,\tau_1) &= \beta[\frac{1}{\lambda} - d] \cdot I_{\{\tau_1 > \frac{1}{\lambda} - d\}}\\ &\quad + [\alpha + \beta\tau_1] \cdot I_{\{\tau_1 < \frac{1}{\lambda} - d\}}\end{aligned} \tag{12}$$

for an observation $d \in \mathcal{D}$. From the postrior expected loss in (12), we have the Bayes rule δ^* as follows:

$$\delta^*(d) = 0 \cdot I_{\{\alpha < \beta[\frac{1}{\lambda} - d]\}} + c \cdot I_{\{\alpha > \beta[\frac{1}{\lambda} - d]\}} \tag{13}$$

where c is any number in $(\frac{1}{\lambda} - d, \infty]$ for all $d \in \mathcal{D}$.

4. Numerical Examples

In this section, for Poisson and determinsitic arrival processes, we demonstrate the effect of traffic load (λ) and loss parameters (α_k and β_k) on the posterior expected loss ($\rho(G,d,u)$).

Figure 2 shows the posterior expected loss with respect to the MAC state sojourn time. In this figure, the number of MAC states is set to be 5. We also set $\alpha_k = k - 1$ (cost units) and $\beta_k = 5 - k$ (cost units/time

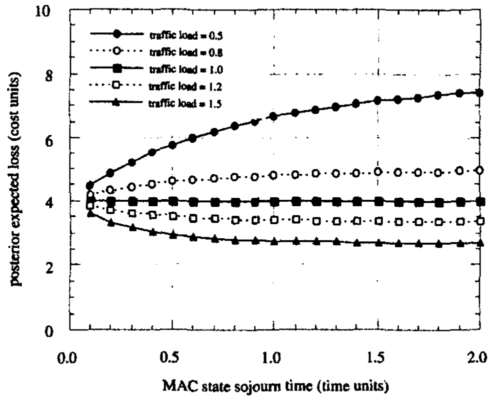

Figure 2: Posterior expected loss with respect to MAC state sojourn time

unit) for $k = 1, \cdots, 5$. The sequence of packet arrival times is assumed to be a Poisson point process with intensity $\lambda \in \{0.5, 0.8, 1.0, 1.2, 1.5\}$ (packets/time unit). For a prior distribution of the length of idle period, we choose an exponential distribution with parameter λ. In figure 2, we observe that the posterior expected loss is either monotone increasing or monotone decreasing with respect to state sojourn time. Thus, we confirm that the Bayes rule for state sojourn time is either 0 or ∞ as stated in section 3. We also note that the Bayes rule for state sojourn time is ∞ for relatively heavy traffic load (when the loss parameters are fixed).

Figure 3 shows the posterior expected loss with

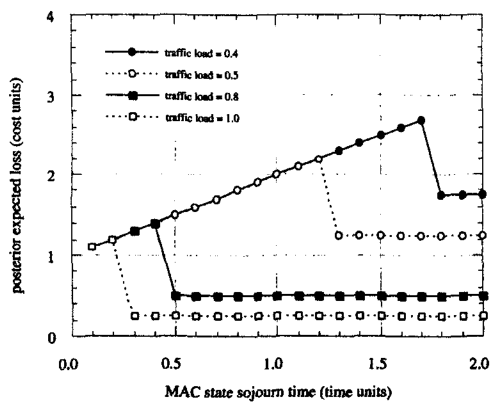

Figure 3: Posterior expected loss with respect to MAC state sojourn time

respect to the MAC state sojourn time. In this figure, the number of MAC states is set to be 2. We also set $\alpha_1 = 0$, $\alpha_2 = 1$ (cost units) and $\beta_1 = 1$, $\beta_2 = 0$ (cost units/time unit). The sequence of packet arrival times is assumed to be a deterministic point process with intensity $\lambda \in \{0.4, 0.5, 0.8, 1.0\}$ (packets/time unit). We also assume that the delay time of the last packet served in a busy period is observed to be 0.75 (time units). In figure 3, we observe that the posterior expected loss linearly increases in proportion to the state sojourn time until it abruptly drops and settles at a constant value. Such observation indicates that the Bayes rule is determined by the comparison of the initial and settled value of posterior expected loss. We also note that the Bayes rule for state sojourn time tends to approach ∞ for heavier traffic load (when the observed delay time is fixed).

5. Conclusions

For packet data service in CDMA wireless cellular networks, we presented a generic MAC state model. Based on the generic model, we revealed a relation of inactive period and the delay time of the last packet served in preceding active period and specified a loss function reflecting two antinomic features that result from a change of state sojourn time. To find an optimal rule for state sojourn timesy, we constructed a decision problem including the proposed loss function. From the decision problem, we present a way of finding Bayes rule for state sojourn time by use of the posterior distribution of idle period for given observation on the delay time of last packet. Especially, we explicitly expresses Bayes rules for Poisson and deterministic arrival processes.

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