An Adaptive Receding Horizon Controller for the Nuclear Steam Generator Water Level

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Abstract: In this work, a recursive parameter estimation algorithm estimates the mathematical model of steam generators every time step and a receding horizon controller is designed by using this estimated linear steam generator model of which parameters change as time goes on. It was shown through application to a linear model of steam generator that the proposed controller has good performance.

1. Introduction

The water level control problem of steam generators has been a main cause of unexpected shutdown of nuclear power plants. In particular, since the swell and shrink phenomena are significantly greater at low power, even to a skilled operator, it is difficult to react effectively in response to such a reverse dynamics of the water level, which is induced by the non-minimum phase effects. Also, the steam generator is a highly complex, non-linear, and time-varying system. Therefore, many advanced control methods have been suggested to resolve the steam generator water level control problem.

The receding horizon control is a kind of model predictive control that has received much attention as a powerful tool for the control of industrial process systems [1-3]. The basic concept of the model predictive control is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. This method presents many advantages over the conventional infinite horizon control because it is possible to handle input and state (or output) constraints in a systematic manner during the design and implementation of the control. In particular, it has been proved to be a suitable control strategy for time varying systems.

Therefore, in this work, the receding horizon control method combined with a parameter estimation algorithm of the extended least-squares method [4] was used to solve the steam generator water level control problems.

2. Receding Horizon Control

The receding horizon control method is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input among the solved optimal control inputs of several time steps. The procedure is then repeated at each subsequent instant. The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and model inaccuracy, both of which cause the measured system output to be different from the one predicted by the model.

In order to achieve fast responses and prevent excessive control effort, the associated performance index for deriving an optimal control input is represented by the following quadratic function:

$$J=\frac{1}{2}\sum_{j=1}^{N}Q(\hat{y}(t+j\mid t)-w(t+j))^2+\frac{1}{2}\sum_{j=1}^{M}\mu(\Delta u_{uv}(t+j-1))^2 \quad (1)$$

subject to constraints

$$\begin{cases} y(t+N+i)=w(t+N+i), \; i=1,\cdots,m \\ \Delta u_{uv}(t+j-1)=u(t+j-1)-v(t+j-1)=0, \quad j>M \end{cases}$$

where Δu_{uv} is the difference between the feedwater flowrate u and the steam flowrate v, and N and M are called the prediction horizon and the control horizon, respectively. The prediction horizon represents the limit of the instant in which it is desired for the output to follow the reference sequence. In order to obtain control inputs, the predicted outputs have to be first calculated as a function of past values of inputs and outputs and of future control signals.

The process is described by the Controlled Auto-Regressive and Integrated Moving Average (CARIMA) model:

$$A(q^{-1})y(t)=B(q^{-1})\Delta u_{uv}(t-1)+D(q^{-1})\xi(t) . \quad (2)$$

The process output at time $t+j$ can be predicted from the measurements of the output and input up to time step t. The j-step-ahead output prediction of a process is derived below.

Multiplying Eq. (2) by $E_j(q^{-1})$ from the left gives (the most usual case of $D(q^{-1})=1$ will be considered here)

$$y(t+j)-E_j(q^{-1})\xi(t+j)=F_j(q^{-1})y(t)+E_j(q^{-1})B(q^{-1})\Delta u_{uv}(t+j-1) , \quad (3)$$

where $E_j(q^{-1})$ and $F_j(q^{-1})$ are polynomials satisfying

$$1=E_j(q^{-1})A(q^{-1})+q^{-j}F_j(q^{-1}) , \quad (4)$$

$$E_j(q^{-1}) = e_{j,0} + e_{j,1}q^{-1} + \cdots + e_{j,j-1}q^{-(j-1)},$$

$$F_j(q^{-1}) = f_{j,0} + f_{j,1}q^{-1} + f_{j,2}q^{-2} + \cdots + f_{j,nA-1}q^{-nA+1}.$$

Equation (4) is called the Diophantine equation, whose solution can be found by an efficient recursive algorithm to be shown later. There exist unique polynomials $E_j(q^{-1})$ and $F_j(q^{-1})$ of order $j-1$ and $nA-1$, respectively, such that $e_{j,0}=1$. By taking the expectation operator on the both side of Eq. (3) and considering the zero mean noise $E\{\xi(t)\}=0$, the optimal j-step-ahead prediction of $y(t+j\mid t)$ satisfies

$$\hat{y}(t+j\mid t) = F_j(q^{-1})y(t) + G_j(q^{-1})\Delta u_{uv}(t+j-1), \quad (5)$$

where

$$G_j(q^{-1}) = E_j(q^{-1})B(q^{-1}).$$

$\hat{y}(t+j\mid t)$ denotes an estimated value of the output at time step $t+j$ based on all the data up to time step t. The output prediction can easily be extended to the nonzero mean noise case by adding a term $E_j(q^{-1})E\{\xi(t+j)\}$ to the output prediction $\hat{y}(t+j\mid t)$.

By dividing the polynomial, $G_j(q^{-1})$ into two terms, like the following equations:

$$G_j(q^{-1}) = \overline{G}_j(q^{-1}) + q^{-j}\widetilde{G}_j(q^{-1}) \quad \text{with } \delta\left(\overline{G}_j(q^{-1})\right) < j, \quad (6)$$

the prediction equation, Eq. (5), can now be written as

$$\hat{y}(t+j\mid t) = \overline{G}_j(q^{-1})\Delta u_{uv}(t+j-1) + \widetilde{G}_j(q^{-1})\Delta u_{uv}(t-1) + F_j(q^{-1})y(t), \quad (7)$$

where $\delta(\cdot)$ denotes the order of a polynomial. The last two terms of the right hand side of Eq. (7) consist of past values of the process input and output variables and correspond to the response of the process if the control input signals are kept constant. On the other hand, the first term of the right hand side consists of present and future values of Δu_{uv} signals and corresponds to the response obtained when the initial conditions are zero $y(t-j)=0$, $\Delta u_{uv}(t-j-1)=0$ for $j>0$ [5]. Equation (7) can be rewritten as

$$\hat{y}(t+j\mid t) = \overline{G}_j(q^{-1})\Delta u_{uv}(t+j-1) + f_j, \quad (8)$$

where

$$f_j = \widetilde{G}_j(q^{-1})\Delta u_{uv}(t-1) + F_j(q^{-1})y(t). \quad (9)$$

Then a set of N j-step-ahead output predictions can be expressed as

$$\hat{\mathbf{y}} = \overline{\mathbf{G}}\Delta\mathbf{u}_{uv} + \mathbf{f}, \quad (10)$$

where

$$\hat{\mathbf{y}} = \left[\hat{y}(t+1\mid t) \;\; \hat{y}(t+2\mid t) \;\; \cdots \;\; \hat{y}(t+j\mid t) \;\; \cdots \;\; \hat{y}(t+N\mid t)\right]^T,$$

$$\Delta\mathbf{u}_{uv} = \left[\Delta u_{uv}(t) \;\; \Delta u_{uv}(t+1) \;\; \cdots \;\; \Delta u_{uv}(t+j) \;\; \cdots \;\; \Delta u_{uv}(t+N-1)\right]^T,$$

$$\mathbf{f} = \left[f_1 \;\; f_2 \;\; \cdots \;\; f_j \;\; \cdots \;\; f_N\right]^T,$$

$$\overline{\mathbf{G}} = \begin{bmatrix} g_0 & 0 & \cdots & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ g_{j-1} & g_{j-2} & \cdots & g_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-2} & \cdots & \cdots & \cdots & g_0 \end{bmatrix},$$

$$\overline{G}_j(q^{-1}) = \sum_{i=0}^{j-1} g_i q^{-i}.$$

If all initial conditions are zero, the response $\mathbf{f}$ is zero. If a unit step is applied to the first input at time t; that is, $\Delta\mathbf{u}_{uv} = [1\,0\cdots 0]^T$, the expected output sequence $[\hat{y}(t+1)\;\hat{y}(t+2)\cdots\hat{y}(t+N)]^T$ is equal to the first column of the matrix $\overline{\mathbf{G}}$.

The computation of the control input involves the inversion of an $N\times N$ matrix $\overline{\mathbf{G}}$ that requires a substantial amount of computation. If the control signal is kept constant after the first M control moves (that is, $\Delta u_{uv}(t+j-1)=0$ for $j>M$), this leads to the inversion of an $M\times M$ matrix, which reduces the amount of computation. If so, the set of predictions affecting the objective function can be expressed as

$$\hat{\mathbf{y}} = \overline{\mathbf{G}}_s\Delta\mathbf{u}_s + \mathbf{f}, \quad (11)$$

where

$$\overline{\mathbf{G}}_s = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-2} & \cdots & g_{N-M} \end{bmatrix},$$

$$\Delta\mathbf{u}_s = \left[\Delta u_{uv}(t) \;\; \Delta u_{uv}(t+1) \;\; \cdots \;\; \Delta u_{uv}(t+M-1)\right]^T.$$

The following relationship can be derived from the foregoing equation:

$$\hat{\mathbf{y}}_f = \overline{\mathbf{G}}_{sf}\Delta\mathbf{u}_s + \mathbf{f}_f, \quad (12)$$

where

$$\hat{\mathbf{y}}_f = \left[\hat{y}(t+N+1\mid t) \;\; \hat{y}(t+N+2\mid t) \;\; \cdots \;\; \hat{y}(t+N+m\mid t)\right]^T,$$

$$\mathbf{f}_f = \left[f_{N+1} \;\; f_{N+2} \;\; \cdots \;\; f_{N+m}\right]^T,$$

$$\overline{\mathbf{G}}_{sf} = \begin{bmatrix} g_N & g_{N-1} & \cdots & g_{N-M+1} \\ g_{N+1} & g_N & \cdots & g_{N-M+2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N+m-1} & g_{N+m-2} & \cdots & g_{N-M+m} \end{bmatrix}.$$

The objective function of Eq. (1) can be rewritten as the following matrix-vector form:

$$J = \frac{1}{2}(\hat{\mathbf{y}}-\mathbf{w})^T\widetilde{\mathbf{Q}}(\hat{\mathbf{y}}-\mathbf{w}) + \frac{1}{2}\Delta\mathbf{u}_s^T\widetilde{\mathbf{R}}\Delta\mathbf{u}_s = \frac{1}{2}\left(\overline{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f}-\mathbf{w}\right)^T\widetilde{\mathbf{Q}}\left(\overline{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f}-\mathbf{w}\right) + \frac{1}{2}\Delta\mathbf{u}_s^T\widetilde{\mathbf{R}}\Delta\mathbf{u}_s, \quad (13)$$

$$\text{subject to constraint } \mathbf{w}_f = \overline{\mathbf{G}}_{sf}\Delta\mathbf{u}_s + \mathbf{f}_f, \quad (14)$$

where

$$\mathbf{w} = [w(t+1\,|\,t) \;\; w(t+2\,|\,t) \;\; \cdots \;\; w(t+N\,|\,t)]^T,$$

$$\mathbf{w}_f = [w(t+N+1\,|\,t) \;\; w(t+N+2\,|\,t) \;\; \cdots \;\; w(t+N+m\,|\,t)]^T.$$

$\tilde{\mathbf{Q}} = diag(Q, \cdots, Q)$ is a diagonal matrix consisting of N diagonal elements, Q , and $\tilde{\mathbf{R}} = diag(\mu \cdots, \mu)$ is a diagonal matrix consisting of M diagonal elements, μ .

The optimal input can be obtained by the well-known Lagrange multiplier approach. To apply the Lagrange multiplier approach, the objective function is rewritten as

$$J' = \frac{1}{2}(\overline{\mathbf{G}}_s \Delta \mathbf{u}_s + \mathbf{f} - \mathbf{w})^T \tilde{\mathbf{Q}} (\overline{\mathbf{G}}_s \Delta \mathbf{u}_s + \mathbf{f} - \mathbf{w}) + \frac{1}{2} \Delta \mathbf{u}_s^T \tilde{\mathbf{R}} \Delta \mathbf{u}_s + \boldsymbol{\lambda}^T (\overline{\mathbf{G}}_{sf} \Delta \mathbf{u}_s + \mathbf{f}_f - \mathbf{w}_f). \tag{15}$$

The optimal control input can be expressed as

$$\Delta \mathbf{u}_s = (\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}})^{-1} \Big[\overline{\mathbf{G}}_s^T \; \tilde{\mathbf{Q}}(\mathbf{w} - \mathbf{f}) + \overline{\mathbf{G}}_{sf}^T \Big[\overline{\mathbf{G}}_{sf} (\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}})^{-1} \overline{\mathbf{G}}_{sf}^T \Big]^{-1} \Big[\mathbf{w}_f - \mathbf{f}_f - \overline{\mathbf{G}}_{sf} (\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}})^{-1} [\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}}(\mathbf{w} - \mathbf{f})] \Big] \Big]. \tag{16}$$

In order to obtain the control input from Eq. (16), it is necessary to calculate the matrices $\overline{\mathbf{G}}_s$, $\overline{\mathbf{G}}_{sf}$, and the vectors $\mathbf{f}$ and $\mathbf{f}_f$. These matrix and vector can be calculated recursively. From now on, the derivation will be described.

By taking into account a new Diophantine equation corresponding to the prediction for $\hat{y}(t+j+1\,|\,t)$, Eq. (4) can also be rewritten as follows:

$$1 = E_{j+1}(q^{-1})A(q^{-1}) + q^{-(j+1)}F_{j+1}(q^{-1}). \tag{17}$$

Subtracting Eq. (4) from Eq. (17) gives

$$0 = [E_{j+1}(q^{-1}) - E_j(q^{-1})]A(q^{-1}) + q^{-j}[q^{-1}F_{j+1}(q^{-1}) - F_j(q^{-1})]. \tag{18}$$

Since the matrix $E_{j+1}(q^{-1}) - E_j(q^{-1})$ is of order j , the matrix can be written as

$$E_{j+1}(q^{-1}) - E_j(q^{-1}) = \tilde{P}(q^{-1}) + p_j q^{-j}, \tag{19}$$

where $\tilde{P}(q^{-1})$ is a polynomial of order smaller than or equal to $j-1$. By substituting Eq. (19) into Eq. (18)

$$0 = \tilde{P}(q^{-1})A(q^{-1}) + q^{-j}[p_j A(q^{-1}) + q^{-1}F_{j+1}(q^{-1}) - F_j(q^{-1})]. \tag{20}$$

Since $A(q^{-1})$ is monic, $\tilde{P}(q^{-1}) = 0$. Therefore, from Eq. (20) the polynomial $E_{j+1}(q^{-1})$ can be calculated recursively by

$$E_{j+1}(q^{-1}) = E_j(q^{-1}) + p_j q^{-j}. \tag{21}$$

The following expressions can easily be obtained from Eq. (21):

$$p_j = f_{j,0}, \tag{22}$$

$$f_{j+1,i} = f_{j,i+1} - p_j a_{i+1} \text{ for } i = 0, \cdots, \delta(F_{j+1}). \tag{23}$$

Also, it can easily be seen that the initial conditions for the recursion equation are given by

$$E_1 = 1, \tag{24}$$

$$F_1 = q(1 - A(q^{-1})). \tag{25}$$

The vectors $\mathbf{f}$ and $\mathbf{f}_f$ can be computed by the following recursive relationship:

$$f_{j+1} = q(1 - A(q^{-1}))f_j + B(q^{-1})\Delta u_{uv}(t+j), \tag{26}$$

with $f_0 = y(t)$, $\Delta u_{uv}(t+j) = 0$ for $j \geq 0$.

Also, the polynomial, $G_j(q^{-1})$, can be obtained recursively as follows:

$$G_{j+1}(q^{-1}) = E_{j+1}(q^{-1})B(q^{-1}) = G_j(q^{-1}) + f_{j,0} q^{-j} B(q^{-1}). \tag{27}$$

3. Parameter Estimation

From Eq. (5) the optimal one-step-ahead prediction of $y(t+1\,|\,t)$ is as follows:

$$\begin{aligned} \hat{y}(t+1\,|\,t) &= F_1(q^{-1})y(t) + G_1(q^{-1})\Delta u_{uv}(t) \\ &= -\hat{a}_1 y(t) - \hat{a}_2 y(t-1) - \cdots - \hat{a}_{nA} y(t-nA+1) \\ &\quad + \hat{b}_0 \Delta u_{uv}(t) + \hat{b}_1 \Delta u_{uv}(t-1) + \cdots + \hat{b}_{nB} \Delta u_{uv}(t-nB). \end{aligned} \tag{28}$$

Equation (28) can be expressed in the following inner product of the parameter vector $\hat{\boldsymbol{\theta}}(t)$ and the measurement vector $\boldsymbol{\varphi}(t)$:

$$\hat{y}(t+1) = \hat{\boldsymbol{\theta}}^T(t) \cdot \boldsymbol{\varphi}(t), \tag{29}$$

where

$$\hat{\boldsymbol{\theta}}^T(t) = [-\hat{a}_1(t) \;\; -\hat{a}_2(t) \;\; \cdots \;\; -\hat{a}_{nA}(t) \;\; \hat{b}_0(t) \;\; \hat{b}_1(t) \;\; \cdots \;\; \hat{b}_{nB}(t)],$$

$$\boldsymbol{\varphi}(t) = [y(t) \;\; y(t-1) \;\; \cdots \;\; y(t-nA+1) \;\; \Delta u_{uv}(t) \;\; \Delta u_{uv}(t-1) \;\; \cdots \;\; \Delta u_{uv}(t-nB)]^T.$$

The parameter vector $\hat{\boldsymbol{\theta}}(t)$ is estimated as follows [4]:

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \mathbf{F}(t)\boldsymbol{\varphi}(t-1)[y(t) - \hat{\boldsymbol{\theta}}^T(t-1) \cdot \boldsymbol{\varphi}(t-1)], \tag{30}$$

$$\mathbf{F}(t) = \frac{1}{\lambda(t)} \left[\mathbf{F}(t-1) - \frac{\mathbf{F}(t-1)\boldsymbol{\varphi}(t-1)\boldsymbol{\varphi}^T(t-1)\mathbf{F}(t-1)}{\lambda(t) + \boldsymbol{\varphi}^T(t-1)\mathbf{F}(t-1)\boldsymbol{\varphi}(t-1)} \right], \tag{31}$$

where the covariance matrix $\mathbf{F}(0) > 0$ and $0 < \lambda(t) \leq 1$. The forgetting factor $\lambda(t)$ is calculated from the following equation [4]:

$$\lambda(t) = \lambda_0 \lambda(t-1) + (1 - \lambda_0) \quad \text{with } \lambda_0 \leq 1 \text{ and } \lambda(0) \leq 1 \tag{32}$$

Figure 1 shows the schematic diagram of the receding horizon controller combined with a parameter estimation algorithm.

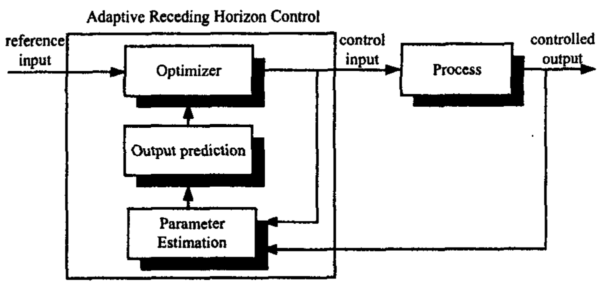

Fig. 1. Schematic diagram of an adaptive receding horizon control method.

4. Application to the Steam Generator Water Level Control

The linear steam generator model derived by Irving [6] was used. Since the parameter values are given at several specific power levels and very different according to the power levels, the parameters of the controlled plant were interpolated versus power. The prediction and control horizons were chosen as 20 and 10, respectively, and the same values were used regardless of power level. The weighting matrices are chosen to be $\mathbf{Q} = \mathbf{I}$ and $\mathbf{R} = \mu\mathbf{I}$. The input-weighting factor μ is different according to power level to accomplish good performance but its value can be chosen easily and decreases exponentially as power level increases.

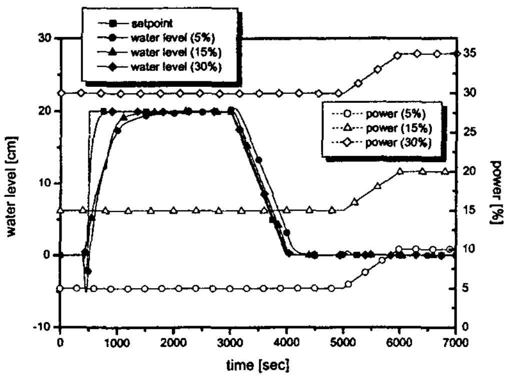

Fig. 2. Performance of the proposed controller for the linear model.

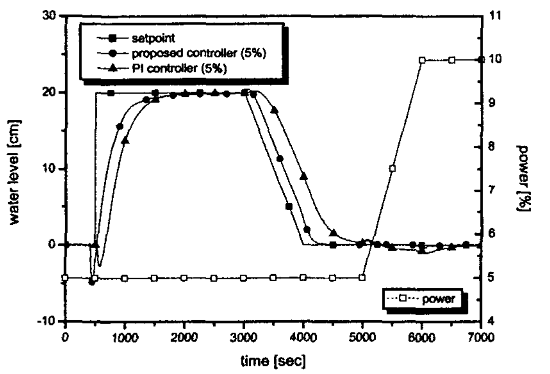

Fig. 3. Comparison of the proposed controller and the PI controller for the linear model at 5% power.

Figure 2 shows the performances of this proposed controller. In these figures, all values represent the difference from the corresponding steady state values. Therefore, all values are zeros at $t = 0$. Also, noise signals are added to describe a real plant. The magnitude of the steam flow disturbance between 5000 sec and 6000 sec corresponds to 5 percent steam flowrate increase at each corresponding power level. The proposed control algorithm tracks well the setpoint and steam flowrate changes. The swell and shrink phenomena are larger at low power levels than those at high power levels. Also, the water level tracks its setpoint faster at high powers than at low powers.

Figure 3 shows the performances of the proposed controller (which corresponds to the 5% power simulation of Fig. 2) and the conventional PI (proportional integral) controller around 5 percent power level. In this simulation, the PI controller gains were optimized by a genetic algorithm. The proposed controller shows better performance under ramp change of the steam flowrate disturbance and the step and ramp changes of the water level setpoint, and also shows a little faster response.

5. Conclusions

In this work, the adaptive predictive controller was developed to control the water level of nuclear steam generators. The steam generator water level controller was designed to effectively cope with water level deviation and steam flow disturbance and especially, computer simulations were conducted to investigate the output tracking performance and swell and shrink characteristics. The proposed controller was compared to the PI controller and was known to have better performance. Since the steam generator has nonlinear characteristics, the proposed algorithm was applied to a nonlinear model of the nuclear steam generator to examine its actual performance. Also, the proposed controller showed good performance for this nonlinear plant against the water level setpoint and steam flowrate (measurable disturbance) changes.

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