For the Association between 3D VAR Model and 2D Features

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Abstract: Although we look at objects as 2D images through our eyes, we can reconstruct the shape and/or depth of objects. In order to realize this ability using computers, it is required that the method which can estimate the 3D features of object from 2D images.

As feature which represents 3D shapes effectively, three dimensional vector autoregressive model is proposed. If this feature is associated other feature of 2D shape, then above aim might be achieved.

On the other hand, as feature which represents 2D shapes, quasi moment features is proposed.

As the first step of association of these features, we constructed real time simulator that computes both of two features concurrently from object data (3D curves). This simulator can also rotate object and estimate the rotation.

The method using 3D VAR model estimates the rotation correctly, but the estimation by quasi moment features includes much errors. This reason would be that projected images are constructed by the points only, and doesn't have enough sizes to estimate the correct 3D rotation parameters.

1. Introduction

Although we look at objects as 2D images through our eyes, we can reconstruct the shape and/or depth of objects. In order to realize this using computers, it is required that the method which can estimate the 3D features of object from 2D images. As feature which represents 3D shapes effectively, three dimensional vector autoregressive model is proposed [1][2][3]. If this feature is associated other feature of 2D shape, then above aim might be achieved.

3D VAR model is constructed on the quaternion, which is the basis of SU(2) (the rotation group in two dimensional complex space). This enables us to define the 3D VAR model as the external products of 3D sequential data and the autoregressive (AR) coefficients, unlike the usual AR models. Therefore the 3D VAR model has some prominent features. For example, the AR coefficients of the 3D VAR model behave like vectors under any three dimensional rotation.

On the other hand, quasi moment features, which is presented Tanaka[7] ([8] also gives the equal features up to the second order), is also the invariant / covariant feature under rotation. Shimai, Kawamoto and Tanaka et al. estimated the rotation from two images captured by active camera (before rotation and after rotation) using quasi moment features[9]. Quasi moment features are expected as effective method for representation of 2D features.

As the first step of association of these features, we constructed real time simulator that computes both of two features concurrently from object data (3D curves). This simulator can also rotate object and estimate the rotation. In this paper, we made some experiment using this simulator to evaluate quasi moment features as which treats 2D features of the three dimensional curves.

2. 3D VAR model and Invariants

When a sequential data set

$$\{\mathbf{z}_j = (x_j, y_j, z_j)^{\mathrm{T}}\}_{i=0}^{N-1}\ (\mathbf{z}_{-j} = \mathbf{z}_{N-j})$$

is given, 3D VAR model of the m-th order is defined by

$$\hat{\mathbf{z}}_j = \sum_{k=1}^{m} \mathbf{z}_{j-k} \times \mathbf{a}_m^k.$$

The coefficients $\{\mathbf{a}_m^k\}_{k=1}^m$ of 3D VAR model are called *3D VAR coefficients* which are determined to minimize the mean square error[6].

We have used the third order 3D VAR model to verify the behavior of 3D VAR coefficients, and the 3×3 matrix $\mathcal{A}$, which the p-th column is $\mathbf{a}_3^p$, that is,

$$\mathcal{A} = (\,\mathbf{a}_3^1 \quad \mathbf{a}_3^2 \quad \mathbf{a}_3^3\,). \tag{1}$$

In this case, the invariants derived from 3D VAR coefficients of the third order is represented by the matrix G, of its (p, q) components are the inner product between $\mathbf{a}_3^p$ and $\mathbf{a}_3^q$, that is,

$$G = \mathcal{A}^{\mathrm{T}}\mathcal{A} = \begin{pmatrix} (\mathbf{a}_3^1)^{\mathrm{T}}\mathbf{a}_3^1 & (\mathbf{a}_3^1)^{\mathrm{T}}\mathbf{a}_3^2 & (\mathbf{a}_3^1)^{\mathrm{T}}\mathbf{a}_3^3 \\ (\mathbf{a}_3^2)^{\mathrm{T}}\mathbf{a}_3^1 & (\mathbf{a}_3^2)^{\mathrm{T}}\mathbf{a}_3^2 & (\mathbf{a}_3^2)^{\mathrm{T}}\mathbf{a}_3^3 \\ (\mathbf{a}_3^3)^{\mathrm{T}}\mathbf{a}_3^1 & (\mathbf{a}_3^3)^{\mathrm{T}}\mathbf{a}_3^2 & (\mathbf{a}_3^3)^{\mathrm{T}}\mathbf{a}_3^3 \end{pmatrix} \tag{2}$$

which has the invariance under 3D rotation and the scaling.

From the experiment processing 3D curves we have made, it is proved that this invariant has following features :

- When a 3D curve is on some plane, that is, the 3D curve is the subset of the plane, it is easy to understand that the rank of G is 1. However, the rank of G can be 1, even if the curve doesn't lie on the plane.

- When the 3D rotation of which transformed the curves identity is unique, the rank of G has to be 3, however, when the curve has high symmetry, there are many orthogonal matrices which transform the curve into the same curve. Therefore, the rank of G has to be less than 3, that is, G is the singular matrix.
- When the points are ordered randomly, the eigen values of G tend to be small and not biased, however, when the points are ordered periodically, the eigen values of G is biased and the rank of G is expected to be 1.

3. 2D Features Using Quasi Moment Features

Quasi moment features, which is presented Tanaka[7] ([8] also gives the equal features up to the second order), can be computed using projected images of objects and its focal length, and this feature is able to treat the rotation of objects because the 2nd order quasi moment feature T behaves tensor under 3D rotation.

Projection image function $F(x,y)$ is obtained through a projection of an object onto 2D plane $Z=f$ (in short, f is the focal length). If the object constructed with point $\mathrm{P}(P=(X,Y,Z)^T)$, point P is projected along following equation.

$$x = f\frac{X}{Z}, \qquad y = f\frac{Y}{Z} \tag{3}$$

The 0th order quasi moment is

$$\mathbf{S} = \int\!\!\int S(x,y)F(x,y)\mathrm{d}m(x,y), \tag{4}$$

$$S(x,y) = 1. \tag{5}$$

the 1th order quasi moment is

$$\mathbf{V} = \int\!\!\int V(x,y)F(x,y)\mathrm{d}m(x,y), \tag{6}$$

$$V(x,y) = \frac{1}{k}\begin{pmatrix} x \\ y \\ f \end{pmatrix}, \tag{7}$$

the 2th order quasi moment is

$$\mathbf{T} = \int\!\!\int T(x,y)F(x,y)\mathrm{d}m(x,y), \tag{8}$$

$$T(x,y) = \frac{1}{k^2}\begin{pmatrix} x^2 & xy & xf \\ yx & y^2 & yf \\ fx & fy & f^2 \end{pmatrix} \tag{9}$$

$$= V(x,y)V(x,y)^{\mathrm{T}}, \tag{10}$$

where

$$\mathrm{d}m(x,y) = \frac{f\mathrm{d}x\mathrm{d}y}{\sqrt{(x^2+y^2+f^2)^3}} \tag{11}$$

is the invariant measure, and

$$k = \sqrt{x^2+y^2+f^2}. \tag{12}$$

4. Experiment and Results

We have an experiment of processing 6 three dimensional curves using our simulator. The experiment along the following procedure:

1. Rotate the curves.
2. Estimate the rotation angle and axis using 3D VAR model and quasi moment features.
3. Compare these two estimated rotation and true rotation.

The results are indicated from figure 1 to figure 6 and table 1. (a) figures indicate the initial pattern of curves, and (b) figures indicate the rotated pattern. (c) figures indicate the projected image of initial pattern, and (d) figures indicate the projected image rotated pattern. The rotation parameters are written in the table 1. "True" rows indicate the true rotation parameters, "3DVAR" rows indicate the estimated rotation parameters using 3D VAR model, and "QMF" rows indicate the estimated rotation parameters using quasi moment features. The error rate comparing the estimated rotation and the true rotation is written at the second row of "QMF" rows. On the "Axis" column, correlation between the true axis and the estimated axis is also written.

Table 1. The result of the simulation

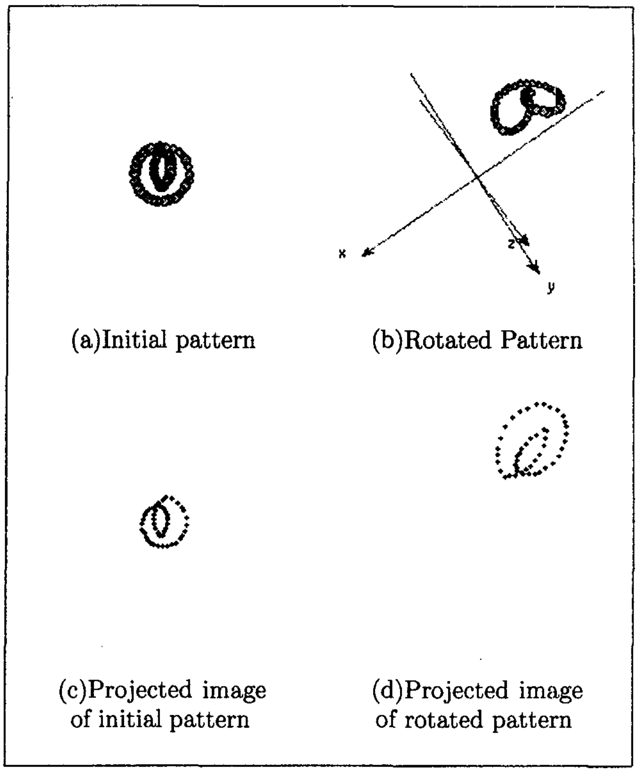
Data	Method	Angle	Axis
Curve1	True	145.7	-0.1131 -0.3424 -0.9327
Twisted	3DVAR	145.7	-0.1131 -0.3424 -0.9327
circle	QMF	98.10	0.4165 -0.8887 0.1915
		(32.66%)	(0.0785,92.15%)
Curve2	True	67.99	0.6284 -0.3035 0.7162
Knot1	3DVAR	67.99	0.6284 -0.3036 0.7162
	QMF	104.2	0.8965 -0.3925 -0.2058
		(53.22%)	(0.5351,46.49%)
Curve3	True	37.76	-0.6315 0.1637 0.7579
Knot2	3DVAR	37.76	-0.6315 0.1637 0.7579
	QMF	44.63	-0.5626 0.0976 0.8209
		(18.19%)	(0.9935,0.6545%)
Curve4	True	43.78	0.3277 0.1233 -0.9367
Random	3DVAR	43.78	0.3277 0.1233 -0.9367
	QMF	44.63	0.3535 0.0892 -0.9312
		(1.948%)	(0.9991,0.0932%)
Curve5	True	48.00	-0.4659 -0.1797 -0.8664
Spiral1	3DVAR	48.00	-0.4659 -0.1797 -0.8664
	QMF	41.91	-0.5079 -0.2094 -0.8356
		(12.70%)	(0.9982,0.1797%)
Curve6	True	66.61	-0.6747 -0.2991 -0.6747
Spiral2	3DVAR	66.61	-0.6747 -0.2991 -0.6747
	QMF	90.01	-0.6803 0.6164 0.3965
		(35.14%)	(0.0072,99.29%)

As we can know from table 1, the estimation using quasi moment features includes much errors. This reason would be that projected images are constructed by the points only, and doesn't have enough sizes to estimate the correct 3D rotation parameters. However, the result of random distributed curve (curve 4) includes small error. This reason would be that points are distributed uniformly in the projected image of random distributed curve.

5. Conclusion

We expect that the association between the feature from the three dimensional space (invariants from 3D VAR coefficients) and the 2D features (quasi moment features) might be useful for object understanding.

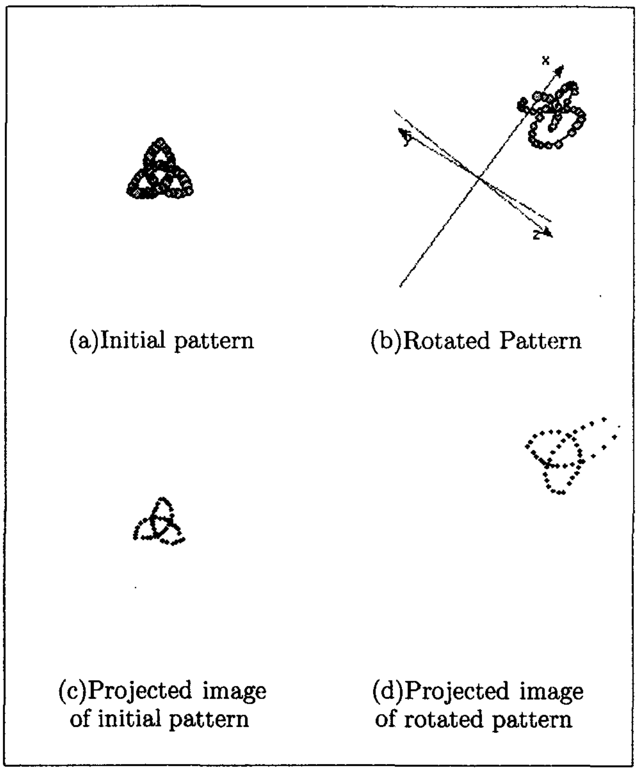
As the first step of it, we constructed real time simulator that computes both of two features from object data (3D curves) concurrently. This simulator can also rotate object and estimate the rotation. The method using 3D VAR model estimates the rotation correctly, but the estimation using quasi moment features includes much errors. This reason would be that projected images are constructed by the points only, and doesn't have enough sizes to estimate the correct 3D rotation parameters. To use quasi moment features as 2D features of projected image of 3D curve, one more step will be needed. For example, making projected images magnifying the points of 3D curves is considered.



(a)Initial pattern (b)Rotated Pattern

(c)Projected image of initial pattern (d)Projected image of rotated pattern

Figure 1. Curve1 ... Twisted circle



(a)Initial pattern (b)Rotated Pattern

(c)Projected image of initial pattern (d)Projected image of rotated pattern

Figure 2. Curve2 ... Knot1

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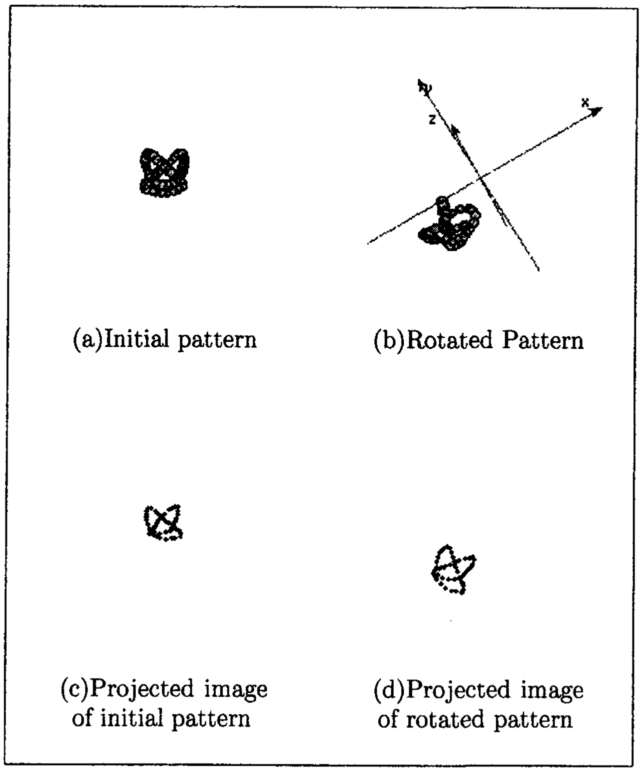

Figure 3. Curve3 ... Knot2

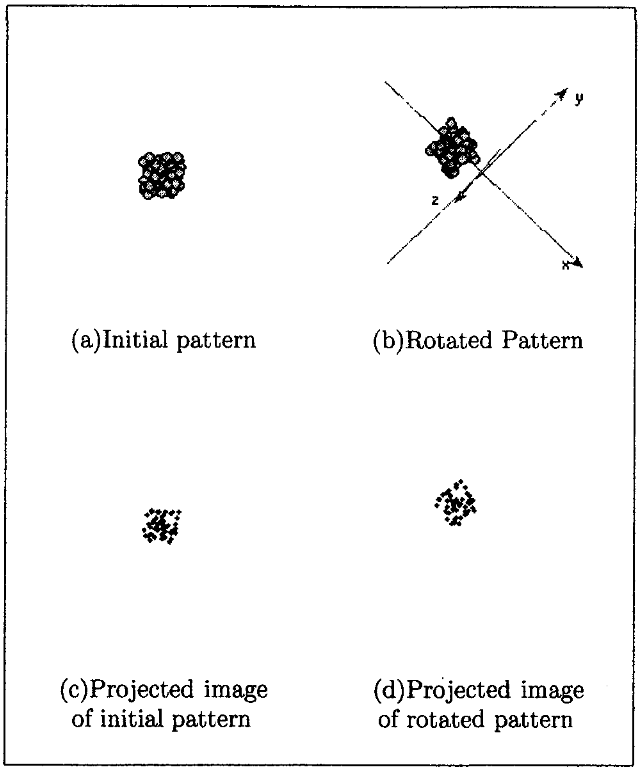

Figure 4. Curve4 ... Random distributed

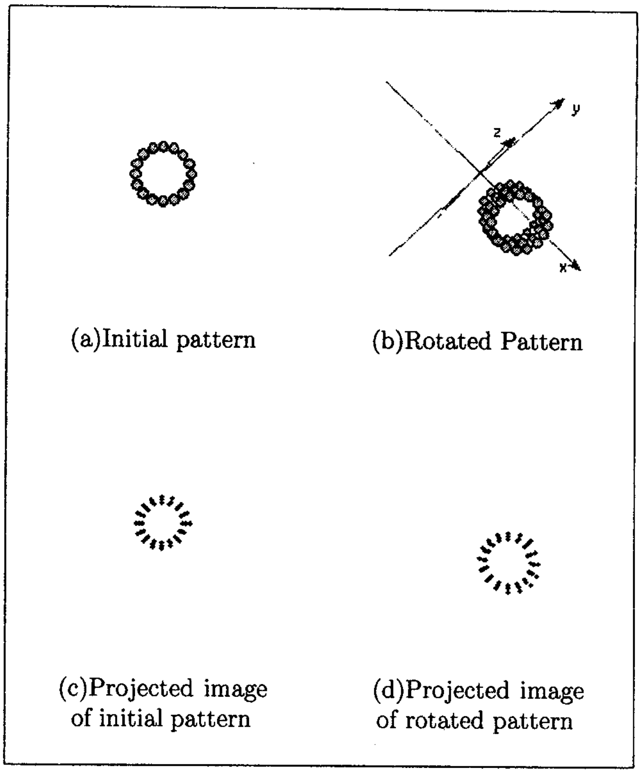

Figure 5. Curve5 ... Spiral1

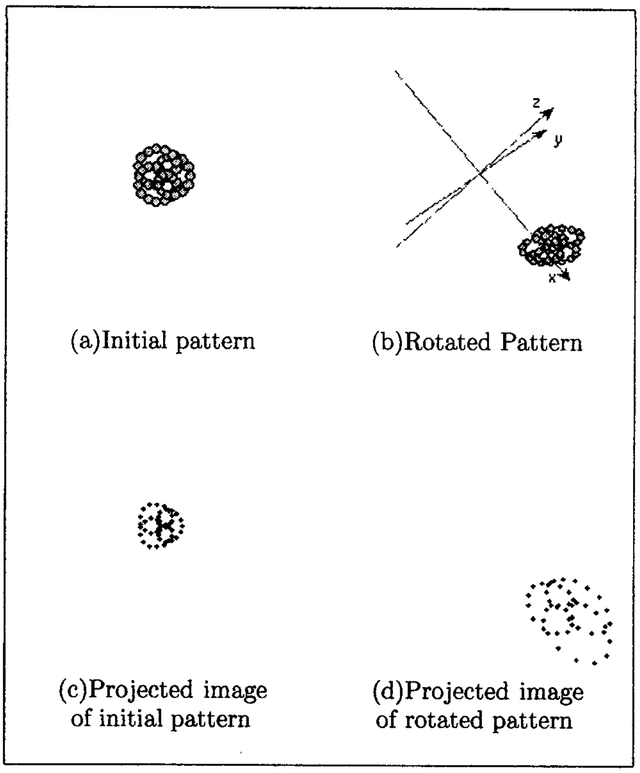

Figure 6. Curve6 ... Spiral2