+4:1 평면수축유동에서의 와류성장기구에 관한 시간-와이젠버그 중첩현상

정창권, 김주민°, 안경현, 이승종
서울대학교 공과대학 응용화학부
°LG 전자

Time-We superposition in a vortex growth mechanism of a 4:1 planar contraction flow

Chang Kwon Chung, Ju Min Kim°, Kyung Hyun Ahn, Seung Jong Lee
School of Chemical Engineering, Seoul National University
°LG Electronics

Introduction

Computational rheologists have tried to achieve the solutions of the 4:1 contraction flow of the highly elastic fluids with various numerical schemes. But it is not easy to obtain the converged solution of highly elastic fluid in the contraction flow, especially in the neighborhood of a singular point. Nevertheless, the critical Weissenberg number of the numerical solutions has been increased persistently due to the improved numerical approaches.

In this work, we obtained transient solutions of the highly elastic fluid using the fractional 4-step finite element formulation with the stabilizing skill such as DEVSS-G (discrete elastic-viscous stress-splitting) / DG (discontinuous Galerkin). As a constitutive equation, the Oldroyd-B model was used. With these improved stabilizing techniques [1,2], the coupled problem of velocity and pressure could be split into several small problems. Here, we could observe a very interesting transient behavior of the highly elastic fluid in the 4:1 planar contraction flow simulation.

Theory

We consider momentum equation including inertia and transient terms with incompressible constraint, and the Oldroyd-B model is used as a constitutive equation.

$$Re\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla \boldsymbol{u}\right)=-\nabla p+\beta\nabla\cdot\left(\nabla \boldsymbol{u}+(\nabla \boldsymbol{u})^T\right)+\nabla\cdot\boldsymbol{\tau}_p \quad (1)$$

$$\nabla\cdot\boldsymbol{u}=0 \quad (2)$$

$$\boldsymbol{\tau}_p+We\boldsymbol{\tau}_{p(1)}=(1-\beta)\left(\nabla \boldsymbol{u}+(\nabla \boldsymbol{u})^T\right) \quad (3)$$

where $Re \equiv HU\rho/\eta_0$ is the Reynolds number, $\beta \equiv \eta_s/\eta_0$ is the ratio of the solvent viscosity (η_s) and the solution viscosity (η_0). H is a characteristic length and ρ is the density. $We \equiv \lambda U/H$ is the Weissenberg number, where λ is the relaxation time of the Oldroyd-B model, and $\tau_{p(1)}$ is the upper-convected derivative of tensor τ_p.

$$\tau_{p(l)}=\frac{\partial \tau_p}{\partial t}+u\cdot\nabla\tau_p-(\nabla u)^T\cdot\tau_p-\tau_p\cdot\nabla u \quad (4)$$

Here we adopt fully implicit 4-step method[3], and carry out FEM formulation for the Navier-Stokes equations[4]. Among the fractional 4-step finite element formulations, the 1st equation can be replaced with DEVSS-G formulation.

$$G_{ij}^{n+1}=\frac{\partial u_i^{n+1}}{\partial x_j} \quad (5)$$

$$Re\left(\frac{\hat{u}_i-u_i^n}{\Delta t}+\frac{1}{2}\frac{\partial}{\partial x_j}\left(\hat{u}_i\hat{u}_j+u_i^n u_j^n\right)\right)=$$
$$-\frac{\partial p^n}{\partial x_i}+\frac{1}{2}\left(\frac{\partial \tau_{ij}^{n+1}}{\partial x_j}+\frac{\partial \tau_{ij}^n}{\partial x_j}\right)+\frac{1}{2}\left(\frac{\partial^2\hat{u}_i}{\partial x_j\partial x_j}+\frac{\partial^2 u_i^n}{\partial x_j\partial x_j}\right)$$
$$-\frac{1}{2}(1-\beta)\left(3\left(\frac{\partial G_{ij}^n}{\partial x_j}+\frac{\partial (G_{ij}^n)^T}{\partial x_j}\right)-\left(\frac{\partial G_{ij}^{n-1}}{\partial x_j}+\frac{\partial (G_{ij}^{n-1})^T}{\partial x_j}\right)\right) \quad (6\text{-}1)$$

$$Re\left(\frac{u_i^*-\hat{u}_i}{\Delta t}\right)=\frac{\partial p^n}{\partial x_i} \quad (6\text{-}2)$$

$$\frac{\partial^2 p^{n+1}}{\partial x_i\partial x_i}=\frac{Re}{\Delta t}\frac{\partial u_i^*}{\partial x_i} \quad (6\text{-}3)$$

$$Re\left(\frac{u_i^{n+1}-u_i^*}{\Delta t}\right)=-\frac{\partial p^{n+1}}{\partial x_i} \quad (6\text{-}4)$$

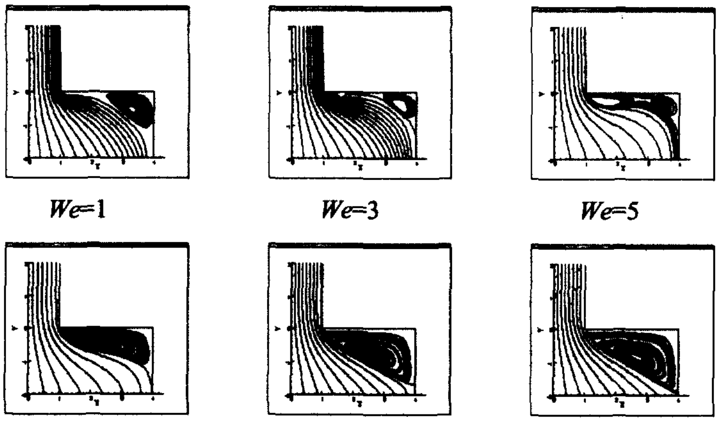
And in order to stabilize the solutions, constitutive equation is formulated by DG.

$$\left\langle \psi, \tau + We\tau_{(1)} - D \right\rangle - \sum_{e=1}^{N} \int_{\Gamma_e} \psi : u \cdot n(\tau - \tau^{ext}) d\Gamma_e = 0 \qquad (7)$$

where $< A, B >$ denotes $\int_{\Omega} ABd\Omega$ on domain Ω, n is a normal vector which has the outward direction at the boundary of finite elements. τ^{ext} takes the upstream additional stress value in the region of $u \cdot n < 0$ [5].

Results & Discussion

We obtained the converged solutions of high *We* flow up to 9.5 with the fine mesh which consists of 1985 elements and 2096 nodes. The time increment was $\Delta t = 2\times 10^{-3}$, *Re* was fixed with 0.1, and the relative error was roughly of the order of ten to -7~-8.



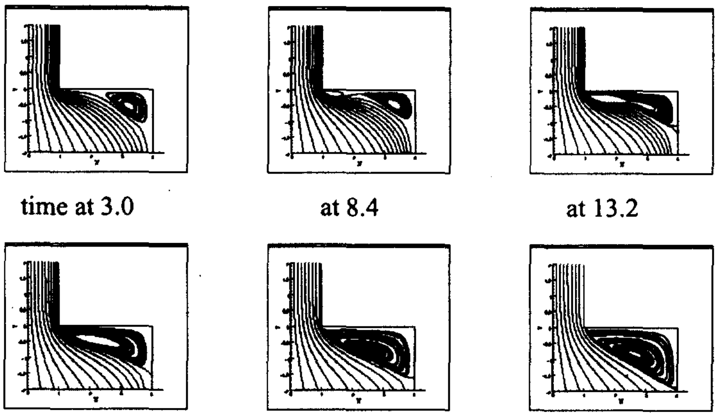
We=1 *We*=3 *We*=5

We=7 *We*=9 *We*=9.5

Fig. 1. the steady solutions at various *We* (*Re*=0.1)

With above convergence criterion, we obtained the steady solutions at various *We* from 1 to 9.5 in Fig. 1. At *We*=1, the corner vortex is observed and no lip vortex appears. But as *We* increases, the lip vortex enhancement mechanism dominates. The lip vortex is getting larger at the re-entrant corner and its intensity is getting stronger as *We* increases, while the corner vortex is getting weaker and is enveloped with the lip vortex.

If we observe the transient process of the highest *We* flow, namely at 9.5, almost the same lip vortex enhancement mechanism is reproduced. As time goes on, the lip vortex appears, grows and devours the corner vortex in the same way as we observe the vortex growth behavior from the steady state solutions with increasing *We*. Therefore, there seems to exist a time-*We* superposition relationship in a vortex growth mechanism of a 4:1 planar contraction flow.



time at 3.0 at 8.4 at 13.2

at 24.2 at 60.8 at 227.4

Fig. 2. transient behavior of highly elastic fluid (*We*=9.5, *Re*=0.1)

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