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On the controllability of fuzzy differential systems with nonlocal initial conditions

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ABSTRACT

In this paper, we find the controllability conditions for the following fuzzy differential systems

$$\begin{cases} \dfrac{dx}{dt} = A(t)x(t) + B(t)U(t) \\ x(0) + g(x) = x_0 \end{cases}$$

where $A(t)$, $B(t)$ are continuous matrices, g is given function, $U(t)$ is fuzzy set and target X^1 is fuzzy set.

Key words and Phrases : controllability, target, fuzzy set, nonlocal initial conditions

1. Introduction

In [10], Z. Ding and A. Kandel studied the controllability of fuzzy dynamical system:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)U(t), \\ x(0) = x_0 \end{cases}$$

where A, B are continuous matrices and $U(t)$ is fuzzy set.

In this paper, we consider the existence of fuzzy solutions and controllability of the following nonlocal initial condition:

$$\text{(E)} \qquad \begin{cases} \dot{x}(t) = A(t)x(t) + B(t)U(t), \\ x(0) + g(x) = x_0 \end{cases}$$

where $A(t), B(t)$ are continuous matrices, $U(t)$ is fuzzy set and g is given function.

2. Preliminary and Definitions

Let A and B be two nonempty bounded subsets of R^n. The distance between A and B is defined by the Hausdorff metric. Denoted by

$$P_k(R^n) = \{\ A(\neq \emptyset) \subset R^n: A \text{ is closed compact convex}\}.$$

Denote by

$E^n = \{u: R^n \to [0,1] \mid u$ satisfies (1)–(4) below $\}$

where

(1) u is normal.

(2) u is fuzzy convex.

(3) u is upper semicontinuous.

(4) $[u]^0 = \overline{\{x \in R^n : u(x) > 0\}}$ is compact.

Define $D: E^n \times E^n \to R \cup \{0\}$ by

$$D(u, v) = \sup_{0 < \alpha < 1} d_H([u]^\alpha, [v]^\alpha)$$

where d_H is the Hausdorff metric.

We see (E^n, D) is a complete metric space.

Theorem 2.1. If $u \in E^n$, then

1. $[u]^\alpha \in P_k(R^n)$ for all $0 \le \alpha \le 1$.
2. $[u]^{\alpha_2} \subset [u]^{\alpha_1}$, for all $0 \le \alpha_1 \le \alpha_2 \le 1$.
3. If (α_k) is a nondecreasing sequence converging to $\alpha > 0$, then

$$[u]^\alpha = \cap_{k \ge 1} [u]^{\alpha_k}.$$

Conversely, if $\{A^\alpha : 0 \le \alpha \le 1\}$ is a family of subsets of R^n satisfying (1)-(3), then there exists a $u \in E^n$ such that $[u]^\alpha = A^\alpha$ for $0 \le \alpha \le 1$ and $[u]^0 = \overline{\cup_{0 \le \alpha \le 1} A^\alpha} \subset A^0$.

Let $F: [0, T] \times E^n \to E^n$. Consider the fuzzy differential equation

(2.1) $\quad \dot{x}(t) = F(t, x),\ x(0) = x_0$

Definition 2.1. A mapping $x: T \to E^n$ is a fuzzy weak solution to (2.1) if it is continuous and satisfies the integral equation

$$x(t) = x_0 + \int_0^t F(s, x(s))ds,\ \forall t \in [0, 1]$$

If F is continuous, then this weak solution also satisfies (2.1) and we call it fuzzy strong solution to (2.1)

3. Main Results

We assume the following hypotheses:

(H1) $M = \max_{t \in [0,T]} \|\phi(t)\|$.

(H2) $N = \max_{t \in [0,T]} \|u(t)\|$

where $u(t) \in [U(t)]^\alpha$.

(H3) $K = \max_{t \in [0,T]} \|B(t)\|$.

(H4) $L = \max_{t \in [0,T]} \|g(x)\|$.

Theorem 3.1. Let $T > 0$. Assume (H1)-(H4). Then the (E) has a fuzzy solution $x(t)$.

Proof. Omitted.

Next we consider the controllability conditions of fuzzy system (E).

The concept of controllability is concerned with the following problem: given system (E), for the initial state $x_0 - g(x)$, the state at time T is a fuzzy set x^1, find the input $u(t), t \in [0, T]$ that transfers $x_0 - g(x)$ (at 0) to x^1 (at T). We need the following definition .

Definition 3.1. The state $x_0 - g(x)$ of system (E) is said to be controllable on the interval $[0, T]$ where T is a finite time if some control U over $[0, T]$ exists which transfers $x_0 - g(x)$ (at 0) to the fuzzy state at T. Otherwise the state $x_0 - g(x)$ is said to be uncontrollable on $[0, T]$.

Lemma 3.1.([10]) Let $f(t) \ne 0$ be a continuous function and U, V are two fuzzy sets. If $\int_0^T f(t)\, U dt = \int_0^T f(t)\, V dt$ then $U = V$.

Theorem 3.2. ([10]) System (E) ($g(x)=0$) is controllable over the interval $[0, T]$, if $\Phi(T-t)\,B(t)$ is nonsingular or equivalently, if the matrix

$$M(0,T)=\int_0^T \Phi(T-t)B(t)B^*(t)\Phi^*(T-t)\,dt$$

is nonsingular. Furthermore, the control $U(t)$ which transfer the state of the system from $x(0)=x_0$ to a fuzzy state $x(T)=x^1$ can be chosen as

$$U(t)=\frac{1}{T}B^{-1}(t)\,\Phi^{-1}(T-t)\,x^1 - B^*(t)\,\Phi^*(T-t)M^{-1}(0,T)\Phi(T)\,x_0$$

Theorem 3.3. System (E) is controllable over the interval $[0, T]$, if $\Phi(T-t)B(t)$ is nonsingular. Furthermore, the control $U(t)$ which transfer the state of the system from $x(0)=x_0-g(x)$ to a fuzzy state $x(T)=x^1$ can be chosen as

$$U(t)=\frac{1}{T}B(t)^{-1}\Phi^{-1}(T-t)\,(x^1-\Phi(T)(x_0-g(x))).$$

Proof.

Since $\Phi(T-t)\,B(t)$ is nonsingular, there exists

$$(\Phi(T-t)\,B(t))^{-1}=B(t)^{-1}\Phi^{-1}(T-t).$$

If $U(t)$ exists such that $U(t)$ transfer $x_0-g(x)$ to x^1 over $[0, T]$, then we get

$$x(T)=x^1=\Phi(T)(x_0-g(x)) + \int_0^T \Phi(T-t)B(t)U(t)\,dt$$

i.e.

$$x^1-\Phi(T)(x_0-g(x)) = \int_0^T \Phi(T-t)B(t)U(t)\,dt.$$

Thus

$$\begin{aligned}&\int_0^T \Phi(T-t)B(t)U(t)\,dt\\ &=\frac{1}{T}\int_0^T \Phi(T-t)B(t)B(t)^{-1}\Phi^{-1}(T-t)\,(x^1-\Phi(T)(x_0-g(x)))\,dt\\ &=\int_0^T \Phi(T-t)B(t)[\frac{1}{T}B(t)^{-1}\Phi^{-1}(T-t)\,(x^1-\Phi(T)(x_0-g(x)))]\,dt\end{aligned}$$

By Lemma 3.1.,

$$U(t)=\frac{1}{T}B(t)^{-1}\Phi^{-1}(T-t)\,(x^1-\Phi(T)(x_0-g(x))).$$

Example 3.1.

Now let us consider the system.

$$\dot{x}(t)=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}x(t)+\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}U(t),$$

$$x(0)=\begin{pmatrix}1\\ 1\end{pmatrix}-\begin{pmatrix}g_1(x)\\ g_2(x)\end{pmatrix}.$$

We assume that α-level sets of fuzzy sets x^1 are

$$[x^1]^\alpha=\begin{pmatrix}[-0.1(1-\alpha),\ 0.1(1-\alpha)]\\ [-0.1(1-\alpha),\ 0.1(1-\alpha)]\end{pmatrix}.$$

$$\Phi(t)=e^{A(t)}=\begin{pmatrix}\cos t & -\sin t\\ \sin t & \cos t\end{pmatrix}.$$

$$\det(\Phi(t)B(t))=\begin{vmatrix}\cos t & -\sin t\\ \sin t & \cos t\end{vmatrix}=1\neq 0.$$

Hence $\Phi(t)B(t)$ is nonsingular.

Final time $T=\frac{\pi}{2}$.

$$\begin{aligned}&[U(t)]^\alpha\\ &=\frac{2}{\pi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\cos(\frac{\pi}{2}-t) & \sin(\frac{\pi}{2}-t)\\ -\sin(\frac{\pi}{2}-t) & \cos(\frac{\pi}{2}-t)\end{pmatrix}\\ &\cdot\left(\begin{pmatrix}[-0.1(1-\alpha),\ 0.1(1-\alpha)]\\ [-0.1(1-\alpha),\ 0.1(1-\alpha)]\end{pmatrix} - \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1-g_1(x)\\ 1-g_2(x)\end{pmatrix}\right).\end{aligned}$$

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