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T-S Model Based Robust Indirect Adaptive Fuzzy Control

Chang-Ho Hyun, Chang-Woo Park, Mignon Park

ICS Lab., Dept. of Electrical and Electronic Eng., Yonsei Univ.
134, Shinchon-dong, Seodaemun-ku, Seoul, Korea
Tel : 82-2-2123-2868 Fax : 82-2-312-2333
E-mail : hhyun6@hotmail.com

Abstract

In this paper, we propose a robust indirect adaptive fuzzy state feedback regulator based on Takagi-Sugeno fuzzy model. The proposed adaptive fuzzy regulator is less sensitive to singularity than the conventional one based on the feedback linearization method. Furthermore, the proposed control method is applicable to not only plants with a perfect model but also plants with an imperfect model, which causes uncertainties. We verify the global stability of the proposed method by using Lyapunov method. In order to support the achievement, the application of the proposed adaptive fuzzy regulator to the control of a nonlinear system under the external disturbance is presented and the performance was verified by some simulation result.

· *Key words : indirect adaptive control, fuzzy control, T-S fuzzy model, robust control*

1. Introduction

Fuzzy control has been successfully applied to many commercial products and industrial systems since Mamdani has applied it to a steam engine [1]-[3]. Fuzzy control is available for controlling systems without accurate mathematical models, and human experts are valuable for providing linguistic fuzzy control rules or linguistic fuzzy descriptions about the systems. However, the stability analysis of the fuzzy control is difficult and often yields a conservative result although fuzzy control does not require accurate mathematical models. Therefore, recent fuzzy control researches have been focused on an indirect adaptive fuzzy control [4]-[5].

Most of the existing indirect adaptive fuzzy control algorithms are based on the feedback linearization method [8],[10]. However, the feedback linearization method cannot be applied to the plant with the singularity in the inverse dynamics. The adaptive fuzzy control algorithms based on the feedback linearization method need the infinite control input, when the state is at singularity of the inverse dynamics or the parameter approximation error diverges to infinity. Pre researches have proposed several ways to avoid the infinite control input such as the assumption that the state is away from the singularity, and the projection algorithm, which prevents the parameter approximation error from diverging to infinity [8].

In this paper, we propose alternative method to solve the singularity problem, a robust indirect adaptive fuzzy state feedback regulator based on Takagi-Sugeno fuzzy model. The proposed method is not based on the feedback linearization method but on a simple state feedback method. Therefore, the proposed algorithm solves the singularity problem in the inverse dynamics without any additional assumption or projection algorithm.

In practice, the perfect modeling is impossible since number of rules are limited and the description is not correct. Therefore, uncertainties on modeling are inevitably produced. To overcome the effect of uncertainties on stability, the various adaptive fuzzy control algorithms have been proposed [6]-[12]. We also considered uncertainties on modeling. It allowed the proposed controller to apply not only to plants with a perfect model but also to plants with an imperfect model. For designing a reliable regulator, we proved the global stability of the closed control systems with the proposed regulator. At last, some simulation results showed the performance of the proposed regulator.

2. Problem Formulation

Consider the regulation problem of the following n-th order nonlinear SISO system.

$$x^{(n)} = f(\underline{x}) + g(\underline{x})u + d(t) \tag{2-1}$$

where, $f(\underline{x})$, $g(\underline{x})$ are unknown but bounded continuous nonlinear functions. $d(t)$ denotes an external disturbance, which is unknown but bounded in magnitude. u is a control input. Let $\underline{x} = [x \ \dot{x} \ \cdots \ x^{(n-1)}]^T \in R^n$ be the state vector of the system which is assumed to be available.

In this paper, Takagi-Sugeno fuzzy model (TS fuzzy model) [15] was used to identify the unknown nonlinear system (2-1). TS fuzzy model can be briefly presented below by the following IF-THEN form or In-Out form.

<Takagi-Sugeno Fuzzy Model>

I) IF-THEN form

Plant rule i:

IF x *is* M_{i1} and $\dot{x}$ *is* M_{i2} and... and $x^{(n-1)}$ *is* M_{in}

THEN $x^{(n)} = a_i^T\underline{x} + b_i u, \quad i=1, 2, \cdots, r$

where $\underline{x} = [x \ \dot{x} \cdots x^{(n-1)}]^T \in R^n$, $a_i \in R^n$, $b_i \in R$. M_{ij} is the fuzzy set and r is the number of rules.

II) Input-Output form

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(\underline{x})\{ a_i^T \underline{x} + b_i u \}}{\sum_{i=1}^{r} w_i(\underline{x})}$$

$$= \sum_{i=1}^{r} h_i(\underline{x})\{ a_i^T \underline{x} + b_i u \} \quad (2\text{-}2)$$

where

$$w_i(\underline{x}) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)}), h_i(\underline{x}) = \frac{w_i(\underline{x})}{\sum_{i=1}^{r} w_i(\underline{x})}$$

$M_{ij}(x^{(j-1)})$ is the grade of membership of $x^{(j-1)}$ in M_{ij}. It is assumed that $w_i(x) \geq 0$,

$\sum_{i=1}^{r} w_i(\underline{x}) > 0, i=1, 2, 3, \cdots, r$

Hence, $h_i(\underline{x}) \geq 0$, $\sum_{i=1}^{r} h_i(\underline{x}) = 1$

In order to apply the proposed regulation algorithm, the given unknown system has to be identified by the TS fuzzy model.

$$x^{(n)} = \hat{f}(x \mid \theta_a) + \hat{g}(x \mid \theta_b)u + d(t)$$

$$= \sum_{i=1}^{r} h_i(\underline{x})\, a_i^T \underline{x} + \sum_{i=1}^{r} h_i(\underline{x})\, b_i u + d(t) \quad (2\text{-}3)$$

Uncertainties on fuzzy modeling have to be considered. At first, for considering uncertainties, optimal parameter vectors used in the fuzzy modeling are assumed as follows.

Assumption 1

a_r^{*T}: *nominal values used in T-S fuzzy modeling*

b_r^* : *nominal values used in T-S fuzzy modeling*

However, in spite of the T-S fuzzy model approximation, uncertainties inevitably occur in the practical application as mentioned above. For that reason, we defined uncertainties as follows.

$$\Delta a = [\Delta a_1 \ \Delta a_2 \ \cdots \ \Delta a_n]^T \in R^{n+}, \Delta b \in R^+$$
$$| \Delta a_j(t) | \leq \Delta a_j, \quad | \Delta b(t) | \leq \Delta b,$$
$$d \in R^+, \quad | d(t) | \leq d$$

Then, unknown nonlinear functions $f(x)$, $g(x)$ in system (1) can be represented as

$$f(\underline{x}) = \sum_{i=1}^{r} h_i(\underline{x})\, a_i^{*T} x + \underline{\Delta a(t)^T \underline{x}} \quad (2\text{-}4)$$

$$g(\underline{x}) = \sum_{i=1}^{r} h_i(\underline{x}) b_i^* + \underline{\Delta b(t)} \quad (2\text{-}5)$$

where, a_i^T, b_i are the original parameter values.

Therefore, we finally identified the unknown nonlinear system with the consideration of modeling uncertainties using TS fuzzy model as follows.

$$x^{(n)} = \sum_{i=1}^{r} h_i(\underline{x})\, a_i^{*T} \underline{x} + \Delta a(t)^T \underline{x} + \sum_{i=1}^{r} h_i(\underline{x}) b_i^* u + \Delta b(t) u + d(t) \quad (2\text{-}6)$$

3. Design of a robust indirect adaptive fuzzy regulator and stability analysis

The proposed regulator has a simple state feedback structure.

$$u = k^T \underline{x} \quad (3\text{-}1)$$

where, $k \in R^n$; $n \times 1$ adaptive state feedback gain vector

By substituting (6-1) for u of (5-5), we can obtain a following closed loop equation.

$$x^{(n)} = \sum_{i=1}^{r} h_i(\underline{x})\, a_i^{*T} \underline{x} + \Delta a(t)^T \underline{x} + \sum_{i=1}^{r} h_i(\underline{x}) b_i^* k^T \underline{x} + \Delta b(t) k^T \underline{x} + d(t) \quad (3\text{-}2)$$

3.1 Derivation of and adaptive law

we derive an adaptive law with the closed loop equation (3-2). An adaptive law is obtained by Lyapunov theory [13].

To use a Lyapunov equation $\underline{A}^T P + P\underline{A} = -Q$, modify system's equation to the form that has a matrix A.

Using $\underline{A}\underline{x} - w\left[\sum_{i=1}^{r} h_i(\underline{x})\, C^T \underline{x} \right]$, (3-2) can be expressed as follows.

$$\dot{\underline{x}} = \underline{A}\underline{x} - w\Big[\sum_{i=1}^{r} h_i(\underline{x}) C^T \underline{x} + \sum_{i=1}^{r} h_i(\underline{x})\, a_i^{*T} \underline{x} + \sum_{i=1}^{r} h_i(x) b_i^* k^T \underline{x} + \Delta a(t)^T \underline{x} + \Delta b(t) k^T \underline{x} + d(t) \Big] \quad (3\text{-}3)$$

where, $C = [-\underline{a_1} \ -\underline{a_2} \cdots -\underline{a_n}]^T \in R^n$ is the transpose vector of the last row of the matrix $\underline{A}$ and $w = [0 \ 0 \cdots 1]^T \in R^n$.

The equation, (3-3) can be adjusted as follows.

$$\dot{\underline{x}} = \underline{A}\underline{x} + w\left[\left[\sum_{i=1}^{r} h_i(\underline{x})\left\{ \frac{k\, \underline{x}^T (a_i^* - C)}{k^T k} + \underline{x} b_i^* \right\} + \frac{k\, \underline{x}^T \Delta a(t)}{k^T k} + \underline{x}\Delta b(t) + \frac{k d(t)}{k^T k} \right]^T k \right]$$
$$= \underline{A}\underline{x} + w r^T k$$

$$\text{where, } r^T = \left[\sum_{i=1}^{r} h_i(\underline{x})\left\{ \frac{k\, \underline{x}^T (a_i^* - C)}{k^T k} + \underline{x} b_i^* \right\} + \frac{k\, \underline{x}^T \Delta a(t)}{k^T k} + \underline{x}\Delta b(t) + \frac{k d(t)}{k^T k} \right]^T$$

Therefore,

$$\dot{x} = \underline{A}\underline{x} + \Gamma(\underline{x}, k, t)^T k \quad (3\text{-}4)$$

where, $\Gamma(\underline{x}, k, t) = r w^T$ then,

We choose a Lyapunov function as follows.

$$V = \underline{x}^T P \underline{x} + k^T \Lambda^{-1} k + \frac{1}{\alpha_1} \sum_{i=1}^{r} \tilde{a}_i^T \tilde{a}_i + \frac{1}{\alpha_2} \sum_{i=1}^{r} \tilde{b}_i^2$$

where, V is positive definite and radially unbounded.

$$\tilde{a}_i = a_i^* - a_i, \quad \tilde{b}_i = b_i^* - b_i \quad (3\text{-}5)$$

Λ : $n\times n$ symmetric positive definite matrix
α_1, α_2 : positive adaptation constant gains

After differentiating V, an adaptation law to make $\dot{V} \le 0$ (negative semi definite) is constructed.

After some calculating, we finally obtain an adaptive law as

$$\dot{\tilde{a}}_i^T = -\alpha_1 h_i(x)\, x^T P w\, x^T \quad (3\text{-}6)$$

$$\dot{\tilde{b}}_i = -\alpha_2 h_i(x)\, x^T P w k^T x \quad (3\text{-}7)$$

$$\dot{k} = -\Lambda \Upsilon(x,k,t) P x \quad (3\text{-}8)$$

where,

$$\hat{r}^T = \left[\sum_{i=1}^{r} h_i(x) \left\{ \frac{k\, x^T (a_i^* - \tilde{a} - C)}{k^T k} + (b_i^* - \tilde{b}_i) x \right\} - \frac{k\, x^T \Delta a(t)}{k^T k} - \Delta b(t) x - \frac{kd}{k^T k} \right]^T$$

$$, \Upsilon(x,k,t) = \hat{r} w^T.$$

Note that $\tilde{a}_i$, $\tilde{b}_i$ and d should be used instead of a_i, b_i and $d(t)$ because only measurable parameters are available.

3.2 Stability Analysis

The stability of systems with adaptive controller is proved through the process of finding an adaptive law. After cancelation,

$$\dot{V} \le -x^T Q x \quad (3\text{-}9)$$

is only remained. Hence, the system (2-1) is stabilized by the derived adaptive law. In order to prove the asymptotic stability, we show that the system with the proposed regulator satisfies the following theorem.

Theorem 1

(a) x, k, $\tilde{a}_i$, $\tilde{b}_i$ are bounded.
(b) The equilibrium point $(x, \tilde{a}_i, \tilde{b}_i) = 0$ and $k = \hat{k}$ is uniformly stable.
(c) $\dot{x}$ is bounded.
(d) $\lim_{t\to\infty} \|x(t)\| = 0$, $\forall x(t_0) \in R^n$, $\forall k(t_0) \in R^p$, $\forall \tilde{a}_i(t_0) \in R^n$, $\forall \tilde{b}_i(t_0) \in R$

Proof

Since V is radially unbounded, (a) is proved. ((a) implies $x \in L_\infty$)
According to the Lyapunov Theory, (b) is also proved. and because of (a), (c) is easily verified.
Let $\lambda_{\min}(Q)$ denote the smallest eigenvalues of Q. Then, since Q is a positive definite matrix, the following property holds.

$$\lambda_{\min}(Q) > 0 \text{ and } \lambda_{\min}(Q)|x|^2 \le x^T Q x$$

From this property, (3-9) becomes

$$\dot{V} \le -x^T Q x \le -\lambda_{\min}(Q)|x|^2 \quad (3\text{-}10)$$

Dividing both sides of (3-10) by $\lambda_{\min}(Q)$ and integrating them with respect to time, we obtain

$$\int_{t_0}^{t} |x(\tau)|^2 d\tau \le \frac{1}{\lambda_{\min}(Q)} (V(t_0) - V(\infty)) < \infty$$

Thus, we have $x \in L_2$. Then, applying Barbalat's Lemma [13] to $x(t)$, we conclude that $\lim_{t\to\infty} \|x(t)\| = 0$, which proves statement (d).

4. Simulation

Let's consider the problem of balancing and swing-up of an inverted pendulum on a cart. The state equation of motion for the pendulum is as follows.

$$\ddot{x} = \frac{g\sin(x_1) - aml\, x_2^2 \sin(2x)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + d(t) \quad (4\text{-}1)$$

where, $x = [x\ \dot{x}]^T$

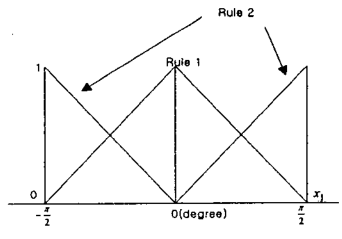
x	angle θ of the pendulum from the vertical;
$\dot{x}$	the angular velocity
g	the gravity constant, $9.8\ m/s^2$;
m	the mass of the pendulum;
M	the mass of the cart;
$2l$	the length of the pendulum;
u	the control force applied to the cart
$d(t)$	the external disturbance

$a = \frac{1}{m + M}$.

We choose $m = 2.0kg$, $M = 8.0kg$, $2l = 1.0m$ in the simulation. For the convenience of simulation, the external disturbance $d(t)$ is assumed to be a square wave with the amplitude ± 0.05 and the period 2π.

We use the following Takagi-Sugeno fuzzy model with two rules.

Rule 1 *IF* x *is about* 0:
THEN $\ddot{x} = a_1^T x + b_1 u$
Rule 2: *IF* x *is about* $\pm\frac{\pi}{2}$ ($|x| < \frac{\pi}{2}$)
THEN $\ddot{x} = a_2^T x + b_2 u$



<Fig. 1> Membership function

In this simulation, the time-varying approximation errors are assumed to be bounded as follows.

$$\Delta a = [\Delta a_1\ \ \Delta a_2]^T = [3\ \ 0]^T \in R^{2+}$$
$$, \Delta b = 0.1 \in R^+$$

To show the validity of the above assumption, we suppose that $f(x)$ and $g(x)$ are known and

$d(t) = 0$ in the system (4-1). Then, the nominal parameter vectors can be obtained as

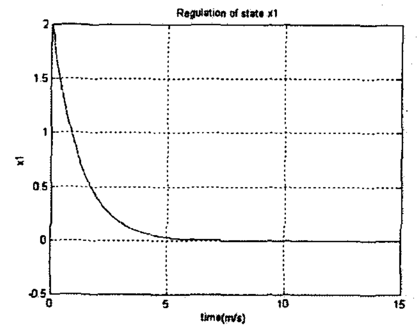
$$a_1^* = [17.29 \quad 0]^T,\ a_2^* = [9.35 \quad 0]^T,$$
$$b_1^* = -0.18,\ b_2^* = -0.01.$$

The conditions used in the computer simulation are summarized as follows.

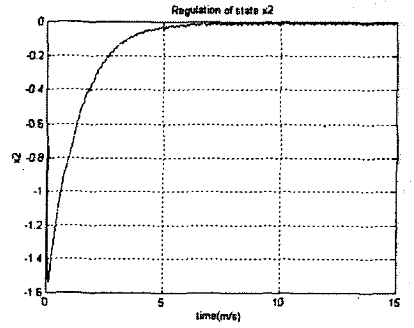
$$\Lambda = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\alpha_1 = \alpha_2 = 0.1$$

5. Conclusion

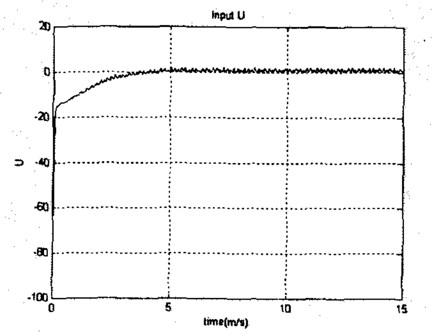
The proposed control method is designed to regulate uncertain SISO nonlinear systems. Where most of adaptive fuzzy control algorithms have been based on the feedback linearization method, the proposed method is based on a simple state feedback structure with the indirect adaptive fuzzy system identification. Therefore, It does not require any assumption on the state variable and guarantee the boundedness of both the control input and the parameter approximation error directly without the additional projection. The proposed method was applicable to plants with an imperfect model, which leads to uncertainties. In addition, the proposed method is robust to an external disturbance. Finally, It was proved that the proposed method guarantee the global stability herein this paper.



<Fig. 2> Regulation of state x



<Fig. 3> Regulation of state $\dot{x}$



<Fig. 4> Control input u

Figure 2 and 3 show the simulation results of the state variable x_1, x_2. From them, we can conclude that the proposed algorithm solve a regulation problem for the uncertain nonlinear system. Also, figure 4 presents that a control input is bounded.

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