A Fast Scheme for Inverting Single-Hole Electromagnetic Data

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Abstract

The extended Born, or localized nonlinear approximation of integral equation (IE) solution has been applied to inverting single-hole electromagnetic (EM) data using a cylindrically symmetric model. The extended Born approximation is less accurate than a full solution but much superior to the simple Born approximation. When applied to the cylindrically symmetric model with a vertical magnetic dipole source, however, the accuracy of the extended Born approximation is greatly improved because the electric field is scalar and continuous everywhere. One of the most important steps in the inversion is the selection of a proper regularization parameter for stability. Occam's inversion (Constable et al., 1987) is an excellent method for obtaining a stable inverse solution. It is extremely slow when combined with a differential equation method because many forward simulations are needed but suitable for the extended Born solution because the Green's functions, the most time consuming part in IE methods, are repeatedly re-usable throughout the inversion. In addition, the IE formulation also readily contains a sensitivity matrix, which can be revised at each iteration at little expense. The inversion algorithm developed in this study is quite stable and fast even if the optimum regularization parameter is sought at each iteration step. In this paper we show inversion results using synthetic data obtained from a finite-element method and field data as well.

Theory

The derivation of sensitivities starts with the vector integral equation for magnetic fields (Hohmann, 1975).

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}^{p}(\mathbf{r}) - \int \underline{\underline{\mathbf{G}}}^{\mathbf{H}}(\mathbf{r},\mathbf{r}')\Delta\sigma\mathbf{E}(\mathbf{r}')d\mathbf{r}'$$

Linearizing this equation using the extended Born approximation (Habashy et al., 1993), we obtain

$$\mathbf{H}(\mathbf{r}) \approx \mathbf{H}^{p}(\mathbf{r}) - \int \underline{\underline{\mathbf{G}}}^{\mathbf{H}}(\mathbf{r},\mathbf{r}')\Delta\sigma\Gamma(\mathbf{r}')\mathbf{E}^{p}(\mathbf{r}')d\mathbf{r}'$$

Since $\underline{\underline{\mathbf{G}}}^{\mathbf{H}}(\mathbf{r},\mathbf{r}')$ and $E^{p}(r)$ are known analytically for a whole space, we can set up an inverse equation to estimate the unknown conductivity distribution.

Key words: single-hole, extended Born approximation, cylindrically symmetry, inversion

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Results

The accuracy of extended Born approximation is checked using a model, which is cylindrically symmetric about the borehole, contains a conductive body with a cross-section of 3 m by 4 m in a whole space of 100 Ω-m. The extended Born approximation is proved to be accurate when the frequency is below 10^6 Hz (Figure 1) and the conductivity contrast σ_2/σ_1 is less than 100 (Figure 2).

A resistivity model employed to evaluate the performance of the extended Born inversion consists of a low-resistivity (1 S/m) and high-resistivity (0.01 S/m) body in a whole space of 0.1 S/m. A finite-element modeling (FEM) scheme (Lee et al., 2002) is used to generate synthetic data. A comparison of this FEM solution to exact IE solution is in good agreement, and the difference between these two solutions is generally less than 1 %. For a M_z source, vertical magnetic fields H_z are computed at five source-receiver offsets of 4 m through 8 m at three frequencies of 12 kHz, 24 kHz and 42 kHz.

Using 3-digit synthetic data generated by FEM the inversion is started with an initial model of 4 Ω-m uniform whole space. In this test we used three forward modelings in each iteration to select parameter update and Lagrange multiplier. After 6 iterations, the two bodies are clearly reconstructed as shown in Figure 3. The recovered conductivity is found to be nearly the same in the conductive body but much smaller than the actual one in the resistive body. The inversion process is quite stable as shown in Figure 4, and rms misfit decreases from the initial guess of 0.478 to under 0.01 after 7 iterations. The rms misfit of 0.01 is assumed to be a target misfit level because the error level in the synthetic responses is estimated to be about 1 %. The data fits between the synthetic and predicted data are excellent at all frequencies.

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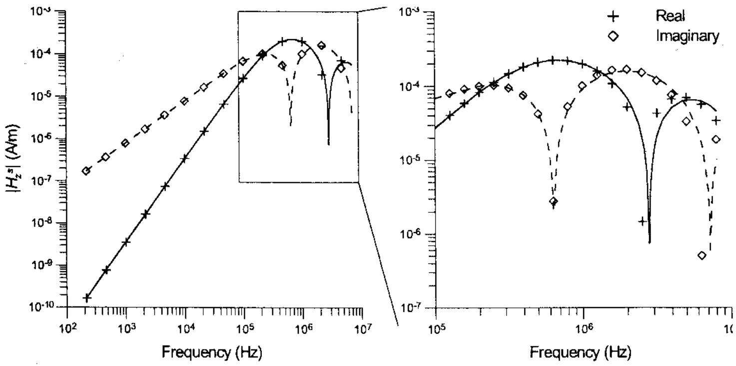

Figure 1.

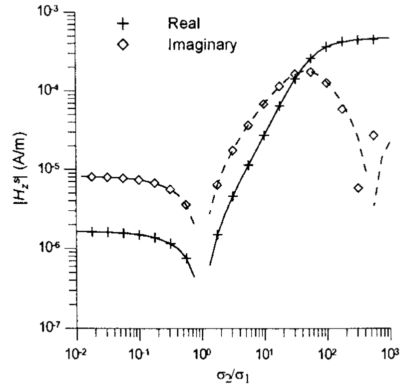

Figure 2.

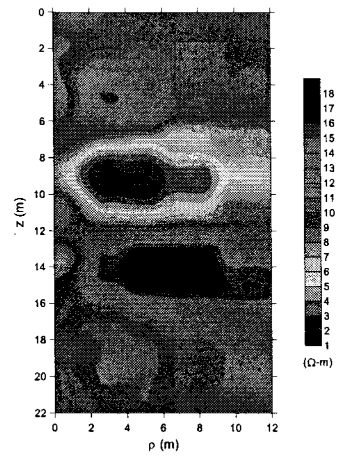

Figure 3.

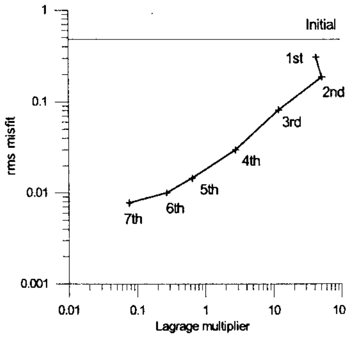

Figure 4.