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Wave Scattering by Multiple Permeable Barriers
다중 투과성 벽에 의한 파랑의 산란

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1. INTRODUCTION

Recently, the decline of water quality in harbors or marinas has become a serious problem. Therefore, the surface-piercing vertical barriers have been used or considered for the purposes of reducing wave heights in the harbor, and facilitating exchange of water inside and outside of the harbor. Permeable barriers have the advantage of reducing wave reflection on the upwave side of the barrier but in order to also reduce wave transmission to an acceptable level it is often necessary to use two or more barriers.

The scattering of water waves even for a single barrier has been well known for the mathematical difficulties encountered within the framework of linearized potential theory. Therefore, vertical barrier performance for more than two barriers is very difficult to be accomplished by theoretical evaluations. In this paper, therefore, the wave/flow fields under multiple barriers are numerically solved by the mild-slope equation model which is expanded by adding the additional terms for wave scattering. Additional terms, which are given in terms of reflection ratio, are added in the momentum equation without inclusion of evanescent modes. Thus the existing of barriers is recognized by that of reflection ratios on the computational domain. In predicting the reflection ratio, the eigenfunction expansion method is used as the most accurate routine.

2. AN IMPERMEABLE BARRIER

The type of a surface-piercing barrier has been examined experimentally by Wiegel(1960) and Jones et al. (1979) and theoretically by Wiegel(1960), who presented an approximate solution for the wave transmission based on the power method, Ursell(1947), who gave an exact formula for the wave transmission in deepwater, Drimer et al. (1992), who presented a simplified model to solve analytically the two-dimensional linearized hydrodynamic problem of a pontoon type floating breakwater, and Losada et al. (1992), Abul-Azm (1993) and Kriebel and Bollmann (1996) who developed numerical solutions based on the method of eigenfunction expansion, while Liu and Abbaspour, (1980) who used a boundary integral equation method(BIEM). In this section, four theoretical approaches above-mentioned except BIEM method will be reviewed for a vertical barrier as shown below.

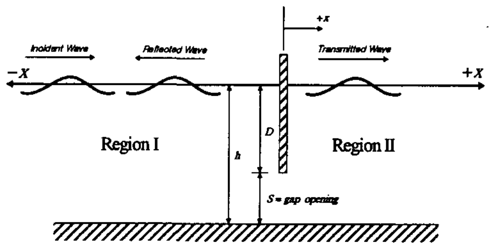

Fig. 1. Schematatic diagram of a vertical barrier

2.1 Theoretical Approaches

Power Transmission Theory (Wiegel, 1960)

A portion of the wave energy incident on the barrier will be reflected as a reflected wave component and

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a portion will pass beneath the barrier and form a transmitted wave component. As a first approximation to determining the height of the transmitted wave component, therefore, Wiegel (1960) considered that all the progressive wave energy being propagated at those levels below the lower edge of the barrier is transmitted past the barrier and results in a transmitted wave.

For the propagation of a series of monochromatic waves of uniform amplitude, when a rigid vertical barrier is partially immersed perpendicular to the wave direction with a gap of " S" in between the bottom of the barrier and the sea bed as shown in Fig. 2, the portion of wave height transmitted between the sea bed and the bottom tip of the vertical barrier for a wave of small amplitude is given by

$$|K_t| = \frac{H_t}{H_i} = \sqrt{\frac{P_S}{P_T}} \quad (1)$$

where H_t is the transmitted wave height and the ratio H_t/H_i is termed the coefficient of transmission $|K_t|$, and by the linear wave theory,

$$\frac{P_S}{P_T} = \frac{\sinh 2k(h+D) + 2k(h+D)}{\sinh 2kh + 2kh} \quad (2)$$

where k is the wave number, h is the water depth, and D is the bottom tip of the vertical barrier measured negative downward from SWL. For an ideal condition the coefficient of reflection $|K_r|$ is given by

$$|K_r| = \frac{H_r}{H_i} = \sqrt{1 - \left(\frac{H_t}{H_i}\right)^2} \quad (3)$$

where H_r is the reflected wave height.

<u>Infinite Depth-Water (Ursell, 1947)</u>

Concerning the performance of vertical barriers in waves, many studies have been carried out since the analytic method with eigenfunction expansion was represented as the basic theory of the wave-structure interaction. Most simple method for the vertical barrier was presented by Ursell (1947) in deep water condition. Ursell(1947) solved for the velocity field everywhere in the case of a thin vertical barrier immersed to a depth D beneath the surface of deep water, in the presence of incident waves of angular frequency σ, and showed that modulus of the ratio reflected to incident wave elevation could be expressed in terms of modified Bessel functions as

$$|K_r| = \frac{\pi I_1(\sigma^2 D/g)}{\sqrt{\left\{\pi^2 I_1^2(\sigma^2 D/g) + K_1^2(\sigma^2 D/g)\right\}}} \quad (4)$$

$$|K_t| = \frac{K_1(\sigma^2 D/g)}{\sqrt{\left\{\pi^2 I_1^2(\sigma^2 D/g) + K_1^2(\sigma^2 D/g)\right\}}} \quad (5)$$

where I_1 is the modified Bessel function of the first kind of order 1, K_1 is the modified Bessel function of the second kind of order 1, and g is the gravitational acceleration.

<u>Simplified Approximate Approach (Drimer et al., 1992)</u>

The scattering waves may be decomposed into symmetric and anti-symmetric components; each can be thought of as caused by two incident waves one from the right and one from the left side of the vertical barrier. The two waves have the same length and amplitude and are phase symmetric scattering waves, respectively. Using the orthonormality, we can express the transmission and reflection coefficients (Drime et al. 1992) as

$$K_t = \frac{iU_0^2/k_o}{iU_0^2/k_o + \sum_{n=1}^{\infty}\frac{U_n^2}{k_n}}, \qquad K_r = 1 - K_t \quad (6)$$

where

$$U_n = \int_h^{-D} f_n dz \qquad (n = 0,1,2,...) \quad (7)$$

$f_n(z)(n=0,1,...)$ is a complete set of orthonormal function in the interval $(-h,0)$ given by

$$f_0(z) = \frac{\sqrt{2}\cosh\left[k_o(z+h)\right]}{\left[h + K^{-1}\sinh^2(kh)\right]^{1/2}},$$
$$f_n(z) = \frac{\sqrt{2}\cos\left[k_n(z+h)\right]}{\left[h - K^{-1}\sin^2(k_n h)\right]^{1/2}}, \quad (n=1,2,...) \quad (8)$$

k_o is the incident wave number which satisfies the dispersion relation ;

$$K = k_o \tanh(k_o h)$$

and k_n are the positive roots of the equation :

$K = -k_n \tan(k_n h)$

Eigenfunction Expansion (Kriebel and Bollmann, 1996)

The method by eigenfunction expansion consists of matching two similar eigenfunctions over the depth across the breakwater gap. The methodology follows that developed by Dalrymple and Martin (1990) in finding the reflection coefficient for a long linear array of offshore breakwater with horizontal gaps between the array under short and long water waves. Such a solution has been given by Losada et al. (1992), Abul-Azm (1993) and Kriebel and Bollmann (1996).

At large distances from the breakwater, ϕ must satisfy a radiation condition, namely

$$\lim_{x \to \pm\infty} \left(\frac{\partial}{\partial x} \mp ik_o \right)(\phi - \phi_i) = 0 \tag{9}$$

where the incident potential ϕ_i is given by

$$\phi_i = Z_o(z) e^{ik_o x} \tag{10}$$

where

$$Z_o(z) = \frac{igH_i}{2\sigma} \frac{\cosh k_o(h+z)}{\cosh k_o h} \tag{11}$$

The eigenfuction expasion method involves solution for the velocity potentials on the up-wave side (I) and on the down-wave side (II) of the vertical barrier(see Fig. 1). These up-wave and down-wave potentials must then be appropriately matched at the location of a vertical barrier. These potentials have a spatial dependence as given by

$$\phi^{(I)} = Z_o(z) e^{ik_o x} + \sum_{n=0}^{\infty} R_n e^{-ik_n x} Z_n(z) \qquad x \le 0 \tag{12}$$

$$\phi^{(II)} = Z_o(z) e^{ik_o x} - \sum_{n=0}^{\infty} R_n e^{ik_n x} Z_n(z) \qquad x \ge 0 \tag{13}$$

The depth-dependent variable $Z_n(z)$ in Eqs. (12) and (13) is defined as

$$Z_n(z) = \frac{igH_i}{2\sigma} \frac{\cosh k_n(h+z)}{\cosh k_n h} \tag{14}$$

where there is then an infinite set of imaginary roots for $n \ge 1$.

The solution for the complex amplitudes R_n must satisfy two additional physical requirements: (a) the velocities must be zero on both sides of the barrier in the upper region where $-D < z < 0$, and (b) the velocity potentials (or equivalently the dynamic pressures) must match in the gap below the barrier where $-h < z < -D$. If the matching conditions are applied, two distinct equations for upper and lower regions are obtained and both are then combined into one mixed boundary condition. This combined function is then multiplied by the orthogonal function Z_n and depth-integrated over the full depth as done by Kriebel and Bollmann (1996). This yields the following single matrix equation that can be solved directly for the unknown amplitudes R_n without the need of a least-squares solution (Losada et al., 1992; Abul-Azm,1993).

$$\sum_{n=0}^{N} R_n (k_n Y_{nm} + 2k_o X_{nm}) = k_o Y_{om} \qquad m = 0,1,...N \tag{15}$$

where the functions Y_{nm} and X_{nm} were defined by Losada et al. (1992) and are given by

$$Y_{nm} = \int_{-D}^{0} Z_n(z) Z_m(z)\, dz \tag{16}$$

$$X_{nm} = \int_{-h}^{-D} Z_n(z) Z_m(z)\, dz \tag{17}$$

Once the matrix equation Eq. (15) is solved for the unknowns R_n, the transmission coefficient is obtained from the first term, R_0. The reflection and transmission coefficients for the progressive wave modes are given by

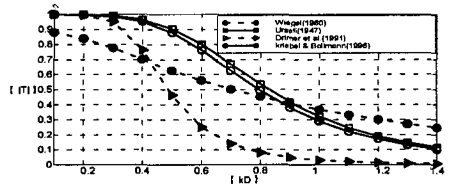
$$K_r = |R_0| \qquad K_t = |1 - R_0| \tag{18}$$

2.2 Comparison and Discussion

The four theories for solving wave reflections against a vertical barrier are now examined and compared one another to determine the most effective routine in predicting the reflection coefficient.

The transmission and reflection coefficients for D/h=0.2, 0.5, and 0.8 are plotted as a function of kD in Figs. 3 and 4, respectively. The eigenfuction theory considered as the most accurate approach, but Ursell's method appeared to provide good results only for small value of D/h, whereas Drimer et al.'s simple method provides good results only for large value of D/h. For conditions where the evanescent modes are important, Drimer et al.'s method has tendency to overestimate the wave transmission. Wiegel's theory yields worst prediction; for near-shallow water overestimates wave transmission while for deeper water underestimates it. The truncation parameter N was increased up to 150 modes for high penetration condition. Fig. 5 shows the variations of computed solutions according to N for D/h=0.1. For low penetration condition, however, N = 50 was generally found to give satisfactory results.

(a)



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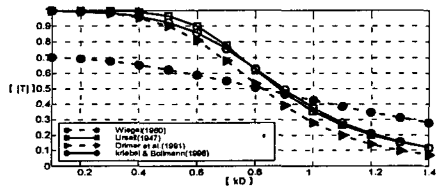

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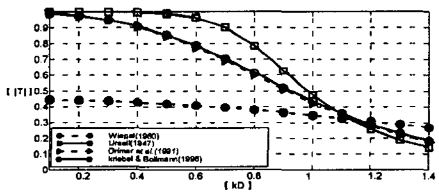
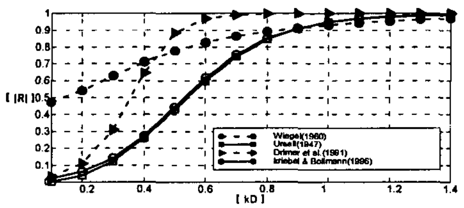


Fig 2. Transmission ratios for a surface-piercing barrier in normally incident waves: (a) D/h=0.2, (b) D/h=0.5, (c)D/h=0.8



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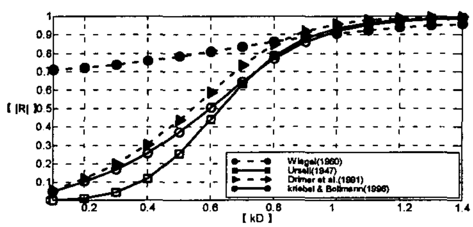


(c)

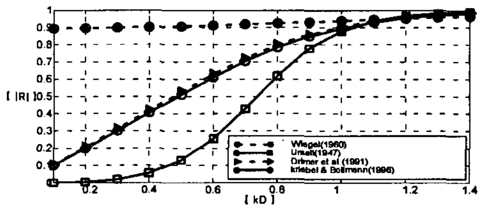


Fig 3. Reflection ratios for a surface-piercing barrier in normally incident waves: (a) D/h=0.2, (b) D/h=0.5, (c)D/h=0.8

3. A PERMEABLE BARRIER

In using the eigenfunction expansion method, the boundary condition along the permeable barrier may be developed on the basis of the formulation of Sollitt and Cross (1972) and as adopted by Yu (1995) for a thin vertical barrier extending to the seabed. This may be expressed along $x=0$ for $-D\leq z\leq 0$ as

$$\frac{\partial\phi_1}{\partial x}=\frac{\partial\phi_2}{\partial x}=-iG'(\phi_2-\phi_1) \qquad (19)$$

where $G'=G/b$, b is the barrier thickness and G is a permeability parameter which is generally complex. Eq. (19) corresponds to the fluid velocity normal to the barrier being proportional to the pressure difference across the barrier, with a complex constant of proportionality so that the real part of G corresponds to the resistance of the barrier and the imaginary part of G corresponds to the phase differences between the velocity and the pressure because of inertial effects.
In the present study, the method of Sollitt and Cross (1972) is followed and G expressed by:

$$G=\frac{\varepsilon}{f-is} \qquad (20)$$

where ε is the porosity of the barrier , f is the friction coefficient and s is the inertia coefficient given by

$$s=1+C_m\left(\frac{1-\varepsilon}{\varepsilon}\right) \qquad (21)$$

In Eq. (21), C_m is the added mass coefficient.

This method is verified with experimental results of the reflection and transmission coefficients for kh=1.5, D/h=1.0, f=2.0 and C_m=0.18. Fig. 5 shows comparisons of between theory and experiments with respect to ε for a breakwater having a single permeable wall as shown in Fig. 4. There is some scatter in the experimental results but the overall agreement satisfactory.

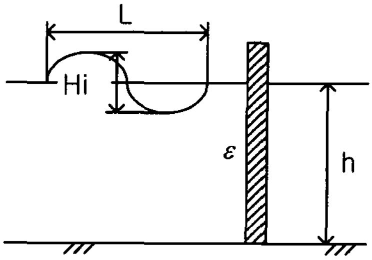


Fig. 4. Breakwater with a single vertical permeable wall

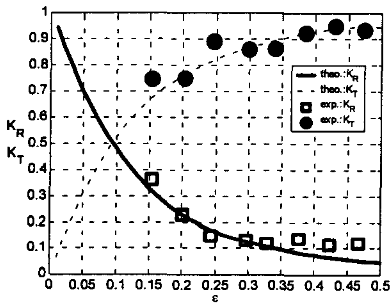


Fig. 5. Comparison of the theoretical and experimental reflection and transmission coefficients for a breakwater with a single permeable wall

4. MULTIPLE BARRIERS

4.1 Time-Dependent Mild-Slope Equation

Based on the equation proposed by Smith and Sprinks (1975), the time-dependent mild-slope equation for waves traveling in x axis can be written as

$$\frac{\partial^2\eta}{\partial t^2}-\frac{\partial}{\partial x}\left[CC_g\frac{\partial\eta}{\partial x}\right]+\left[\sigma^2-k^2CC_g\right]\eta=0 \qquad (22)$$

where, η is the free surface displacement, σ is the angular frequency, k is the wave number, C is the phase speed, and C_g is the group velocity. From the dynamic free surface boundary, $\partial\eta/\partial x$ can be expressed by velocity vector, u_o defined at the free surface as follows.

$$\frac{\partial\eta}{\partial x}=-\frac{1}{g}\frac{\partial}{\partial t}\left(\frac{\partial\hat{\phi}}{\partial x}\right)=-\frac{1}{g}\frac{\partial u_o}{\partial t} \qquad (23)$$

in which $\hat{\phi}$ implies the velocity potential at the free surface. Therefore, Eq. (22) becomes

$$\frac{\partial^2\eta}{\partial t^2}+\frac{\partial}{\partial x}\left[\frac{CC_g}{g}\frac{\partial u_o}{\partial t}\right]+\left[\sigma^2-k^2CC_g\right]\eta=0 \qquad (24)$$

The above equation is combined with Eq. (23) expressed as

$$\frac{\partial u_o}{\partial t}+g\frac{\partial\eta}{\partial x}=0 \qquad (25)$$

A set of differential equations (24) and (25) could be applied in most effective engineering practice to assess the wave conditions in existing or proposed new harbours.

As done by Madsen and Larsen (1987) for the regular waves, the above equations are reformulated extracting the harmonic time variation with letting $\eta=S\exp(-i\sigma t)$ and $u_o=U_o\exp(-i\sigma t)$:

$$\frac{\partial^2 S}{\partial t^2}-2i\sigma\frac{\partial S}{\partial t}+\frac{\partial}{\partial x}\left[\frac{CC_g}{g}\left(\frac{\partial U_o}{\partial t}-i\sigma U_o\right)\right]-k^2CC_gS=S_s \qquad (26)$$

$$\frac{\partial U_o}{\partial t}-i\sigma U_o+g\frac{\partial S}{\partial x}=U_s \qquad (27)$$

where S_s is the source term which generates the incoming wave while the U_s is the local scattering term which is employed for wave scattering by a vertical barrier. This approach speeds up the solution considerably since one does not need to resolve the wave period any longer. It is notable that Eqs. (26) and (27) allow the wave energy propagating with a group velocity differently from Madsen and Larsen (1987)'s wave model.

The source term S_s is given in terms of the internal wave condition S_I on the grid size of Δx:

$$S_s=2iC_gS_I\frac{1}{\Delta x} \qquad (28)$$

where S_I is the height function of internal waves given as

$$S_I = \frac{H_I}{2}\exp(ikx) \tag{29}$$

where H_I is the wave height.

The local wave scattering term, U_s, is given in terms of the local wave flux, U, on the grid size of Δx similarly as the source term (impermeable barrier):

$$U_s = -2\frac{\gamma}{\sqrt{1-\gamma^2}}CU\frac{1}{\Delta x} \tag{30}$$

where is the reflection coefficient. For dissipative waves(permeable barrier),

$$U_s = -2i\frac{\gamma}{1-\gamma}CU\frac{1}{\Delta x} \tag{31}$$

4.2 Numerical Modeling

The numerical formulation for the mild-slope equation given here is now given for the completeness of the treatment. The governing equations (26) and (27) are solved by the implicit finite difference schemes using the tridiagonal algorithm. Both flux variables U_1 and U_{N+1} at boundary sides are given in terms of S posed at the center of the grid according to the boundary conditions:

$$U_1 = \frac{1-\gamma}{1+\gamma}C_x F_l S_1, \quad U_{N+1} = \frac{1-\gamma}{1+\gamma}C_x F_r S_N \tag{36}$$

where

$$F_l = \frac{ik_x + 4/\Delta x}{ik_x - 4/\Delta x}, \quad F_r = \frac{ik_x + 4/\Delta x}{-ik_x + 4/\Delta x} \tag{37}$$

Fig. 6 shows the performance of wave reflecting for given reflection coefficients. For a vertical barrier posed at $x=0$, the spatial variations of reflective and transmissive waves are shown in Fig. 7 and the corresponding perspective views are shown in Fig. 8. Decomposition into reflection and transmission components was performed by Lee(1998)'s approach.

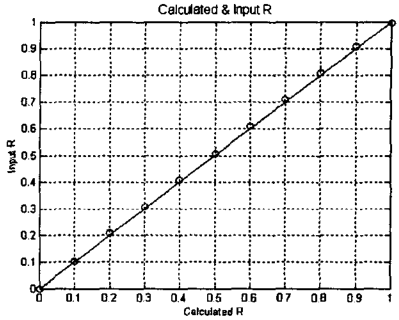
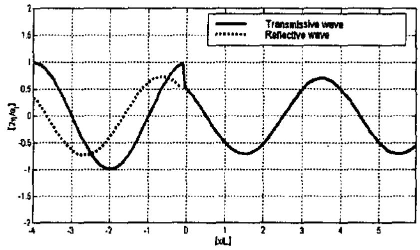


Fig. 6. Performance test for wave reflecting from a vertical barrier

(a) Wave profile



(b) Wave height

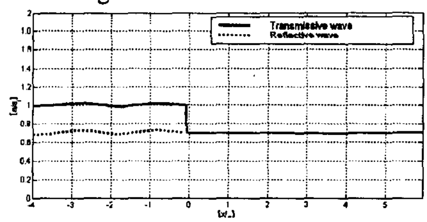
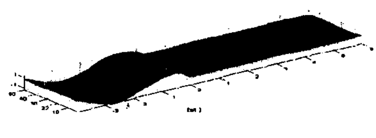


Fig. 7. Reflective and transmissive wave components against a vertical barrier.(x=0); (a) wave profile , (b) wave height

(a) Reflective component



(b) Transmissive component

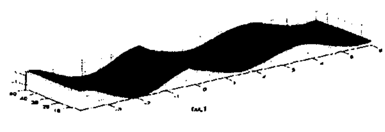


Fig. 8. Perspective views of reflective and transmissive waves. : (a) Reflective component, (b) Transmissive component

For the breakwater with double vertical permeable walls as shown in Fig. 9, the calculated reflection and transmission coefficients are compared with Hagiwara (1984)'s experimental data in Fig. 10. Although the calculated reflection coefficients are underestimated in general, they show better agreement with experiments than Hagiwara (1984)'s theoretical results (see Fig. 11). In a similar approach, Isaacson et al. (1999) obtained theoretical results closely fit with Hagiwara (1984)'s ones. Therefore, the present method provides possibility of practical approach although we can't say the better results always prove the more accurate method.

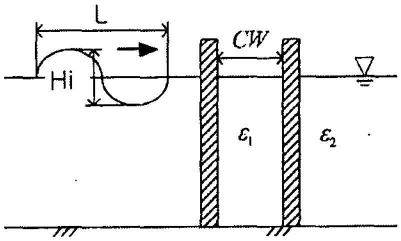


Fig. 9. Breakwater with double vertical permeable walls

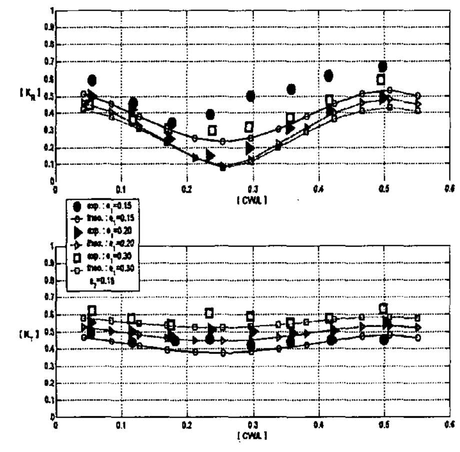


Fig. 10. Comparison with Hagiwara (1984)'s experimental data

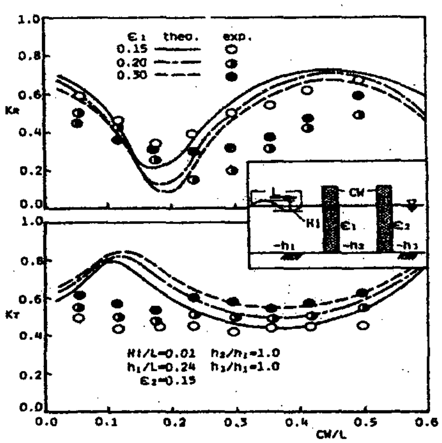


Fig. 11. Hagiwara (1984)'s theoretical results

5. CONCLUSION

The modified mild-slope equation has been derived as a governing equation for predicting waves in a harbour protected with a vertical impermeable or permeable barrier. For the regular waves, the modified mild-slope equations were reformulated extracting the harmonic time variation. This approach yields much higher computational efficiency as compared to conventional approaches where one does not need to resolve the wave period any longer. The major conclusions are listed below.

1) The representative two-dimensional problems involving the scattering of water waves by thin vertical barriers were examined and compared each other to determine the most effective routine in predicting the reflection coefficient. As a result, the eigenfunction expansion method by Abul-Azm (1993) was considered to yield the most accurate solutions.
2) In order to provide an extremely simple method, the traditional mild-slope equation has been modified by adding additional terms without inclusion of evanescent modes. Differently from the source term, the local scattering term was added in the other equation.
3) When applied to multiple permeable barriers, the present numerical results are in better agreement than Hagiwara (1984)'s and Isaacson et al. (1999)'s theoretical results when compared with Hagiwara (1984)'s experimental data.

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REFERENCES

Abul-Azm RG., 1993. Water diffraction through submerged breakwaters. *J. Waterway, Port, Coastal and Ocean Eng.* : 119(6), 587-605.

Berkhoff, J.C.W., 1972. Computation of combined refraction diffraction. *Proc. 13th Conf. Coastal Eng.*, pp.471-490.

Copeland, G.J.M., 1985. A practical alternative to the mild slope wave equation. *Costal Engrg.*, 9, 125-149.

Drimer et al., 1992. A simplified analytical model for a floating breakwater in water of finite depth. *Applied Ocean Res.*, 14, 33-41.

Hagiwara, K., 1984. Analysis of upright structure for wave dissipation using integral equation. *ICCE,* 2810-2426.

Isaacson, M., Baldwin, J., Premasiri, S., Yang, G., 1999. Wave interactions with double slotted bariers. *Applied Ocean Res.,* 21,81-91.

Jones, D.B., Lee, J-J and Raichlen, F., 1979. A transportable breakwater for near shore applications. *Proc. Specialty Conference Civil Engineering in Ocean IV,* 433-456.

Kriebel, D.L. and Bollmann, C.A., 1996. Wave Transmission past vertical wave barriers. *ICCE,* 2470-2483.

Lee, J.L., 1998. Decomposition of reflecting waves by hyperbolic model. *J. Korean Society of Coastal and Ocean Engneers,* 10(4), 197-203.

Liu, P.L.F. and Abbaspour, M., 1982. Wave scattering by a rigid thin barrier. *J. Waterway, Port, Coastal and Ocean Div.*, ASCE, 108, 479-91.

Losada, I, Losada, M., and Roldan, A., 1992. Propagation of oblique incident waves past rigid vertical thin barrier. *Applied Ocean Res.*, 14, 191-199.

Madsen, P.A., and Larsen, J., 1987. An efficient finite-difference approach to the mild-slope equation. *Coastal Engrg,* 11, 329-351.

Smith, R., and Sprinks, T., 1975. Scattering of surface waves by a conical island. *J. Fluid Mech.*, 72, 373-384.

Sollit, C.K., and Cross, R.H, 1972. Wave Transmission through permeable breakwater. *Proc. 13th Int. Conf. On Coastal Engr.,* 1827-1846.

Ursell, F., 1947. The effect of a fixed vertical barrier on surface waves in deep water. *Proc. Camb. Phil: Soc.* 43, 374. 25(8), 621-637.

Wiegel, R. L., 1960. Transmission of waves past a rigid vertical barrier. *J. Waterways and Harbours Div.*, ASCE WW1, 1-12.