

## 코깅 토크를 포함한 광역 속도 영역상의 BLDCM의 토크 리플 최소화를 위한 기준 프레임 접근기법

박한웅\* · 조성배\*\* · 원태현\*\*\* · 권순재\*\*\*\* · 함병운\* · 김철우\*\*\*\*\*

\* 해군사관학교 · \*\* 동의대학교 · \*\*\* 동의공업대학 · \*\*\*\* 부경대학교 · \*\*\*\*\* 부산대학교

### Reference Frame Approach for Torque Ripple Minimization of BLDCM over Wide Speed Range Including Cogging Torque

Han-Woong Park\* · Sung-Bae Cho\*\* · Tae-Hyun Won\*\*\* · Soon-Jae Kwon\*\*\*\* · Byung-Woon Ham\* · Cheul-U Kim\*\*\*\*\*  
\*Korea Naval Academy · \*\*DongEui Univ. · \*\*\*DongEui College · \*\*\*\*Pukyung Nat. Univ. · \*\*\*\*\*Pusan Nat. Univ.

**Abstract** - Torque ripple control of brushless DC motor has been the main issue of the servo drive systems in which the speed fluctuation, vibration and acoustic noise should be minimized. Most methods for suppressing the torque ripples require Fourier series analysis and either the iterative or least mean square minimization. In this paper, the novel approach to achieve the ripple-free torque control with maximum efficiency based on the  $d-q-0$  reference frame is presented. The proposed method optimize the reference phase current waveforms including even the case of 3 phase unbalanced condition, and the motor winding currents are controlled to follow up the optimized current waveforms by delta modulation technique. As a result, the proposed approach provides a simple and clear way to obtain the optimal motor excitation currents. The validity and practical applications of the proposed control scheme are verified through the simulations and experimental results.

## 1. Introduction

Brushless DC motors (BLDCMs), in which the most popular back EMF waveform is trapezoidal, are increasingly being used in high performance applications due to the simplicity in their control. In many of these applications, the production of ripple-free torque in the motors is of primary concern. There are three main sources of torque production in BLDCMs; cogging torque, reluctance torque, mutual torque. Cogging torque is created by the stator slots interacting with the rotor magnetic field and is independent of the stator current excitation. Reluctance torque is caused by the variation in the phase inductance with respect to the position. Mutual torque is created by the mutual coupling between the stator winding current and rotor magnetic field. Therefore, if the waveforms of the phase back EMF and phase current are perfectly matched to produce the required load torque and eliminate the first two torque component, torque ripple can be minimized.

A great deal of study has been devoted to identifying the sources, characteristics and minimization of torque ripple[1-3]. In particular, the interaction between the back EMF and the current excitation has been described and analyzed by a number of authors[1-3]. LeHuy, Perret and Feuillit[1] investigated that torque ripple can be minimized by appropriately selecting the current harmonics to eliminate both excitation and cogging torque ripple components. Hung and Ding[2] used the complex exponential decomposition to find a closed form solution for the current harmonics that eliminate the torque ripple and maximize the efficiency

simultaneously. Hanselman[3] extended these prior works to the case of a finite supply voltage and resulting finite  $di/dt$  capability. All these works have made several unnecessary assumptions such that all three phases have an identical back EMF waveforms offset  $2/3$ [rad] electrical angle with respect to one another and the back EMF and motor excitation current exhibit half-wave symmetry, etc. Among them, [2] and [3] easily extend to the more general case where those assumptions need not to be made. In practice, due to the several reasons like manufacturing imperfection, deterioration of permanent magnets or unbalanced stator windings etc, those assumptions can make an undesirable error. In addition, if the motor has four or more poles, the phase back EMF waveform from one cycle to another may be different because of the unbalanced magnetization of the magnets and/or manufacturing error.

This paper deals with the new torque control scheme of BLDCM at low speeds with maximum efficiency based on the  $d-q-0$  reference frame, including the cases when the 3 phase stator windings are unbalanced and the phase back EMF waveform from one cycle to another is different. The phase back EMF waveforms in natural  $a-b-c$  reference frame are transformed to the  $d-q-0$  reference frame. Only the quadrature component which contributes to the production of the mutual torque, then, is derived by equating the electrical power absorbed by the motor to the mechanical power that includes the reluctance and cogging torque components. Consequently, the optimum phase current waveforms can be obtained by transforming the  $d-q-0$  variables to  $a-b-c$  ones inversely. The motor winding currents are forced to track the optimal current waveforms by delta modulation technique. Simulation and experimental results are presented to prove the validity and practical availability of the proposed control scheme.

## 2. Proposed approach

Figure 1 shows the configuration of the BLDCM under test. To analyze and develop the proposed torque control scheme, the following simple assumptions are made in this paper.

- 1) The three phase stator windings are Y-connected.
- 2) The mutual torque produced by the motor is linearly proportional to the phase current.
- 3) The mutual inductance between phases is negligible.
- 4) DC source voltage is infinite and is capable of delivering infinite  $di/dt$ .

Given the above assumptions, Fig. 1 shows the measured phase back EMF waveforms of BLDCM under test. As

shown in the figure, three back EMFs are a little different in magnitude and shape from each other, so the corresponding phase currents should be different. In addition, if the motor has four or more poles,

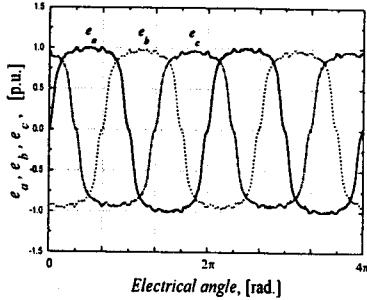


Fig. 1. The measured phase back EMF waveforms

the back EMF waveforms from one cycle to another may be different because of the dimension and magnetization unbalance of the magnets and/or manufacturing error. So the back EMF waveform of one cycle of the mechanical angle, which is equal to two cycles of the electrical angle for four pole motor, should be incorporated to control the instantaneous torque.

Therefore, a new torque ripple minimization approach with maximum efficiency based on the  $d-q-0$  reference frame is presented in this paper. In general, the stator windings of BLDCM are concentrated and the air-gap flux density by the rotor magnet is similar to square wave. Consequently, it can be argued that since the stator to rotor mutual inductance does not vary sinusoidally, the synchronously rotating  $d-q$  reference frame analysis is no longer valid except only when the harmonic components of field distribution and inductance variation are considered. However,  $d-q-0$  reference frame approach adopted in this paper will be used only to obtain the optimum current waveform for the ripple-free torque based on the minimum input power, not to perform a modeling and simulation of the motor itself. It can be considered as an another harmonic current injection method that is different from the previous Fourier analysis works. So the proposed method can be a proper choice. In addition, the zero sequence variables must be taken into account because of the unbalances among the phase windings and the discrepancy between the cycles of the phase back EMF. The proposed approach has the following features.

- 1) An alternative simple method is presented.
- 2) Phase commutation needs not to be considered.
- 3) Assumptions such as the shape, magnitude and half-wave symmetry of each phase back EMF need not to be made.

The optimum current waveforms for each phase by the proposed method can be obtained as follows. Firstly, the phase back EMF waveforms in natural  $a-b-c$  reference frame are transformed to the  $d-q-0$  reference frame.

$$\begin{bmatrix} e_d \\ e_q \\ e_0 \end{bmatrix} = C \cdot \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (1)$$

$$C = \frac{2}{3} \begin{bmatrix} \sin(\theta_e - \theta_0) & \sin(\theta_e - \theta_0 - \frac{2\pi}{3}) & \sin(\theta_e - \theta_0 + \frac{2\pi}{3}) \\ \cos(\theta_e - \theta_0) & \cos(\theta_e - \theta_0 - \frac{2\pi}{3}) & \cos(\theta_e - \theta_0 + \frac{2\pi}{3}) \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

where  $\theta_e = \omega_e t$ ,  $e$  is an electrical angular frequency and  $\theta$  is an angular displacement between the stator and rotor flux linkage and  $C$  is the transformation matrix of 3 phase to synchronously rotating  $d-q-0$  reference frame. In  $d-q-0$  reference frame, the mutual torque is derived by equating the electrical power absorbed by the motor to the mechanical power produced as follows.

$$\tau_L \frac{\omega_e}{P} = \frac{3}{2} (e_d i_d + e_q i_q + e_0 i_0) \quad (2)$$

where  $L$  is an ripple-free torque required by load, and  $P$  is the number of pole pairs and  $e/P$  is the motor mechanical speed. Considering that the magnetic field of the BLDCM is provided by the permanent magnet in the rotor and also that the sum of phase current must be zero at any instant, the current components  $i_d$  and  $i_0$  associated with field and zero sequence flux generation respectively must be zero. Therefore Eq.(3) can be rewritten by

$$\tau_L \frac{\omega_e}{P} = \frac{3}{2} e_q i_q \quad (3)$$

From the above equation,  $i_q$  component can be obtained as follows.

$$i_q = \frac{2}{3} \tau_L \frac{\omega_e}{P} \frac{1}{e_q} \quad (4)$$

Each optimum phase current waveform can be derived by transforming the  $d-q-0$  variables to the  $a-b-c$  ones inversely.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = C^{-1} \cdot \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \quad (5)$$

It should be emphasized that in order to minimize losses and maximize the torque/ampere, each phase current must be excited in phase with the corresponding phase back EMF, i.e.,  $\theta_0 = 0$  must be substituted to  $C$  matrix in Eq. (5). This will be discussed in the next section.

### 3. Numerical Results

The phase back EMF waveform is measured by the constant speed test for 400 [W], 4 pole, Y-connected, 100[V] BLDCM. Each back EMF waveform is normalized by the peak value of the phase  $a$  having the highest value among the phases. The optimization procedure using Eq. (1)~(5) is performed to generate the current profiles which produce the desired torque with minimum pulsation.

Fig. 2 shows the measured cogging torque component  $\tau_{cog}$  around the airgap (solid line). To eliminate the ripple torque caused by the cogging torque, an additional mutual torque component  $\tau_{m cog}$  are required as shown in the same figure (dotted line).

Fig. 3 shows the simulation results of the proposed approach using the measured back EMF waveforms without considering the cogging torque. Fig. 3(a) shows the back EMF components of the  $d-q-0$  reference frame obtained by using Eq. (1). The torque current  $i_q$  is obtained from  $e_q$  by Eq. (4) and  $i_d$  and  $i_0$  must be zero as shown in Fig. 3(b). The instantaneously optimized phase currents that are obtained from Eq. (5) are shown in Fig. 3(c). The optimum phase current considering the cogging torque can be obtained by simply adding the cogging torque component to the required load torque component in Eq. (4). As a result, the resultant torque  $\tau_{total}$  is produced as shown in figure 3(d) which also shows the waveforms of the phase torque  $a, b, c$

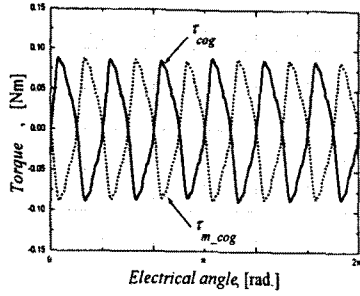
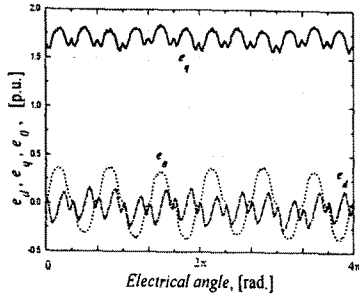
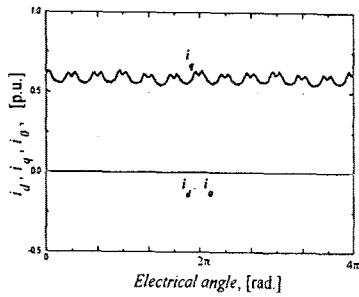


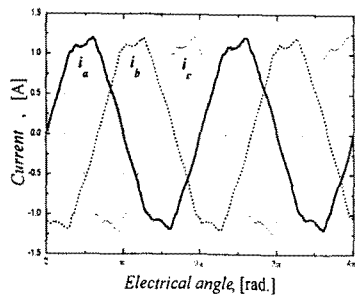
Fig. 2. Measured cogging torque  $\tau_{cog}$  and mutual torque  $\tau_{m\_cog}$  for compensating the cogging torque.



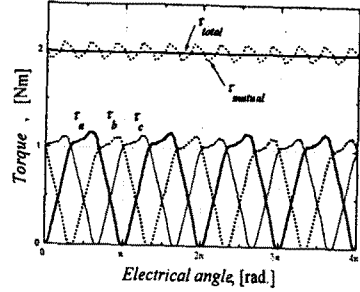
(a)  $e_d$ ,  $e_q$ ,  $e_\theta$  waveforms



(b) Desired  $i_d$ ,  $i_q$ ,  $i_\theta$  waveforms



(c) Desired  $i_a$ ,  $i_b$ ,  $i_c$  waveforms



(d) Waveforms of the optimized torque per phase  $a$ ,  $b$ ,  $c$ , and total mutual torque  $\tau_{mutual}$  and resultant developed torque  $\tau_{total}$  at the motor shaft

Fig. 3. Simulation results of the proposed control scheme of BLDCM with considering the cogging torque.

and the resultant mutual torque  $\tau_{mutual}$ . In this case, the phase torque profiles are also different each other as expected from the current and back EMF waveforms, but total torque exhibits no pulsation as shown in the figure. From these results, the proposed control scheme for the torque ripple elimination is very simple and clear.

#### 4. Experimental Results

The proposed method is implemented using the high performance digital signal processor (DSP) TMS320C40. The sampling time of the torque controller is 100  $\mu$ s, and the switching frequency is 5 kHz. The motor currents are compared and forced to track the reference currents within the predetermined bandwidth by the delta modulation technique. The peripheral devices with high speed switching are selected to utilize the DSP effectively, the controller and its interfaces are insulated to prevent the electric shock and noise. The I/O controllers are implemented by erasable programmable logic device. The measured motor current, speed and rotor position are measured and transferred to the DSP controller from the insulated sensors via the 12 bit A/D converter (MAX 120 CNG) at every sampling period.

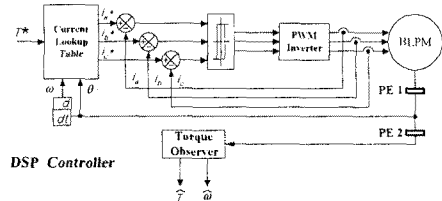
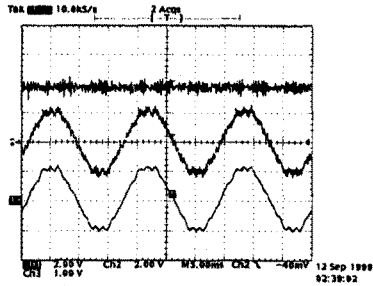


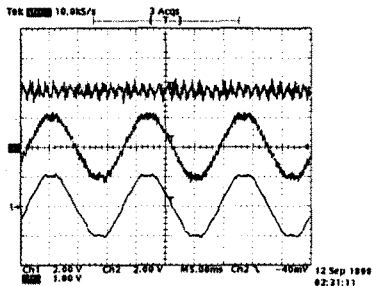
Fig. 4. Block diagram of the speed control scheme of the proposed approach

A speed control scheme of the proposed approach is shown in Fig. 4. The back EMF data according to the rotor position are measured and set up in the look-up table. The optimized reference current waveforms are obtained from the look-up table using the position and speed information from the shaft encoder PE1.



current : 3 [A/div.]  
torque : 0.5 [Nm/div.]

(a) When the cogging torque is considered



current : 3 [A/div.]  
torque : 0.5 [Nm/div.]

(b) When the cogging torque is not considered

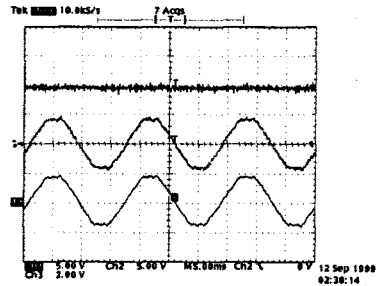
Fig. 5. The experimental results of the proposed control scheme of BLDCM when the motor speed is 1,800 rpm, the load torque is 1 Nm

We show the experimental results to confirm the validity of the proposed method. Experiments are performed at the speed of 1,800 rpm and the load torque of 1 and 2 N.m. Fig. 5(a) and (b) show the developed torque measured at the observer terminals together with the reference and motor current waveforms when the cogging torque is considered and not considered, respectively. It is clear from the figures that both the motor currents are tracking the reference currents satisfactorily and they can deliver the load torque of 1 Nm. The torque ripple caused by the high frequency switching is included in both two cases. In particular, the effect of the cogging torque on the torque ripple can be shown.

Fig. 6(a) and (b) show the experimental results when the load torque is increased to 2 Nm with the speed unchanged for two cases. It is also clear from the figures that both torque waveforms have a same tendency to the former case and due to the increased load torque the ratio of the cogging torque to the load torque is relatively reduced.

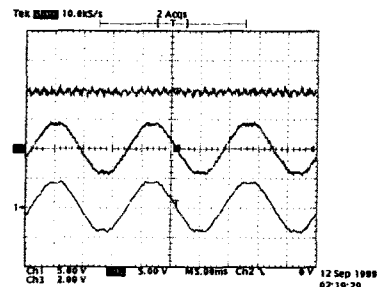
These instantaneous torque profiles are calculated using measured phase currents and back EMFs, mechanical speed and rotor position in the DSP, and are measured from the DSP terminals. The obtained torque profiles are not a true shaft torque, because the other torque components caused by the noise, a mechanical damping and so on are not included. However, the effects of these torque components can be neglected. The torque profiles are almost in good agreement with that of Fig. 3(d) and only the ripple component by the current deviation from the reference is added to average torque. However, its magnitude is also insignificant compared with the total torque and can be reduced by setting the

current bandwidth to be confined to a small value.



current : 7.5 [A/div.]  
torque : 1 [Nm/div.]

(a) When the cogging torque is considered



current : 7.5 [A/div.]  
torque : 1 [Nm/div.]

(b) When the cogging torque is not considered

Fig. 6. The experimental results of the proposed control scheme of BLDCM when the motor speed is 1800 rpm, the load torque is 2 Nm

## 5. Conclusion

In this paper, the novel optimal current excitation scheme of BLDCM producing loss-minimized ripple-free torque based on the  $d-q-0$  reference frame is presented. The optimized phase current waveforms that are obtained by the proposed method can be a reference values and the motor winding currents are forced to track it by delta modulation technique. The proposed control scheme for the torque ripple elimination is proved to be very simple and clear. The validity of the proposed control scheme is proved by the simulation and experimental results.

## References

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